SLOVAK UNIVERSITY OF TECHNOLOGY IN BRATISLAVA
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OFDM and CP as errror control codes

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Contents

- OFDM overview
  - Basic principles
  - Multipath channel
- Forms of Redundancy in OFDM
- Decoding Cyclic Prefix
  - Extracting prefix information
  - Modified OFDM SDR Receiver

OFDM adoption

- OFDM modulation in:
- WAN
  - 3GPP LTE DL
- MAN
  - WiMax IEEE 802.16e
  - ADSL
- LAN
  - WiFi IEEE 802.11g, n

OFDM – Basic Principles

- FDM
  - Multicarrier
- Orthogonality
  - Spectral efectivity
- DFT
  - Reduced Implementation complexity
- DSP
  - Flexibility of software processing

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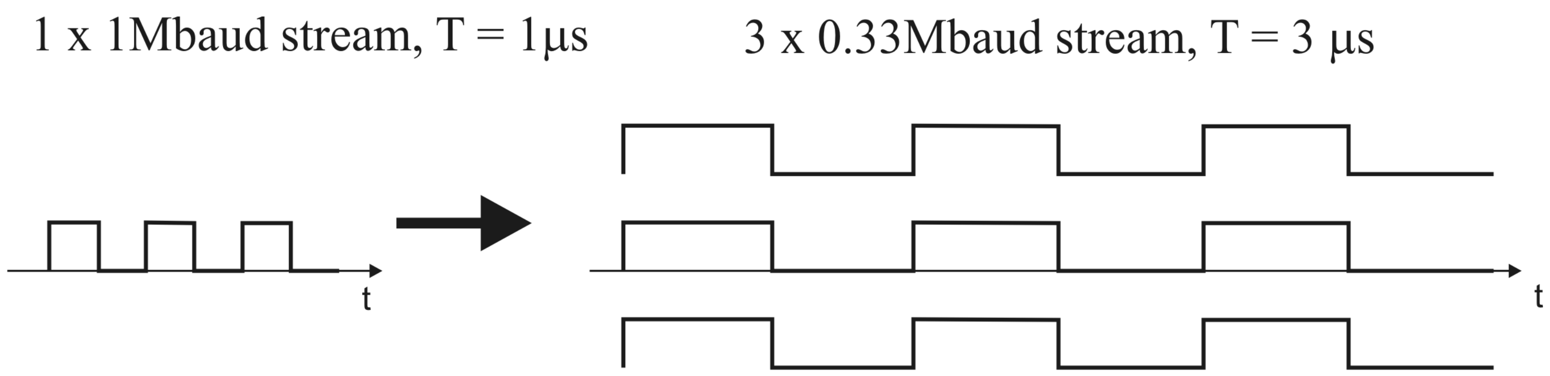
Multiplexing

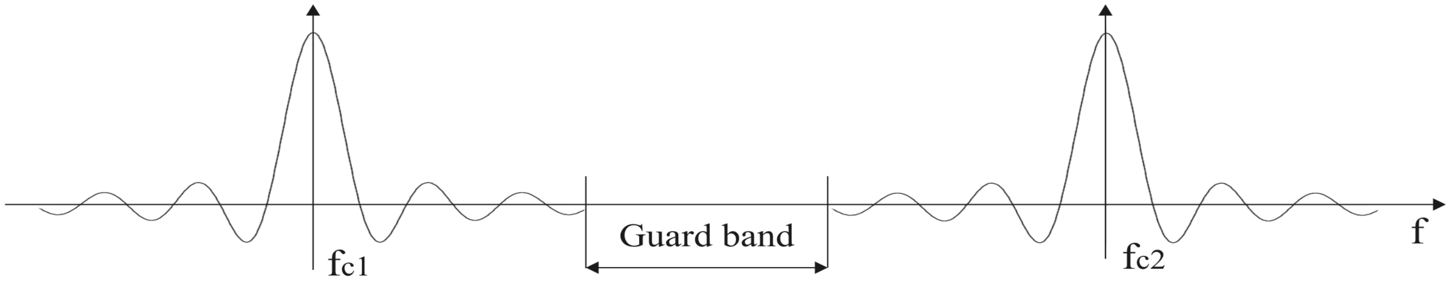
- Demultiplexing
  - Fast data stream divided to many slow data streams (up to 2048)
  - Resistance to Inter Symbol Interference (ISI)



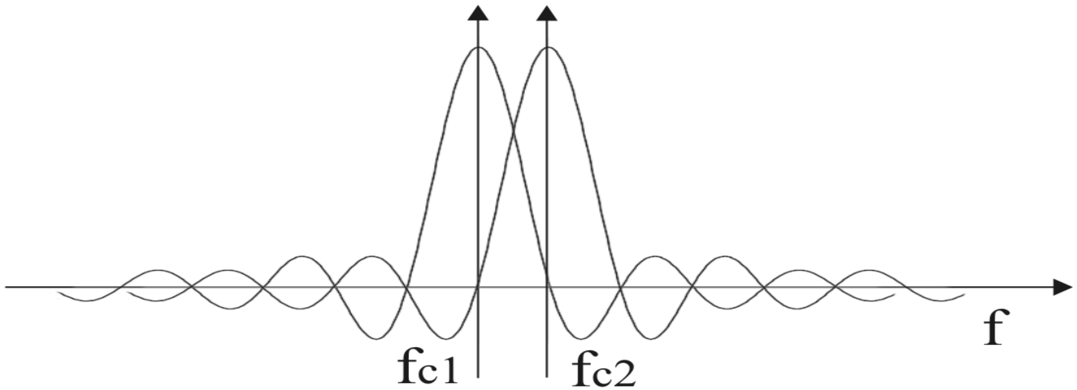
- Fixed ISI interval $\tau = 0.5\ \mu s$
  - 50% of symbol time in fast stream
  - 16 % of symbol time on slow stream

Orthogonal FDM

- FDM wastes frequency band

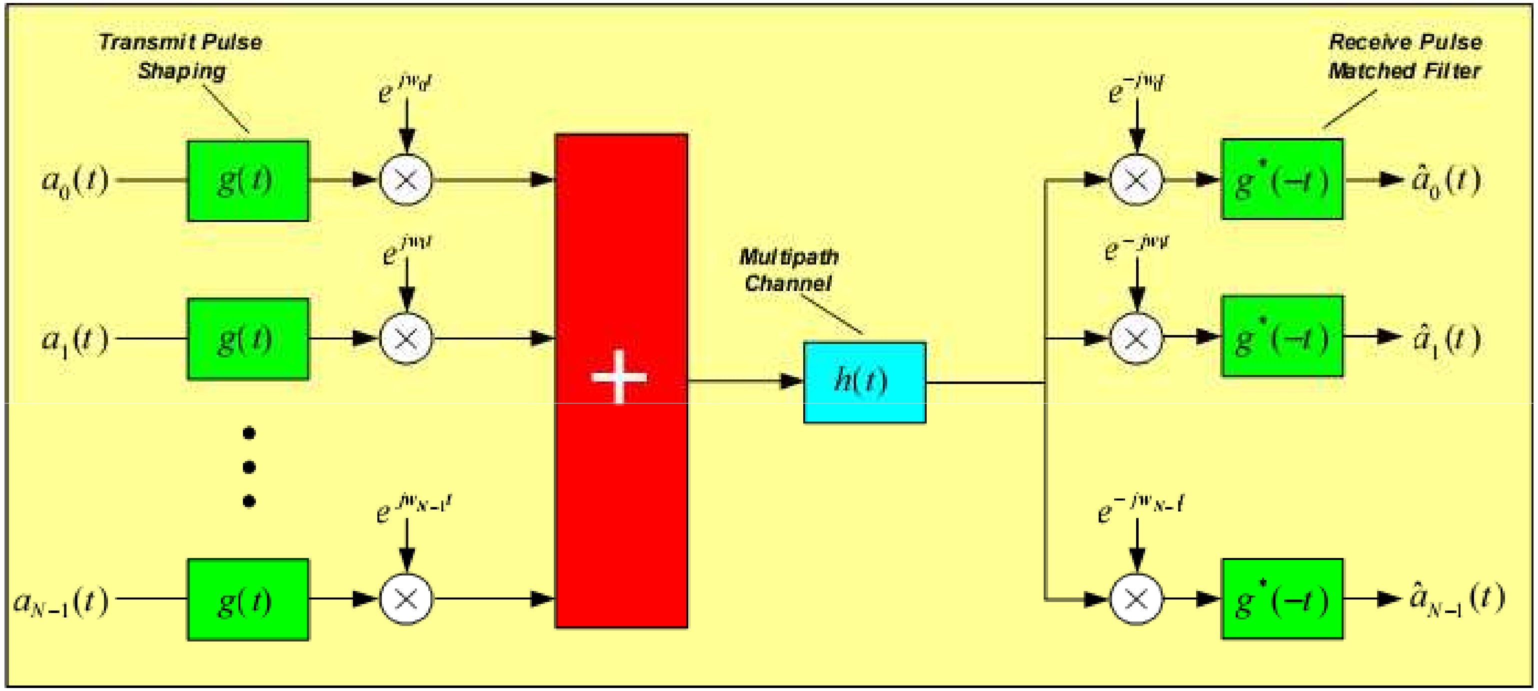


- OFDM: orthogonal spacing of carriers – high spectral effectivity



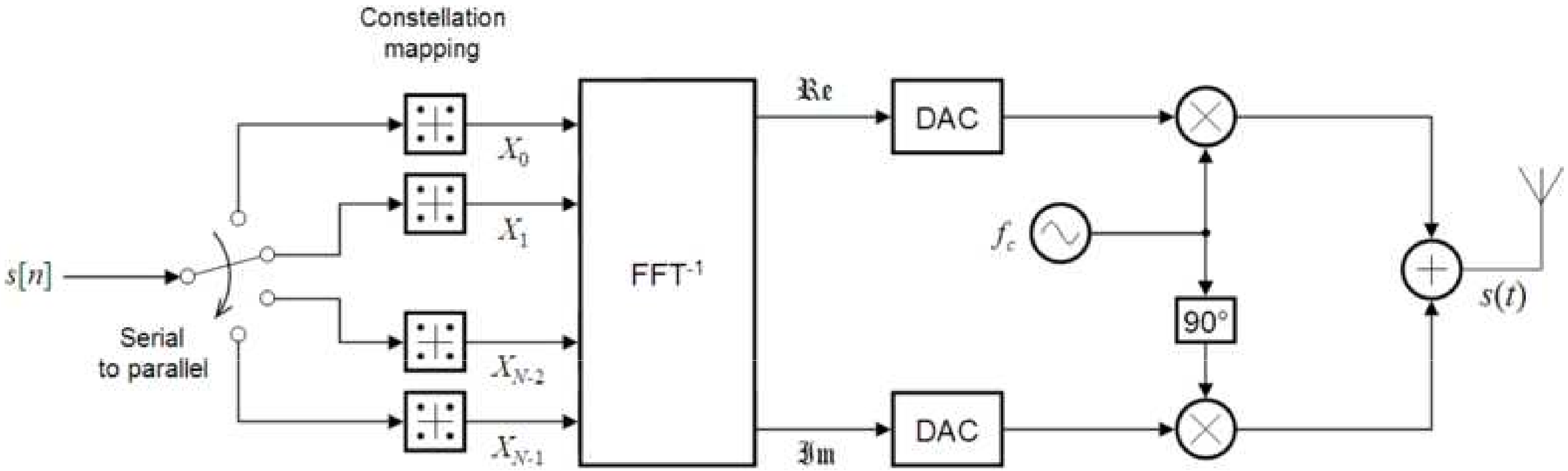
- Only discrete frequencies can be utilized ► use of DFT

Basic OFDM system architecture



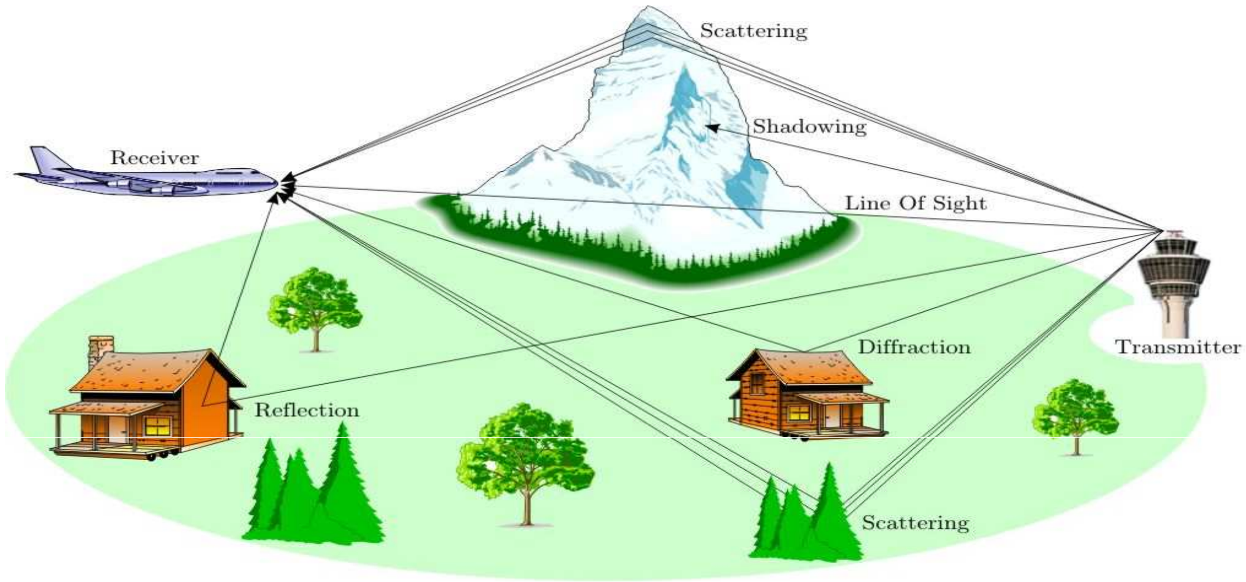
- Carriers are equally spaced by Δf = 1/T
- Amplitude and phase carries information

More Practical OFDM transmitter



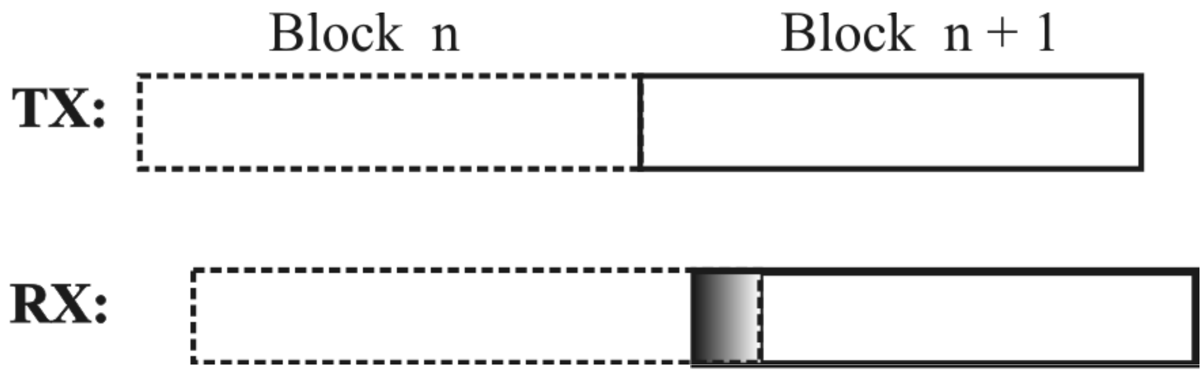
- Oscillator Bank replaced by IDFT
- OFDM symbol is a block of samples

Multipath propagation



- RX observes sum of delayed copies of transmitted symbol
- Channel impulse response $h(t)$
- Transmission modeled by convolution $r(t) = h(t) * s(t)$

Inter Symbol & Inter Block Interference



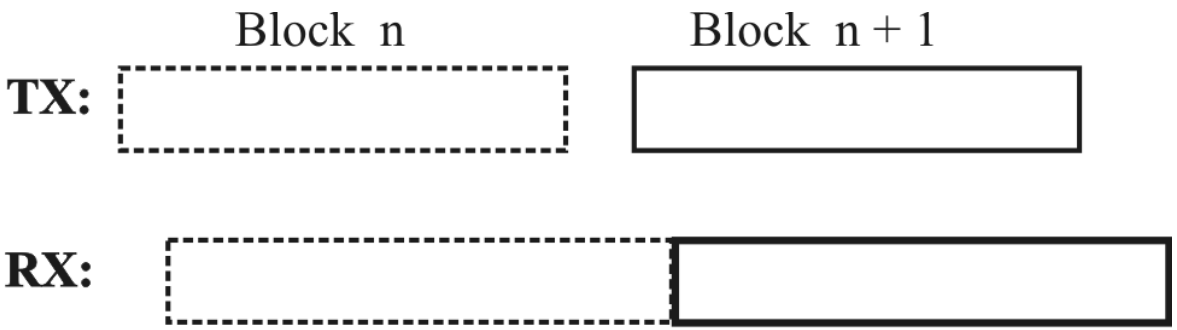
- Channel convolution prolongs transmitted blocks
- Blocks are transmitted in serial way
- In RX
  - blocks overlap – IBI
  - Samples inside one block interfere - ISI

OFDM ISI / IBI tolerance

- GSM max. cell radius 35 km means max. delay 233.3 μs
- Urban open space max. delay 1760 ns
- IEEE 802.16e OFDM parameters:
  - Usefull symbol time 91.4 μs
  - Number of samples per symbol 1024 (f_s = 11.2 MHz)
  - Blocks can overlap by 19 to 2621 samples
    - approx. 1.85 up to theoretically 255 %
- Additional tolerance necessary ► prefix

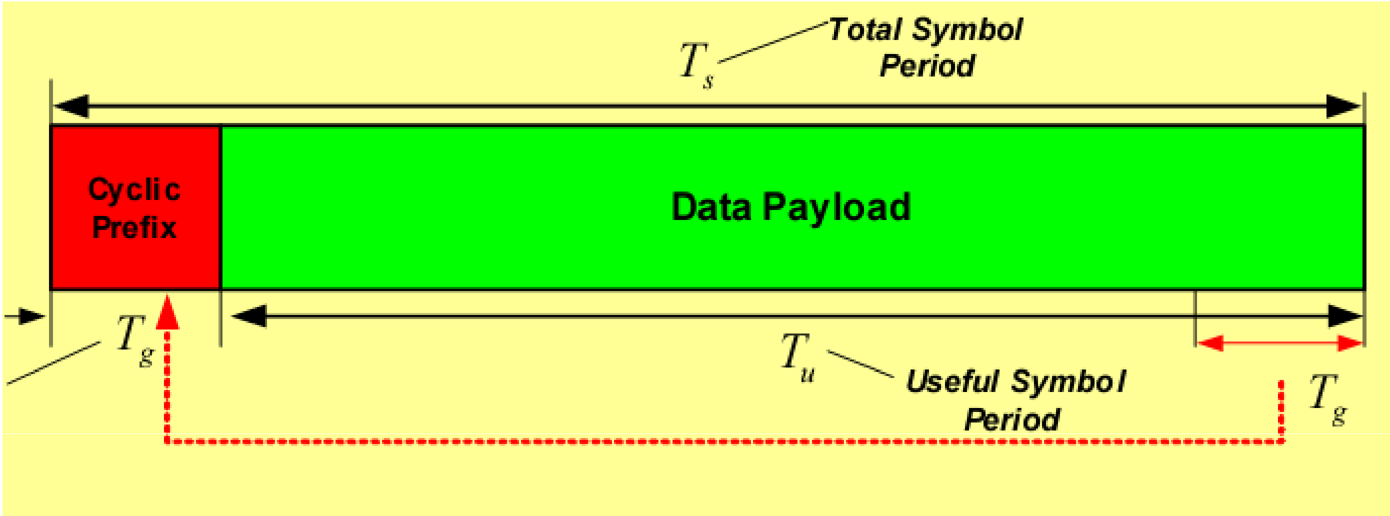
Improving ISI/IBI tolerance

- Use of guard interval
  - Zero prefix



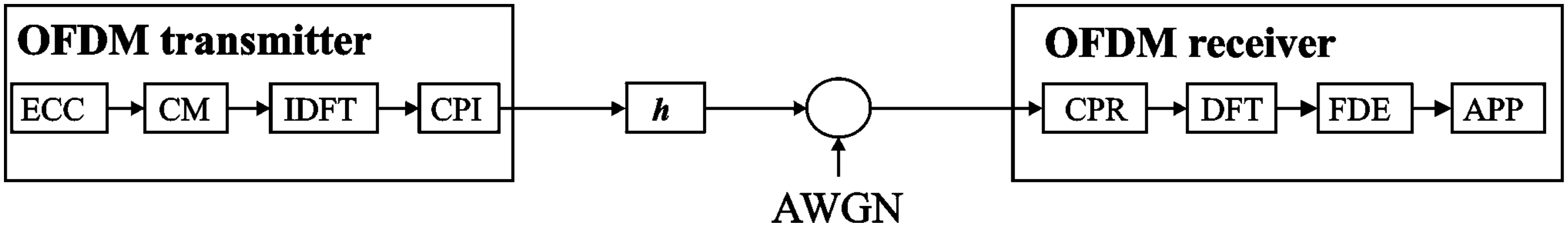
- In real OFDM systems
  - Cyclic prefix is used

Cyclic prefix in OFDM



- Mobile WiMax (IEEE 802.16e):
  - T_g = 1/4, 1/8, 1/16, 1/32 of T_u

Practical OFDM system



- ECC – Error Control Code
- CM – Constellation Mapping
- CPI – Cyclic Prefix Insertion
- CPR – Cyclic Prefix Removal
- FDE – Frequency Domain Equalization
- APP – A-posteriory Probability Decoder

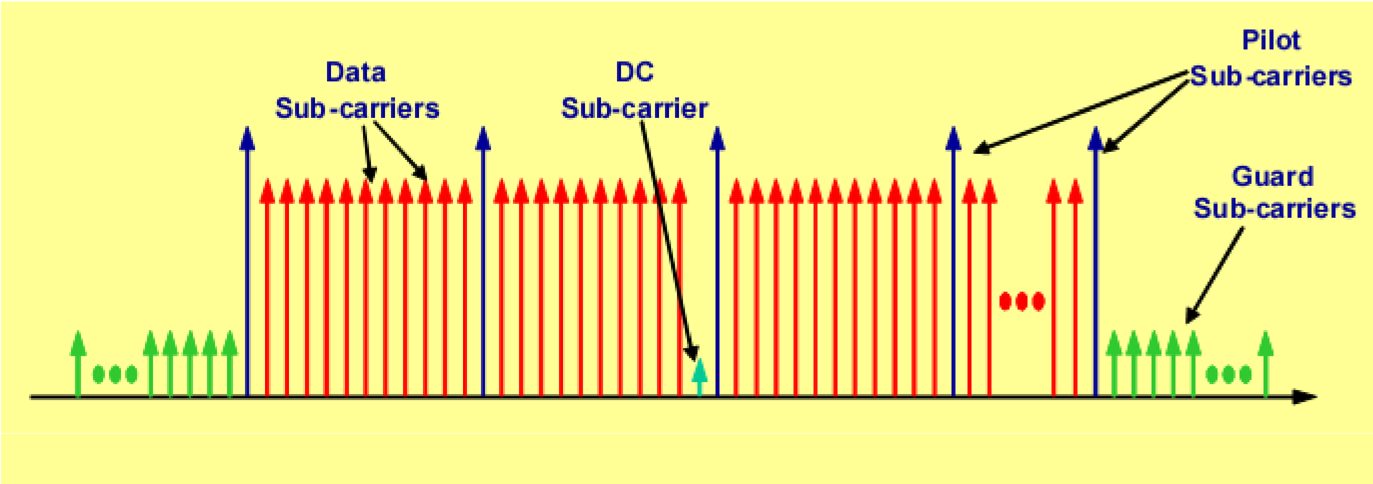
Redundancies in OFDM

- Cyclic prefix
  - Designed for easy FDE
- Pilots & guard subcarriers
  - used for Channel Estimation (CE)
- Upper layer headers
  - IP & TCP headers

PDU redundancies

- Parts of Headers of higher layers NWK+ protocols are the same during one session
- If known, this can be utilised in the lower PHY.
- This information is randomly distributed inside the OFDM symbol
- Not available if header compression is used

Redundancy in Pilot & Guard carriers



- IEEE 802.16e allocation for a 1024 subcarrier channel:
- sub-carriers grouped to clusters 48 data carriers + 8 pilots
  - 120 pilots DL, 280 UL
- Many zero guard sub-carriers
  - 184 null sub-carries in both DL & UL

Redundancy in Null Carriers

- Idea:
  - FARKAŠ, P. : OFDM is an Error Control Code. Journal of Electrical Eng. 54, No. 11-12, (2003)
  - OFDM with consecutive suppressed carriers can be understood as an Reed-Solomon code
  - RS code not over GF(2^r) but over Complex numbers
- Practical decoder:
  - Dlhan, S. Farkas, P. : IMPULSIVE NOISE CANCELLATION IN SYSTEMS WITH OFDM MODULATION. Journal of Electrical Eng. 59, No. 6, (2008)

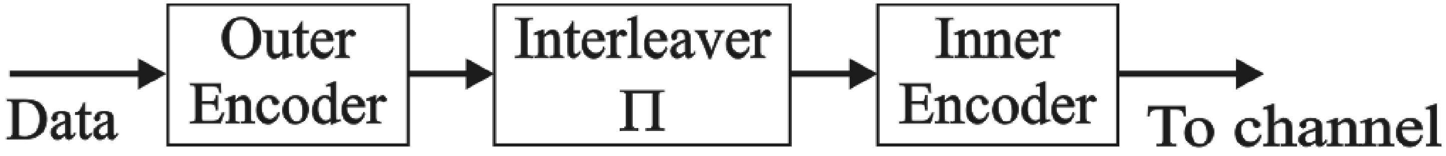
Redundancy in cyclic prefix

- Mobile WiMaxT_g = 1/4, 1/8, 1/16, 1/32 of T_u
- Cyclic prefix insertion is partial repetition code
  - High code rate R = 4/5, 8/9, 16/17, 32/33
  - Very weak code
- Diversity: Max Ratio Combining
- Another possibility

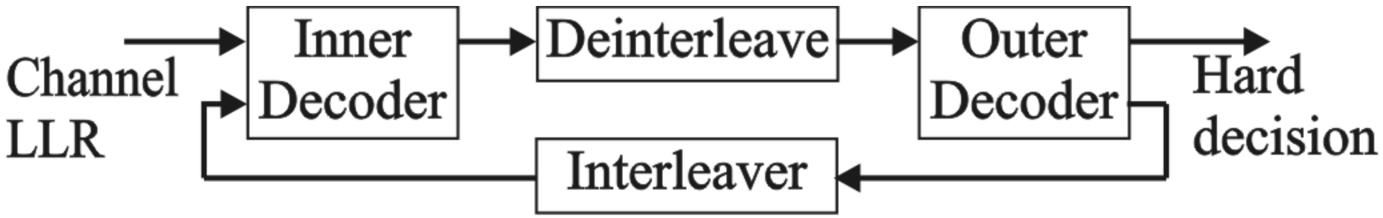
Concatenation of codes

- Hagenauer, J., Offer, E., Papke, L. Iterative decoding of Binary Block and Convolutional Codes. In *IEEE Transactions on Information Theory*, Vol. 42, No. 2, Mar. 1996, pp. 429-445
- Turbo codes
- **Serial concatenation of codes:**

Encoder:



Iterative SISO decoder :



COFDM transmitter with CP is a serially concatenated ECC encoder

- Strong ECC is present defined by standard
- IDFT can be viewed as Interleaver
- The insertion of cyclic prefix is a partial repetition code

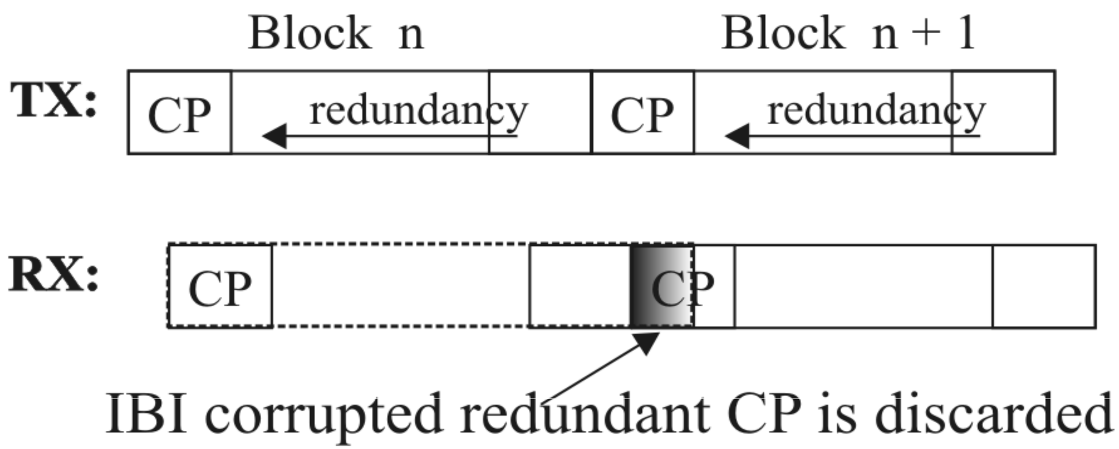
- OFDM Receiver can be extended to implement iterative decoding
- Partial SISO decoders must exchange extrinsic information

Problems

- Each Symbol in time domain is a linear combination of many symbols (IDFT)
- LLR metric necessary for exchanging extrinsic information - No LLR defined for time-domain repetition code decoding
- IBI corrupts the CP and must be eliminated

- Solution – return to current FDE algorithm

IBI in a prefixed OFDM system



- Blocks with CP are corrupted by IBI
- If delay spread < CP size => all OK
- CP designed for worst case
- Why do we use CP ?

Frequency-Domain Equalisation

- Channel convolution is *linear* convolution
- DFT theory:
  - Works wih periodic discrete-time signals
  - Transfers *circular* convolution in time-domain to *simple per-component multiplication* in frequency-domain
- Cyclic Prefix insertion in TX and removal in RX ensures that the linear channel convolution appears as circular convolution.

Linear Convolution Matrix

$\mathbf{H_c}$ =

1	0	0	0	0	0	0	0	0	0
0.9	**1**	0	0	0	0	0	0	0	0
0.4	**0.9**	**1**	0	0	0	0	0	0	0
0	**0.4**	**0.9**	**1**	0	0	0	0	0	0
0	0	**0.4**	**0.9**	**1**	0	0	0	0	0
0	0	0	**0.4**	**0.9**	**1**	0	0	0	0
0	0	0	0	**0.4**	**0.9**	**1**	0	0	0
0	0	0	0	0	**0.4**	**0.9**	**1**	0	0
0	0	0	0	0	0	**0.4**	**0.9**	**1**	0
0	0	0	0	0	0	0	**0.4**	**0.9**	**1**
0	0	0	0	0	0	0	0	**0.4**	**0.9**
0	0	0	0	0	0	0	0	0	**0.4**

size($\mathbf{H_c}$) = 12 rows, 10 columns

- $\boldsymbol{h}(n)$ = {1, 0.9, 0.4}
- **t** – input vector (transmitted block)
- **r** – output vector (received block)
- $\mathbf{H_c}$ – convolution matrix
- Linear convolution:

$$r_n = \sum_{m=1}^{\nu} h_m \times t_{n-m+1}$$

- can be modelled by matrix multiplication:

$$\mathbf{r} = \mathbf{H_c} \times \mathbf{t} \qquad (1)$$

Size of $\mathbf{H_c}$ dictated by **t** and $\boldsymbol{h}(n)$

Cyclic Convolution Matrix

$\mathbf{H_{circ}}$ =

1	0	0	0	0	0	0	0	**0.4**	**0.9**
0.9	**1**	0	0	0	0	0	0	0	**0.4**
0.4	**0.9**	**1**	0	0	0	0	0	0	0
0	**0.4**	**0.9**	**1**	0	0	0	0	0	0
0	0	**0.4**	**0.9**	**1**	0	0	0	0	0
0	0	0	**0.4**	**0.9**	**1**	0	0	0	0
0	0	0	0	**0.4**	**0.9**	**1**	0	0	0
0	0	0	0	0	**0.4**	**0.9**	**1**	0	0
0	0	0	0	0	0	**0.4**	**0.9**	**1**	0
0	0	0	0	0	0	0	**0.4**	**0.9**	**1**

- Cyclic convolution:

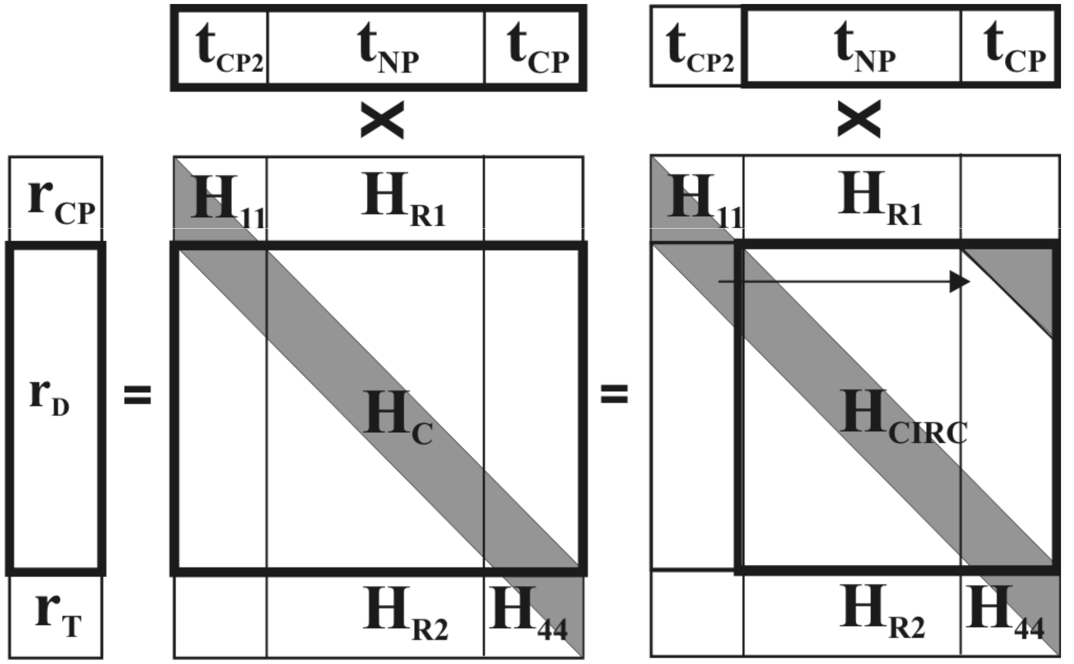
$$r_n = \sum_{m=1}^{\nu} h_m \times s_{[(n-m) \bmod N]+1}$$

- can be modelled by matrix multiplication:

  $\mathbf{r} = \mathbf{H_{circ}} \times \mathbf{t}$ (2)
- Vectors **r** & **t** have the same size
- Matrix $\mathbf{H_{circ}}$ has *circulant* property

Linear to cyclic convolution

- $\mathbf{t}_D = [\mathbf{t}_{NP} \| \mathbf{t}_{CP}]$ - original data withouth the cyclic prefix
- $\mathbf{r} = [\mathbf{r}_{CP} \| \mathbf{r}_D \| \mathbf{r}_T]$ - received vector
- $\mathbf{r}_D$ - subblock of **r**, selected for further processing



$$\boldsymbol{t}_{CP} = \boldsymbol{t}_{CP2} \qquad (3)$$

$$\mathbf{r}_D = \mathbf{H}_\mathbf{C} \times \mathbf{t} = \mathbf{H}_\mathbf{CIRC} \times \mathbf{t}_D \qquad (4)$$

Matrix models of multipath channel

- OFDM transmission can be modelled by matrix multiplication:

$$\mathbf{r}_D = \mathbf{H}_\mathbf{C} \times \mathbf{t} = \mathbf{H}_\mathbf{CIRC} \times \mathbf{t}_D \qquad (5)$$

- A circulant matrix can be diagonalized [Toepl]

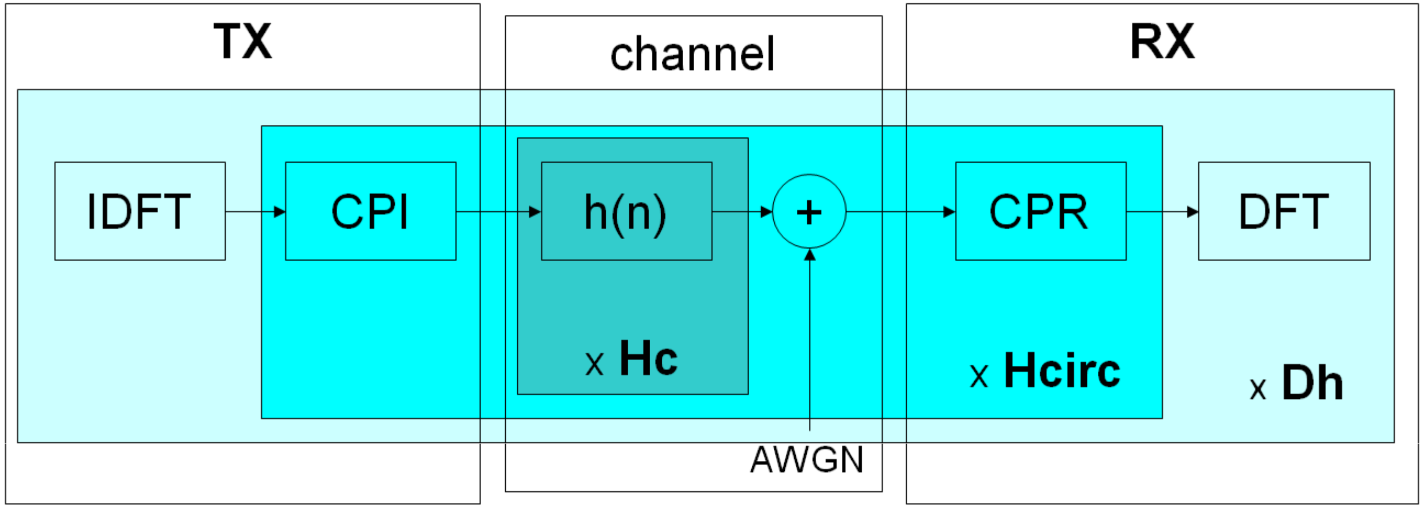
$$\mathbf{D}_\mathbf{h} = \mathbf{F} \times \mathbf{H}_\mathbf{circ} \times \mathbf{F}^{-1} \qquad (6)$$

- **F** - Fourier transform matrix
- $\mathbf{F}^{-1}$ - inverse Fourier transform matrix
- $\mathbf{D}_\mathrm{h}$ - diagonal matrix:

$$\mathrm{diag}(\mathbf{D}_\mathrm{h}) = \boldsymbol{H}(k) \qquad (7)$$

- ***H*(*k*)** is N-point channel frequency response

OFDM transmission matrix models



- Time domain models:

$$\mathbf{r}_D = \mathbf{H}_{\mathbf{C}} \times \mathbf{t} = \mathbf{H}_{\mathbf{CIRC}} \times \mathbf{t}_D \qquad (8)$$

- Frequency domain model:

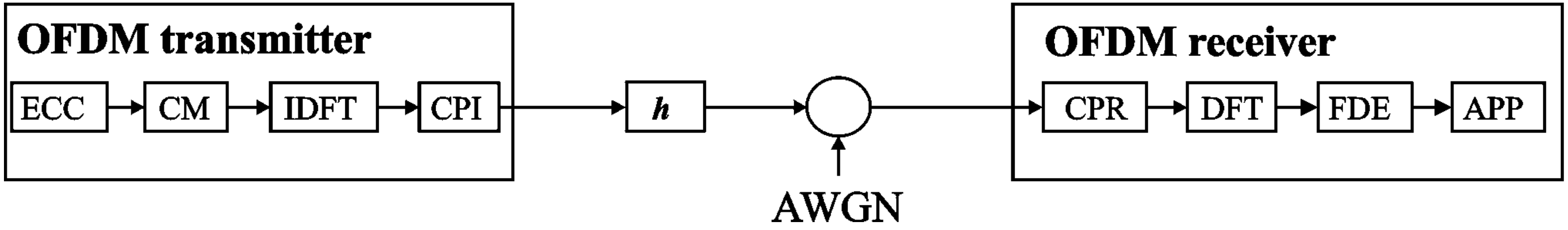
$$\mathbf{R}_D = \mathbf{D}_{\mathrm{h}} \times \mathbf{T}_D = \boldsymbol{H}_{(k)} \times \boldsymbol{T}_{(k)} = \boldsymbol{R}_{(k)} \qquad (9)$$

Simple **F**requency **D**omain **E**qualization (FDE)

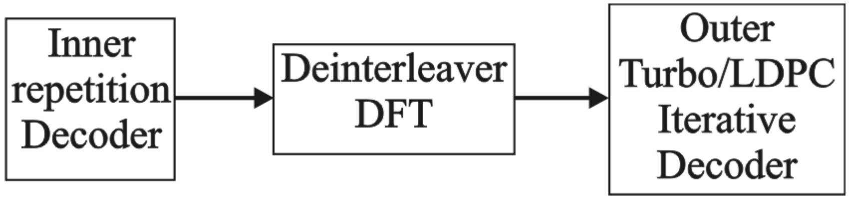
- In time domain models - deconvolution
  - each RX sample is a linear combination of several TX samples
- In frequency domain model
  - Each RX sample depends on one TX sample
  - If CSI (***H****(k))* is know in RX, equalization is multiplication:

$$\hat{\boldsymbol{T}}_{(k)} = \left(\boldsymbol{H}_{(k)}\right)^{-1} \times \boldsymbol{R}_{(k)} \qquad (10)$$

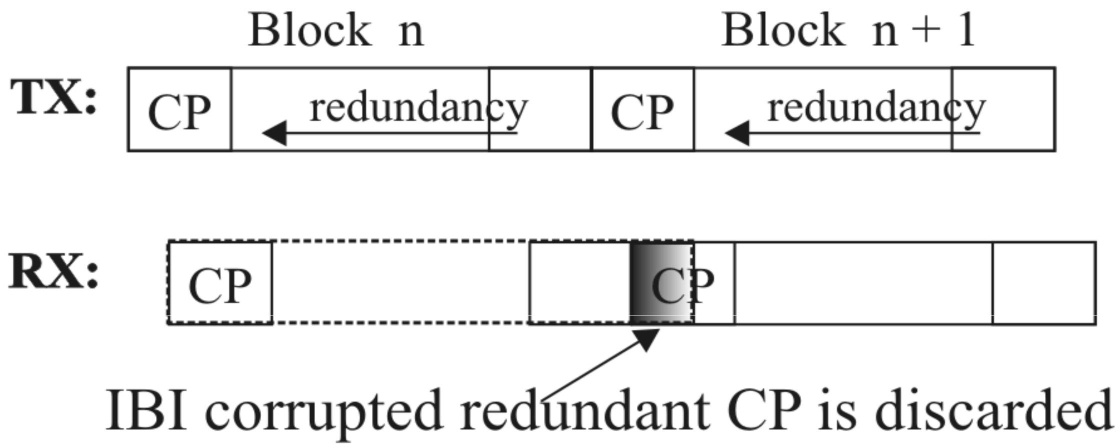
Exploiting of CP redundancy in a new way



- Cyclic Prefix
  - is present for purpose of FDE
  - Contains redundancy – partial repetition code
- Modification of receiver – iterative decoder
  - Exploiting CP information
  - More simple modification – just non-iterative setup:

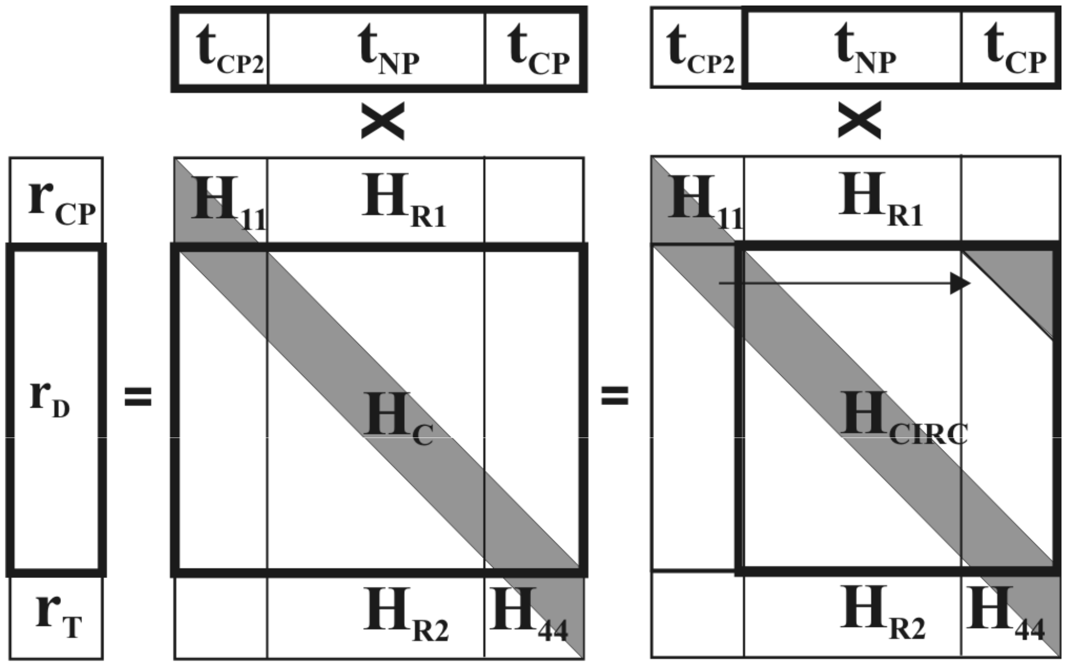


IBI must be eliminated to extract CP data



- Samples of CP are corrupted by IBI

Matrix models revisited



- Currently $\mathbf{r}_{\mathrm{CP}}$ & $\mathbf{r}_{\mathrm{T}}$ are discarded in RX because of IBI

$$\mathbf{r}_O = \mathbf{r}_{CP(n)} + \mathbf{r}_{T(n-1)} = \mathbf{H}_{\mathbf{11}(n)} \times \mathbf{t}_{CP2(n)} + \mathbf{H}_{\mathbf{44}(n-1)} \times \mathbf{t}_{CP(n-1)} \qquad (11)$$

Recovering the redundant samples

- Assuming CSI (**H** matrix) is known perfectly:

$$\mathbf{r}_O = \mathbf{r}_{CP(n)} + \mathbf{r}_{T(n-1)} = \mathbf{H}_{\mathbf{11}(n)} \times \mathbf{t}_{CP2(n)} + \mathbf{H}_{\mathbf{44}(n-1)} \times \mathbf{t}_{CP(n-1)} \tag{12}$$

- It is possible to apply subtractive correction:

$$\mathbf{r}_{cor1(n-1)} = \mathbf{H}_{\mathbf{44}(n-1)} \times \mathbf{t}_{CP(n-1)} \tag{13}$$

- In theory the transmitted samples can be recovered:

$$\mathbf{t}_{CP2(n)} = (\mathbf{H}_{\mathbf{11}}{}^{-1}) \times (\mathbf{r}_O - \mathbf{r}_{cor1(n-1)}) \tag{14}$$

- This will do no good:
  - For APP decoding, the LLR of samples in frequency domain are necessary!