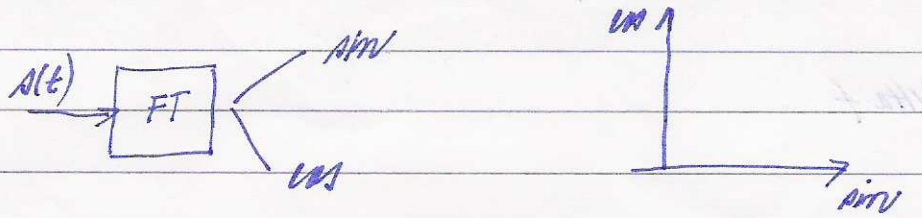
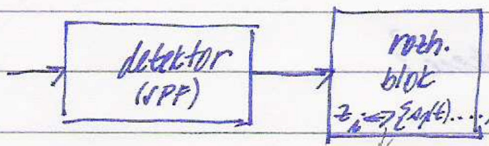
MSK 1
cv.

záp. písomka 5.12. piatok 16^{00}

GFT

FT – 2 báz. funkcie sin, cos kolmé





$z_i \Rightarrow \{s_1(t) \dots s_M, t\}$

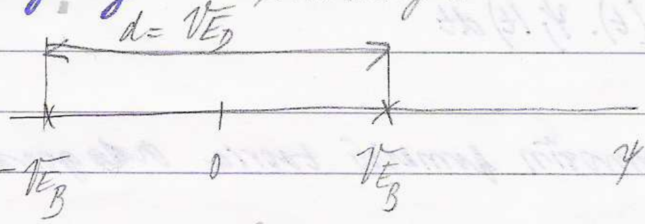
v tom prípade sme potrebovali M blokov

alebo

$z_i \Rightarrow \{a_{11}, \dots a_{1M}\}$

sme porovnávali s nejakými prototypmi

antipodálne signály



$$E_d = \int_0^T (s_1(t) - s_2(t))^2 \, dt$$

$$P_B = Q\left(\sqrt{\frac{E_d}{2N_0}}\right)$$ argument väčší ⇒ menšia chybovosť

ľub. signál v sig. priestore možno vyjadriť pomocou jeho súradníc v príslušnej báze

báza: $\{\psi_j(t)\}$ $j = 1, \dots, N$ — báz. funkcie musia byť nezávislé

sig. abeceda bude mať: $\{s_1(t), \dots, s_M(t)\}$

M prvkov

$N \leq M$

podmienka ortogonality:

$$\int_0^T \psi_j(t)\,\psi_k(t)\,dt = k_j \cdot \delta_{jk} \qquad 0 \le t \le T$$

$$j,k = 1, \dots N$$

k_j - konštanta

δ_{jk} - kroneckerova delta f

$$\delta_{jk} = \begin{cases} 1 & j = k \\ 0 & j \neq k \end{cases}$$

$k_j = \int_0^T \psi_j^2(t)\,dt$ {energia báz. funkcie}

ak $k_j = 1$ ⇒ ortonormálna báza

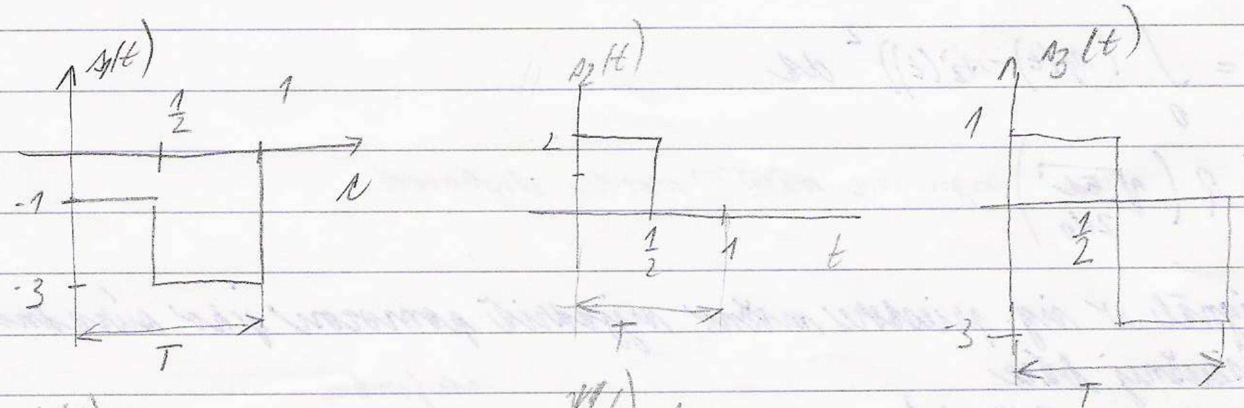
$$s_i(t) = \sum_{j=1}^{N} a_{ij} \cdot \psi_j(t) \qquad i = 1, \dots, M$$

súradnice signálov v danej báze

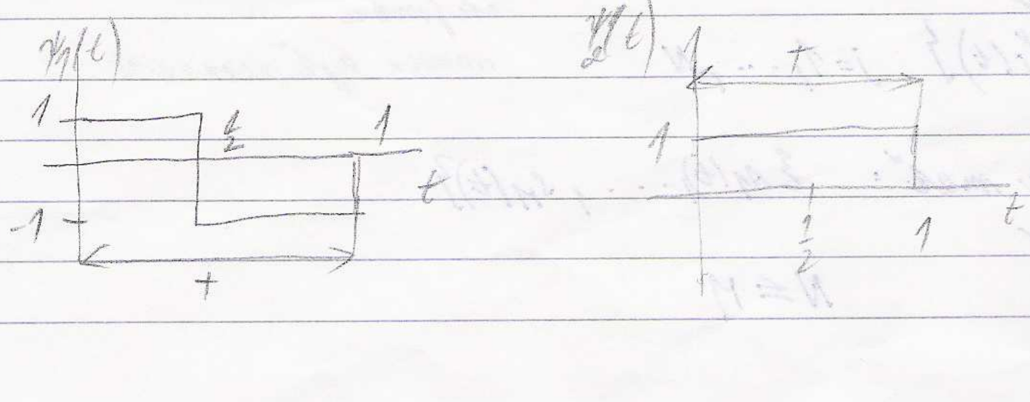
$$a_{ij} = \frac{1}{k_j} \int_0^T s_i(t) \cdot \psi_j(t)\,dt$$

Pr. zistite, či z nasled. množín funkcií tvoria ortogonálnu množinu

a)

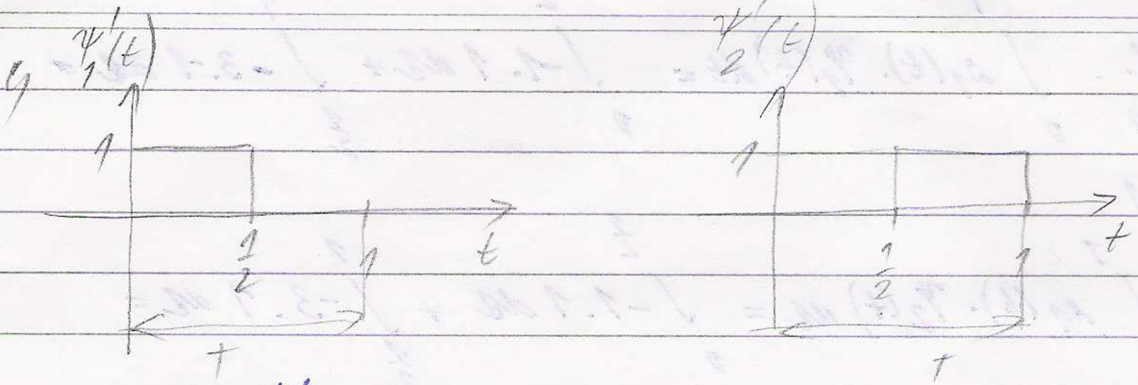


b)



MSK 1

cv.

c) 

aby bola báza musí platiť podmienka ortogonality

a) $\int_0^T s_1(t) \cdot s_2(t)\, dt = \int_0^{\frac{1}{2}} -1 \cdot 2\, dt + \int_{\frac{1}{2}}^{1} -3 \cdot 0\, dt =$

$-2 \cdot \frac{1}{2} = -1 \neq 0 \Rightarrow$ nie sú ortogonálne

b) $\int_0^{\frac{1}{2}} 1 \cdot 1\, dt + \int_{\frac{1}{2}}^{1} -1 \cdot 1\, dt = \frac{1}{2} - \frac{1}{2} = 0 \Rightarrow$ ortog.

c) $\int_0^{\frac{1}{2}} 1 \cdot 0\, dt + \int_{\frac{1}{2}}^{1} 0 \cdot 1\, dt = 0 \Rightarrow$ ortog.

b) $s_{ij} = \frac{1}{K_j} \int_0^T s_i(t) \cdot \psi_j(t)\, dt$

$k_j = \int_0^T \psi_j^2(t)\, dt$

$k_1 = \int_0^{\frac{1}{2}} (+1)^2\, dt + \int_{\frac{1}{2}}^{1} (-1)\, dt = 1$

$k_2 = \int_0^1 1^2\, dt = 1$

$\Rightarrow$ ORTONORMÁLNA BÁZA

$$a_{11} = \frac{1}{K_1} \cdot \int_0^T s_1(t) \cdot \psi_1(t)\, dt = \int_0^{\frac{1}{2}} -1 \cdot 1\, dt + \int_{\frac{1}{2}}^{1} -3 \cdot 1\, dt =$$

$$= 1$$

$$a_{12} = \int_0^T s_1(t) \cdot \psi_2(t)\, dt = \int_0^{\frac{1}{2}} -1 \cdot 1\, dt + \int_{\frac{1}{2}}^{1} -3 \cdot 1\, dt =$$

$$= -\frac{1}{2} - \frac{3}{2} = -2 \qquad \bar{s}_1 = (1, -2)$$

$$s_1 = \psi_1 - 2 \cdot \psi_2$$

$$a_{21} = \int_0^T s_2(t) \cdot \psi_1(t)\, dt =$$

$$= \int_0^{\frac{1}{2}} 2 \cdot 1\, dt + \int_{\frac{1}{2}}^{1} 0 \cdot (-1)\, dt = 1$$

$$a_{22} = \int_0^T s_2(t) \cdot \psi_2(t)\, dt = \int_0^{\frac{1}{2}} 2 \cdot 1\, dt + \int_{\frac{1}{2}}^{1} 0 \cdot \quad = 1$$

$$\bar{s}_2 = (1, 1)$$

$$a_{31} = \int_0^{\frac{1}{2}} 1 \cdot 1\, dt + \int_{\frac{1}{2}}^{1} -3 \cdot (-1)\, dt = \frac{1}{2} + \frac{3}{2} = 2$$

$$a_{32} = \int_0^{\frac{1}{2}} 1 \cdot 1\, dt + \int_{\frac{1}{2}}^{1} -3 \cdot 1\, dt = \frac{1}{2} - \frac{3}{2} = -1$$

$$\bar{s}_3 = (2, -1)$$

GRAMM-SCHMIDTOVA METÓDA HĽADANIA BÁZ. FUNKCIÍ

Sig. abec. $s = \{s_1(t), \dots s_M(t)\}$

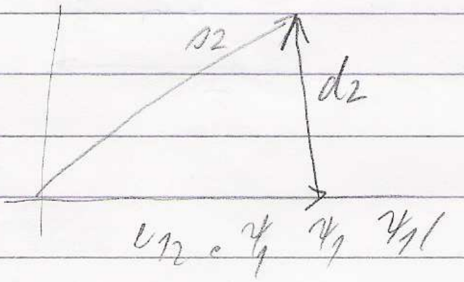
1. báz. funkcia

$$\psi_1(t) = \frac{s_1(t)}{\sqrt{E_1}} \qquad E_1 = \int_0^T s_1^2(t)\, dt$$

MSK 1
cv.

2. bázová f.

$$\psi_2(t) = \frac{d_2(t)}{\sqrt{E_2}} \qquad E_2 = \int_0^T d_2^2(t)\,dt$$

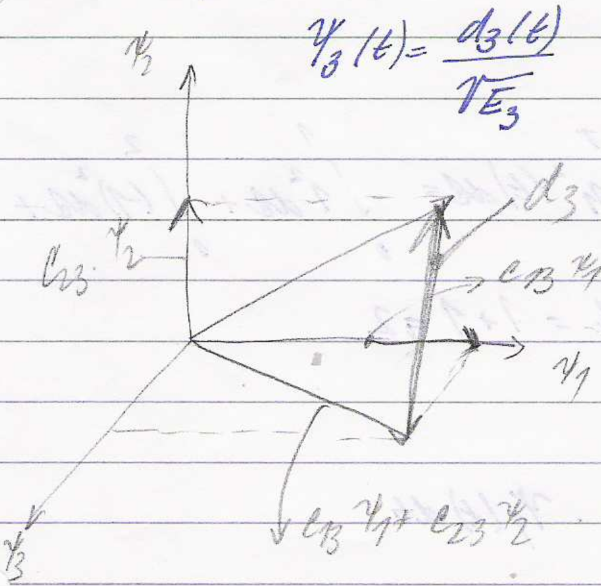


$$d_2(t) = s_2(t) - c_{12} \cdot \psi_1(t)$$

$$c_{12} = \int_0^T s_2(t) \cdot \psi_1(t)\,dt$$

3. báz. funkcia

$$\psi_3(t) = \frac{d_3(t)}{\sqrt{E_3}} \qquad E_3 = \int_0^T d_3^2(t)\,dt$$



$$d_3 = s_3(t) - c_{13} \cdot \psi_1(t) - c_{23} \cdot \psi_2(t)$$

$$c_{13} = \int_0^T s_3(t) \cdot \psi_1(t)\,dt$$

$$c_{23} = \int_0^T s_3(t) \cdot \psi_2(t)\,dt$$

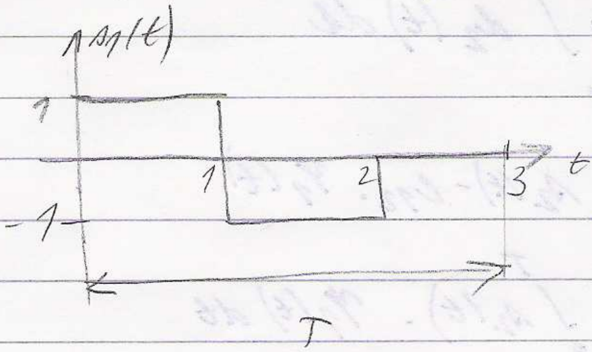
$\vdots$

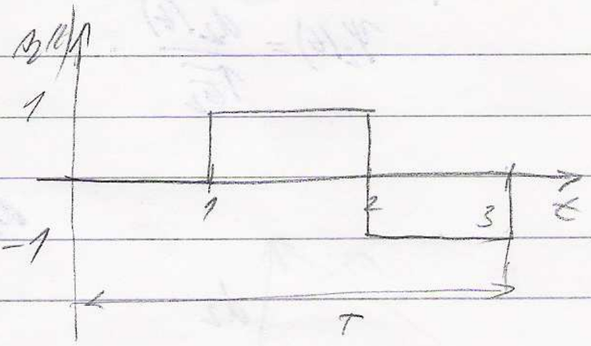
$$\psi_l(t) = \frac{d_l(t)}{\sqrt{E_l}} \qquad E_l = \int_0^T d_l^2(t)\,dt$$

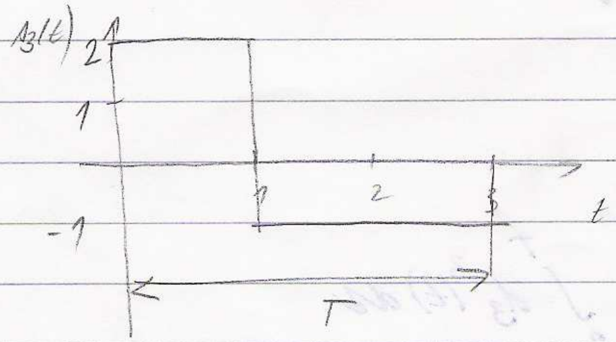
$$d_l(t) = s_l(t) - \sum_{j=1}^{l-1} c_{jl}\,\psi_j(t)$$

Pr nájdite báz. funkcie týchto signálov

Pr. Najdite baz. f. tijekto s.







$$\psi_1(t) = \frac{s_1(t)}{\sqrt{E_1}} = \frac{s_1(t)}{\sqrt{2}}$$

$$E_1 = \int_0^T s_1^2(t)\,dt = \int_0^1 1^2\,dt + \int_1^2 (-1)^2\,dt + \int_2^3 0^2\,dt = 1 + 1 = 2$$

$$\psi_2(t) = \frac{d_2(t)}{\sqrt{E_2}}$$

$$c_{12} = \int_0^T s_2(t) \cdot \psi_1(t)\,dt$$

$$c_{12} = \int_0^1 0 \cdot \frac{1}{\sqrt{2}}\,dt + \int_1^2 1 \cdot \frac{-1}{\sqrt{2}}\,dt + \int_2^3 -1 \cdot \frac{0}{\sqrt{2}}\,dt = -\frac{1}{\sqrt{2}}$$

$$d_2(t) = s_2(t) - c_{12}\,\psi_1(t) = s_2(t) - \frac{1}{\sqrt{2}}\,\psi_1(t)$$

$$E_2 = \int_0^T d_2^2(t)\,dt = \int_0^1 \left(0 - \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}\right)^2 dt + \int_1^2 \left(1 - \frac{1}{\sqrt{2}} \cdot -\frac{1}{\sqrt{2}}\right)^2 dt + \int_2^3 \underbrace{-1 - \frac{1}{\sqrt{2}} \cdot \frac{0}{\sqrt{2}}}_{0}\,dt = \left(-\frac{1}{2}\right)^2 + \left(1 + \frac{1}{2}\right)^2 = \frac{1}{4} + \frac{9}{4} = \frac{10}{4} = 2{,}5$$

MSK 1
w.

$$E_2 = \int_0^1 \left(\frac{1}{2}\right)^2 dt + \int_1^2 \left(\frac{2}{2}\right)^2 dt + \int_2^3 (-1)^2 dt = \underline{\underline{\frac{3}{2}}}$$

$$\psi_2(t) = \frac{d_2(t)}{\sqrt{E_2}} = \frac{s_2(t) - \frac{1}{\sqrt{2}}\psi_1(t)}{\sqrt{1{,}5}} = \frac{s_2(t) + \frac{1}{\sqrt{2}} \cdot \frac{s_1(t)}{\sqrt{2}}}{\sqrt{1{,}5}} =$$

$$= \frac{s_2(t) + \frac{1}{2} s_1(t)}{\sqrt{\frac{3}{2}}} = s_1(t) \cdot \frac{1}{2\sqrt{\frac{3}{2}}} + s_2(t) \cdot \frac{1}{\sqrt{\frac{3}{2}}} = s_1(t) \cdot \frac{1}{\sqrt{6}} +$$

$$+ s_2(t) \cdot \frac{2}{\sqrt{6}}$$

$$E_3 = \int_0^T d_3^2(t)\, dt$$

$$d_3 = s_3(t) - c_{13}\psi_1(t) - c_{23}\psi_2(t)$$

$$\psi_3(t) = \frac{d_3(t)}{\sqrt{E_3}}$$

$$c_{13} = \int_0^T s_3(t) \cdot \psi_1(t)\, dt$$

$$c_{23} = \int_0^T s_3(t) \cdot \psi_2(t)\, dt$$

$$c_{13} = \int_0^1 2 \cdot \frac{1}{\sqrt{2}}\, dt + \int_1^2 -1 \cdot \frac{-1}{\sqrt{2}}\, dt + \int_2^3 -1 \cdot \frac{0}{\sqrt{2}}\, dt = \frac{2}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{3}{\sqrt{2}}$$

$$c_{23} = \int_0^1 2 \cdot \overbrace{\left(1 \cdot \frac{1}{\sqrt{6}} + 0 \cdot \frac{2}{\sqrt{6}}\right)}^{\frac{2}{\sqrt{6}}} dt + \int_1^2 -1 \cdot \overbrace{\left(-1 \cdot \frac{1}{\sqrt{6}} + 1 \cdot \frac{2}{\sqrt{6}}\right)}^{-\frac{1}{\sqrt{6}}} dt +$$

$$+ \int_2^3 -1 \cdot \overbrace{\left(0 \cdot \frac{1}{\sqrt{6}} + -1 \cdot \frac{2}{\sqrt{6}}\right)}^{\frac{2}{\sqrt{6}}} dt = \frac{2}{\sqrt{6}} - \frac{1}{\sqrt{6}} + \frac{2}{\sqrt{6}} = \frac{3}{\sqrt{6}}$$

$$d_3 = s_3(t) - \frac{3}{\sqrt{2}} \cdot \frac{s_1(t)}{\sqrt{2}} - \frac{3}{\sqrt{6}} \cdot \left(s_1(t) \cdot \frac{1}{\sqrt{6}} + s_2(t) \cdot \frac{2}{\sqrt{6}}\right) =$$

$$= s_3(t) - \frac{3 \cdot s_1(t)}{2} - \frac{[illegible]\, s_1(t)}{[illegible]\,2} - s_2(t) =$$