

# Fourierov rad

$$x(\lambda) = \frac{1}{2}a_0 + a_1 \cos \lambda + a_2 \cos 2\lambda + a_3 \cos 3\lambda + \dots + b_1 \sin \lambda + b_2 \sin 2\lambda + b_3 \sin 3\lambda + \dots \quad (\text{A.1})$$

exponenciálny tvar:

$$x(t) = \frac{a_0}{2} + \frac{1}{2} \sum_{n=1}^{\infty} [(a_n - jb_n)e^{j2\pi n f_0 t} + (a_n + jb_n)e^{-j2\pi n f_0 t}] \quad (\text{A.11})$$

komplexné koeficienty:

$$c_n = \begin{cases} \frac{1}{2}(a_n - jb_n) & n > 0 \\ \frac{a_0}{2} & n = 0 \\ \frac{1}{2}(a_n + jb_n) & n < 0 \end{cases} \quad (\text{A.12})$$

$$c_n = |c_n| e^{j\theta_n} \quad (\text{A.17})$$

$$c_{-n} = |c_n| e^{-j\theta_n} \quad (\text{A.18})$$

potom:

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_0 t} \quad (\text{A.13})$$

$$c_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{-j2\pi n f_0 t} dt \quad (\text{A.14})$$

nakolko:

$$\frac{1}{T_0} \int_{-T_0/2}^{T_0/2} e^{j(n-m)2\pi f_0 t} dt = \delta_{nm} = \begin{cases} 1 & n = m \\ 0 & n \neq m \end{cases} \quad (\text{A.15})$$

$$\frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{-j2\pi n f_0 t} dt = \sum_{m=-\infty}^{\infty} c_m \delta_{nm} = c_m \quad (\text{A.16})$$

# Fourierova transformácia

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df$$

$x(t)$	$X(f)$
1. $\delta(t)$	1
2. 1	$\delta(f)$
3. $\cos 2\pi f_0 t$	$\frac{1}{2}[\delta(f - f_0) + \delta(f + f_0)]$
4. $\sin 2\pi f_0 t$	$\frac{1}{2j}[\delta(f + f_0) - \delta(f - f_0)]$
5. $\delta(t - t_0)$	$\exp(-j2\pi f t_0)$
6. $\exp(j2\pi f_0 t)$	$\delta(f - f_0)$
7. $\exp(-at), a > 0$	$\frac{2a}{a^2 + (2\pi f)^2}$
8. $\exp\left[-\pi\left(\frac{t}{T}\right)^2\right]$	$T \exp[-\pi(fT)^2]$
9. $u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
10. $\exp(-at)u(t), a > 0$	$\frac{1}{a + j2\pi f}$
11. $t \exp(-at)u(t), a > 0$	$\frac{1}{(a + j2\pi f)^2}$
12. $\text{rect}\left(\frac{t}{T}\right)$	$T \text{sinc } fT$
13. $\cos 2\pi f_0 t \left[\text{rect}\left(\frac{t}{T}\right)\right]$	$T \text{sinc}(f - f_0)T$
14. $W \text{sinc } Wt$	$\text{rect}\left(\frac{f}{W}\right)$
15. $\begin{cases} 1 - \frac{ t }{T} &  t  \leq T \\ 0 &  t  > T \end{cases}$	$T \text{sinc}^2 fT$
16. $\sum_{m=-\infty}^{\infty} \delta(t - mT_0)$	$\frac{1}{T_0} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$
$x(at)$	$\frac{1}{ a } X\left(\frac{f}{a}\right)$
$x(t - t_0)$	$X(f) \exp(-j2\pi f t_0)$
$x(t) \exp(j2\pi f_0 t)$	$X(f - f_0)$
$\frac{d^n x}{dt^n}$	$(j2\pi f)^n X(f)$
$(-jt)^n x(t)$	$\frac{d^n X}{df^n}$
$\int_{-\infty}^t x(\tau) d\tau$	$\frac{1}{j2\pi f} X(f) + \frac{1}{2} X(0) \delta(f)$
$x_1(t) * x_2(t)$	$X_1(f) X_2(f)$
$x_1(t)x_2(t)$	$X_1(f) * X_2(f)$

# Často používané vzťahy

$$\text{rect}(f/2W) = \text{sak} - W < f < W, 0 \text{ ak } |f| > W, \\ \text{sinc } x = (\sin \pi x) / \pi x, \\ \text{Si } x = \text{Si} \pi x / \pi$$

$$\cos x \cos y = \frac{1}{2} \cos(x + y) + \frac{1}{2} \cos(x - y) \quad (\text{D.1})$$

$$\sin x \sin y = -\frac{1}{2} \cos(x + y) + \frac{1}{2} \cos(x - y) \quad (\text{D.2})$$

$$\sin x \cos y = \frac{1}{2} \sin(x + y) + \frac{1}{2} \sin(x - y) \quad (\text{D.3})$$

$$\cos x \sin y = \frac{1}{2} \sin(x + y) - \frac{1}{2} \sin(x - y) \quad (\text{D.4})$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y \quad (\text{D.5})$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y \quad (\text{D.6})$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x) \quad (\text{D.7})$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \quad (\text{D.8})$$

$$\sin x \cos x = \frac{1}{2} \sin 2x \quad (\text{D.9})$$

$$\sin x + \sin y = 2 \sin \frac{1}{2}(x + y) \cos \frac{1}{2}(x - y) \quad (\text{D.10})$$

$$\sin x - \sin y = 2 \cos \frac{1}{2}(x + y) \sin \frac{1}{2}(x - y) \quad (\text{D.11})$$

$$\cos x + \cos y = 2 \cos \frac{1}{2}(x + y) \cos \frac{1}{2}(x - y) \quad (\text{D.12})$$

$$\cos x - \cos y = -2 \sin \frac{1}{2}(x + y) \sin \frac{1}{2}(x - y) \quad (\text{D.13})$$

$$\sin x = \frac{e^{jx} - e^{-jx}}{2j} \quad (\text{D.14})$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2} \quad (\text{D.15})$$

$$\left. \begin{aligned} \int_{-\pi}^{\pi} \sin m\lambda d\lambda &= 0 \\ \int_{-\pi}^{\pi} \cos m\lambda d\lambda &= 0 \end{aligned} \right\} m \wedge n \in \mathbb{Z} \quad (\text{A.2})$$

$$\left. \begin{aligned} \int_{-\pi}^{\pi} \sin m\lambda \cos n\lambda d\lambda &= 0 \\ \int_{-\pi}^{\pi} \sin m\lambda \sin n\lambda d\lambda &= 0 \\ \int_{-\pi}^{\pi} \cos m\lambda \cos n\lambda d\lambda &= 0 \end{aligned} \right\} m \neq n \quad (\text{A.3})$$

$$\left. \begin{aligned} \int_{-\pi}^{\pi} (\sin m\lambda)^2 d\lambda &= \pi \\ \int_{-\pi}^{\pi} (\cos m\lambda)^2 d\lambda &= \pi \end{aligned} \right\} m = n \quad (\text{A.4})$$