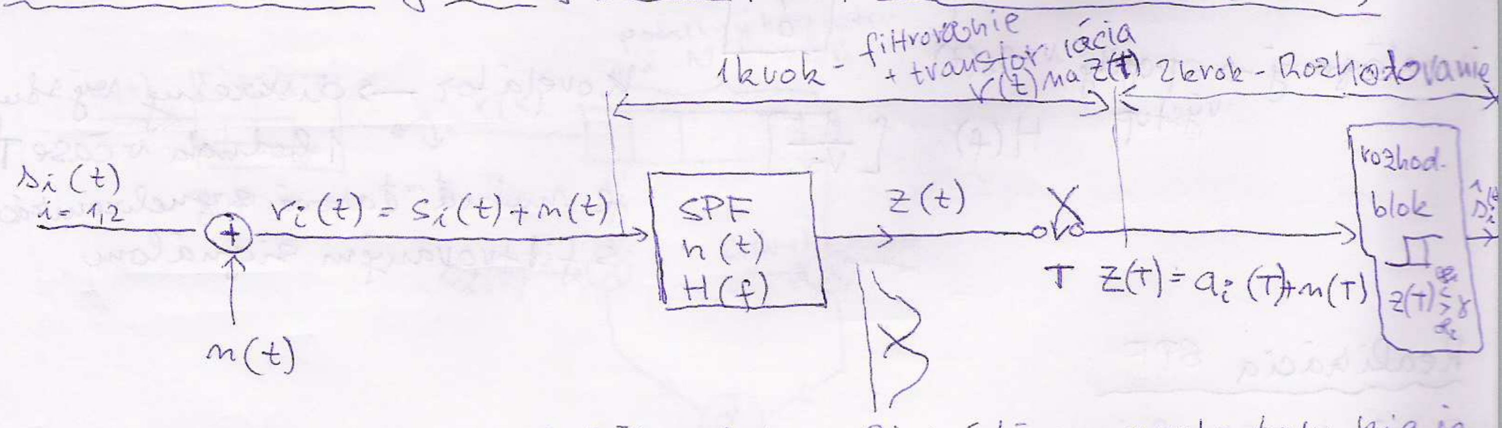
24. 10. 08

Detekcia binárnych signálov pri pôsobení AWGN (šumu)



MF - filter poskytuje najväčší odstup signál/šum, preto tam nie je napr. DP-filter

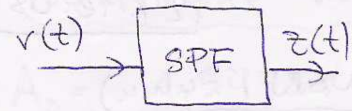
SPF

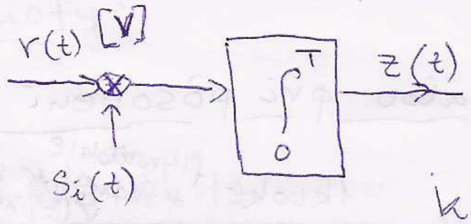
$$H(f) = K \cdot S^*(f) \cdot e^{-j2\pi fT}$$

$$\Updownarrow \text{FT}$$

$$h(t) = \begin{cases} k \cdot s(T-t) & t \leq T \\ 0 & t > T \end{cases}$$

pre $t = T \Rightarrow$ je výstup filtra spriahnutého a korelátora zhodný





analógový → spojitý výstup

$H(f)$ $\left[\frac{1}{Vs}\right]$

korelátor → diskrétny výstup
1 hodnota v čase T
je nutná časová synchronizácia
s filtrovaným signálom

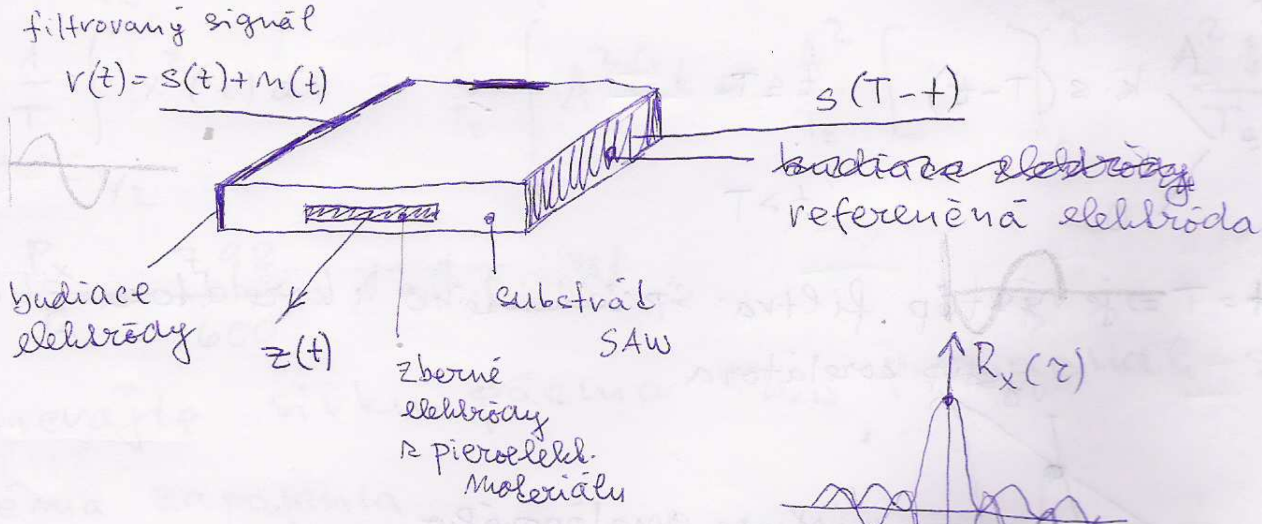
Realizácia SPF

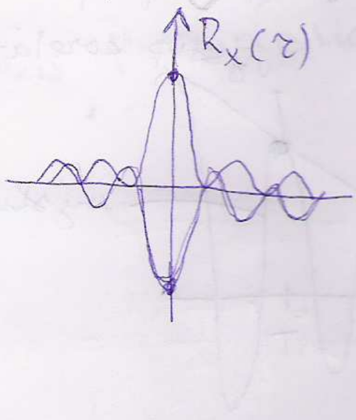
Podľa konštrukcie

1.) Analógové

2.) Digitálne

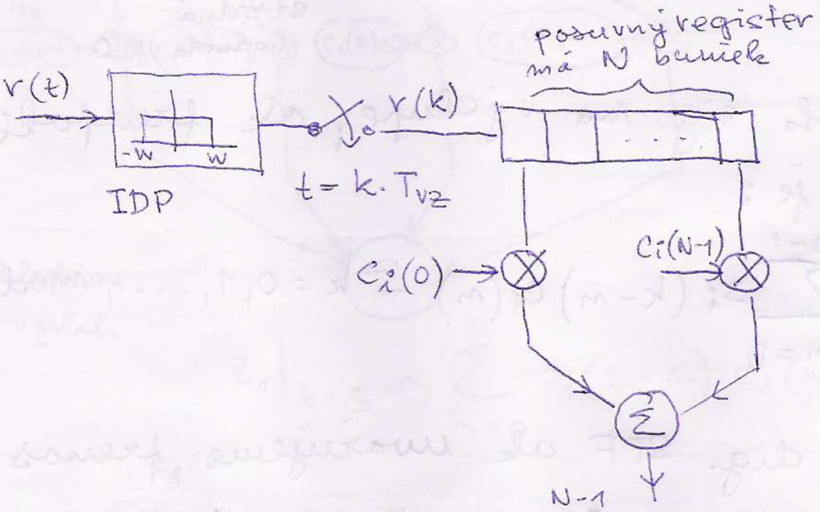
1.) Analógové SPF





2.) Digitálne SPF

blokova schéma



$$z_i(k) = \sum_{n=0}^{N-1} r(k-n)\, c_i(n)$$

šírka pásma prijatého signálu: $W = \frac{1}{2T}$

Nyquistova vzorkovacia frekvencia $f_0 = 2W = \frac{1}{T}$

v reálnych systémoch $f_{VZ} \geq f_0$

vstup: spojitý signál

výstup IDP + vzorkovací obvod → diskrétny výstup $r(t)$ →

→ $r(k) = s_i(k) + n(k)$

binárny prípad $i = 1, 2$

k - index vzorkovacieho času $k = 0, 1, \dots$

váhy $c_i(n)$ aproximujú $h(t)$, kde $n = 0, \dots N-1$ časový index a bunieк pos. registra

N - počet buniek registra a počet vzoriek/symbol

$$c_i(n) = s_i\left[(N-1) - n\right]$$

výstup v čase k tej vzorky $z_i(k) = \sum_{n=0}^{N-1} r(k-n)\, c_i(n)$

$k = 0, 1, \dots \bmod N$

za predpokladu, že systém je zosynchronizovaný, T je známe, predp. AWGN šum v kanáli ($\mathcal{E}\{n(t)\}=0$)

stredná hodnota je 0

potom očakávaná hodnota sig. na výstupe, ak predpokl. že bol vyslaný $s_i(t)$ je:

$$\mathcal{E}\{z_i(k)\} = \sum_{n=0}^{N-1} s_i(k-n)\, c_i(n) \qquad k = 0, 1, \dots, \mod N$$

Pr. aký bude výstup dig. SPF ak uvažujeme prenos BPSK signálu navzorkovaného 4 vzorkami na symbol a použije sa 3-úrovňové vzorkovanie

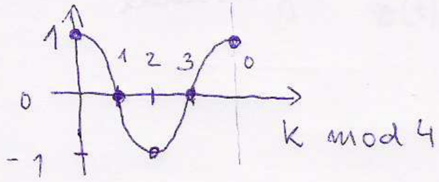
vysielané signály:

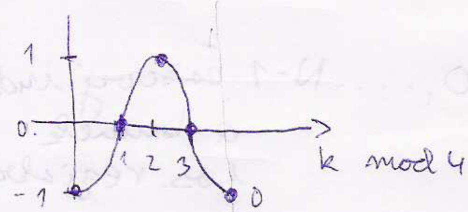
a.) $s_1(t) = \cos \omega_0 t$

b.) $s_2(t) = -\cos \omega_0 t$

Riešenie:

4 vzorky/symbol $\rightarrow N = 4$





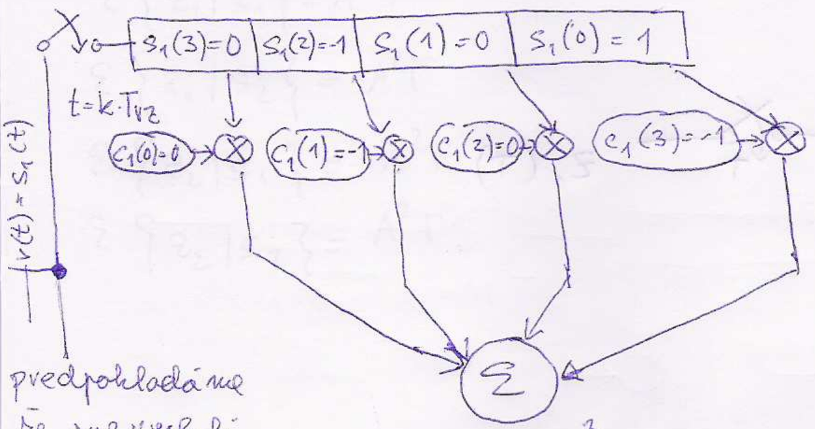
po navzorkovaní

$s_1(0) = 1 \qquad c_1(0) = 0$

$s_1(1) = 0 \qquad c_1(1) = -1$

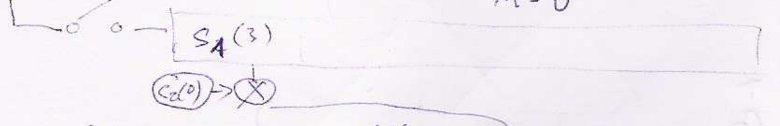
$s_1(2) = -1 \qquad c_1(2) = 0$

$s_1(3) = 0 \qquad c_1(3) = +1$



predpokladáme že sme vyslali

$$z_1\Big|_{k=3} = \sum_{n=0}^{3} s_1(3-n)\cdot c_i(n) = z_1(k=3) = +2$$



b.)

$s_1(0) = -1$ $\qquad$ $c(0) = 0$

$s_1(1) = 0$ $\qquad$ $c(1) = +1$

$s_1(2) = +1$ $\qquad$ $c(2) = 0$

$s_1(3) = 0$ $\qquad$ $c(3) = -1$

$$z_2\Big|_{k=3} = \sum_{n=0}^{3} s_1(3-n)\, c_2(n) = z_2(k=3) = -2$$

$\varepsilon\{z_1 | s_1\} = +2$

$\varepsilon\{z_2 | s_2\} = -2$

$\varepsilon\{z_1 | s_2\} = -2$

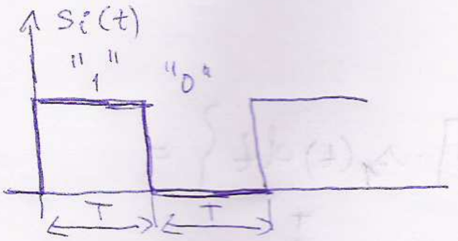
$\varepsilon\{z_2 | s_2\} = +2$

Pr.) Aký bude výstup SPF ak použijeme:

a.) UNRZ s amp. A

b.) BPNRZ

a.)

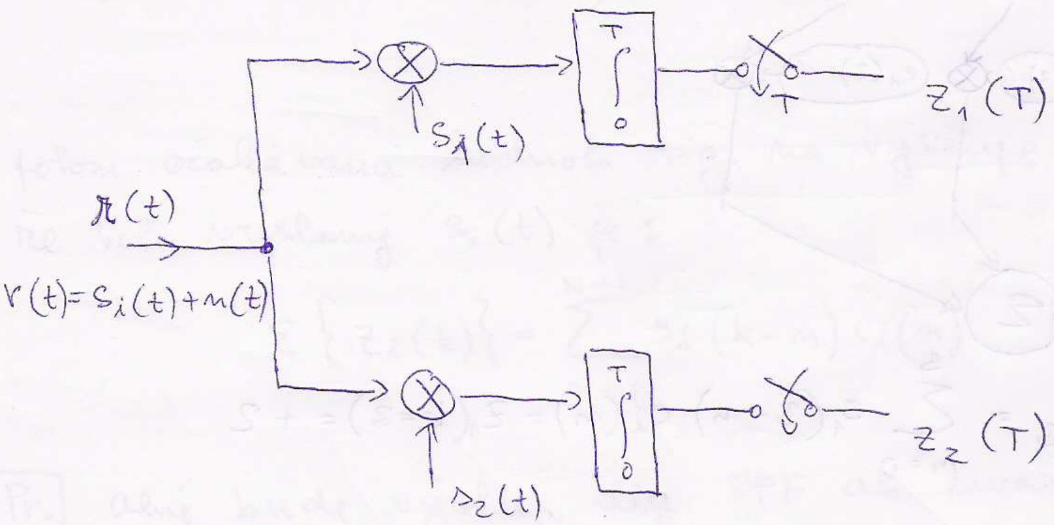


$$s_1(t) = \begin{cases} A & 0 \le t \le T \\ 0 & t > T \end{cases}$$

$s_2(t) = 0$

$s_1(t)$ a $s_2(t)$ sú ortogonálne (preto lebo sa nepodobajú tie signály
autokorelácia je 0

blokovā schēma



$$a_1(T) = \mathcal{E}\left\{ z_1(T) / s_1(t) \right\} = \mathcal{E}\left\{ \int_0^T [s_1(t) + n(t)] \cdot s_1(t)\, dt \right\} =$$

$$= \mathcal{E}\left\{ \int_0^T (A^2 + n(t) \cdot A)\, dt \right\} = \mathcal{E}\left\{ A^2 \int_0^T dt \right\} + A \int_0^T \underbrace{\mathcal{E}\{n(t)\}}_{\Downarrow\ 0} dt = A^2 T$$

AWGN →

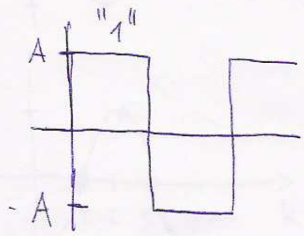
$\mathcal{E}\{z_1 | s_1\} = A^2 T$

$\mathcal{E}\{z_1 | s_2\} = 0$

$\mathcal{E}\{z_2 | s_1\} = 0$

$\mathcal{E}\{z_2 | s_2\} = 0$

b.)



$$a_2(T) = \mathcal{E}\left\{ z_2(T) | s_2(T) \right\} = \mathcal{E}\left\{ \int_0^T [s_2(t) + n(t)] \cdot s_1(t)\, dt \right\} =$$

$$= \mathcal{E}\left\{ \int_0^T (-A^2 + n(t) \cdot A)\, dt \right\} = \mathcal{E}\left\{ -A^2 \int_0^T dt \right\} + A \int_0^T \underbrace{\mathcal{E}\{n(t)\}}_{\Downarrow\ 0} dt =$$

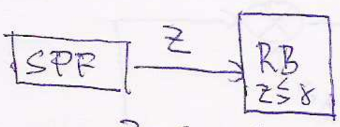
$\mathcal{E}\{z_1|s_1\} = A^2T$

$\mathcal{E}\{z_1|s_2\} = -A^2T$

$\mathcal{E}\{z_2|s_1\} = -A^2T$

$\mathcal{E}\{z_2|s_2\} = A^2T$

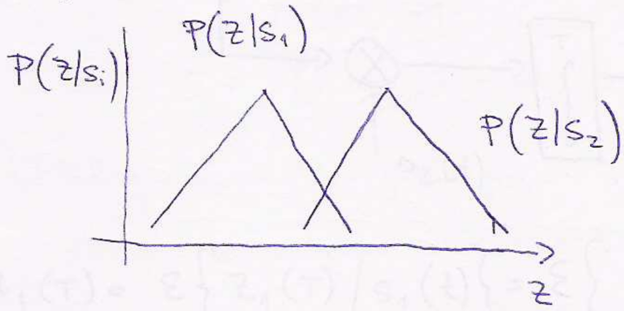
Cviko 6
31.10.08

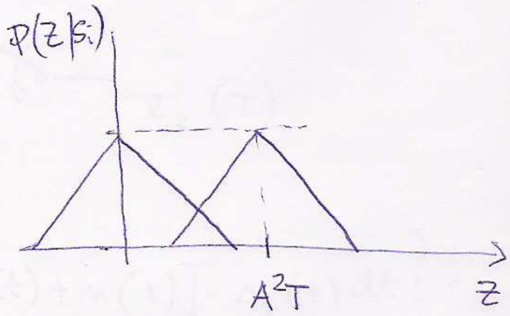


$z = a_i + n_i$

$s_1(t) = A$

$s_2(t) = 0$





$s_1(t) = A \qquad a_1(t) = A^2T$

$s_2(t) = 0 \qquad a_2(t) = 0$

2 kritéria

MAP → Maximum Aposterial Probability

ML → Maximum Likelihood

berie do úvahy aj „štatistiku" zdroja

MAP

pravdepodobnosť nastatia z že bol vyslaný signál s_1

$$\frac{P(z|s_1)}{P(z|s_2)} \underset{\mathcal{H}_2}{\overset{\mathcal{H}_1}{\gtrless}} \frac{P(s_2)}{P(s_1)}$$

štatistika zdroja

$$P(s_1|z) \underset{\mathcal{H}_2}{\overset{\mathcal{H}_1}{\gtrless}} P(s_2|z)$$

Bayesov vzorec

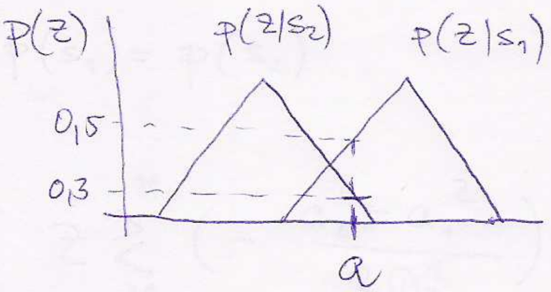
$$P(s_i|z) = \frac{P(z|s_i) \cdot P(s_i)}{P(z)}$$

$\boxed{ML}$

$$\frac{p(z|s_1)}{p(z|s_2)} \underset{H_2}{\overset{H_1}{\gtrless}} 1$$

$\boxed{Pr.}$ Systém používa 2 signály $s_1(t)$ a $s_2(t)$. Výstup z detektora v prijímači je $z = a$

Dané sú podmienené pravdepodobnosti



pričom

$p(s_1) = 0,3$

$p(s_2) = 0,7$

Určte ktorý signál bude vororhaný: a.) MAP

b.) ML

a.) MAP

$$p(s_1|z) = \frac{p(z_a|s_1)\cdot p(s_1)}{p(z)} = \frac{p(z_a|s_1)\, p(s_1)}{p(s_1)\cdot p(z_a|s_1) + p(s_2)\cdot p(z_a|s_2)} =$$

$p(z)$ – úplna pravdep.

$$p(s_2|z) = \frac{p(z_a|s_2)\cdot p(s_2)}{p(z)} = \frac{p(z_a|s_2)\cdot p(s_2)}{p(s_1)\cdot p(z_a|s_2) + p(z_a|s_2)} =$$

$$p(s_1|z) = \frac{0,5\cdot 0,3}{0,5\cdot 0,3 + 0,3\cdot 0,7} = 0,416$$

$$p(s_2|z) = 1 - 0,416 = 0,584$$

2 spôsob MAP

$$\frac{0,5}{0,3} \underset{H_2}{\overset{H_1}{\gtrless}} \frac{0,7}{0,3}$$

$$0,5 < 0,7$$

b.) ML

$$\frac{0{,}5}{0{,}3} \underset{H_2}{\overset{H_1}{\gtrless}} 1 \qquad s_1(t)$$

---

r
↓
optimálna rozhodovacia úroveň

MAP: $$\frac{p(z|s_1)}{p(z|s_2)} \underset{H_2}{\overset{H_1}{\gtrless}} \frac{p(s_2)}{p(s_1)}$$

$$\frac{\frac{1}{a_0\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left[\frac{z-a_1}{a_0}\right]^2}}{\frac{1}{a_0\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left[\frac{z-a_2}{a_0}\right]^2}} \underset{H_2}{\overset{H_1}{\gtrless}} \frac{p(s_2)}{P(s_1)}$$

a_0 – disperzia šumu

a_1, a_2 -> výstupné zložky zo sprialnutého filtra spôsobené sig. s_1, s_2

$z \gtrless \ldots\ldots \; \gamma$

$$-\frac{1}{2}\left[\frac{z-a_1}{a_0}\right]^2 + \frac{1}{2}\left[\frac{z-a_2}{a_0}\right]^2 \underset{H_2}{\overset{H_1}{\gtrless}} \ln\frac{P(s_2)}{P(s_1)}$$

$$\frac{-z^2+2a_1z-a_1^2}{2a_0^2} + \frac{z^2-2a_2z+a_2^2}{2a_0^2} \underset{H_2}{\overset{H_1}{\gtrless}} \ln\frac{P(s_2)}{P(s_1)}$$

$$\frac{a_2^2-a_1^2+2a_1z-2a_2z}{2a_0^2} \gtrless \ln\frac{P(s_2)}{P(s_1)}$$

$$z\frac{a_1-a_2}{2\sigma_0^2}+\frac{a_2^2-a_1^2}{2\sigma_0^2}\underset{H_2}{\overset{H_1}{\gtrless}}\ln\frac{P(s_2)}{P(s_1)}$$

$$z\underset{H_2}{\overset{H_1}{\gtrless}}\left\{\ln\frac{P(s_2)}{P(s_1)}-\frac{a_2^2-a_1^2}{2\sigma_0^2}\right\}\cdot\frac{\sigma_0^2}{a_1-a_2}$$ γ - pre MAP-kritéria

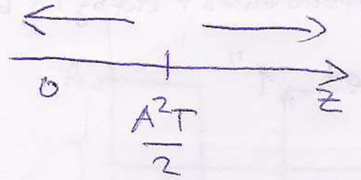
$\boxed{ML}$: $\gamma = ?$

$p(s_1)=p(s_2)$

$$z\underset{H_2}{\overset{H_1}{\gtrless}}\left(-\frac{a_2^2-a_1^2}{2\sigma_0^2}\right)\left(\frac{\sigma_0^2}{a_1-a_2}\right)$$

$$z\underset{H_2}{\overset{H_1}{\gtrless}}-\frac{(a_2+a_1)(a_2-a_1)}{2(a_1-a_2)}$$

$$z\underset{H_2}{\overset{H_1}{\gtrless}}\frac{a_2+a_1}{2}$$



$\boxed{Pr.}$ Na signalizáciu je použitý signál UPNRZ s parametrami

$A = 5V$

$\sigma_0^2 = 10^{-6}$, $\boxed{10^{-4}}$

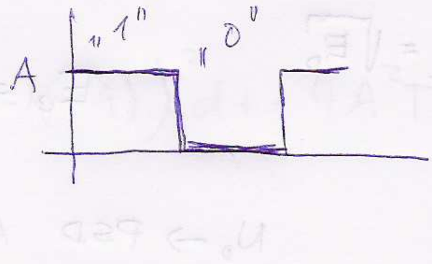
$R_B = 1\,kbps$

$P(0) = 0{,}3$

$P(1) = 0{,}7$

$s_1(t) = A$

$s_2(t) = 0$



$\gamma = ?$

a.) MAP

b.) ML

a.) $a_1 = A^2 T$ $T = \frac{1}{R_B} =$

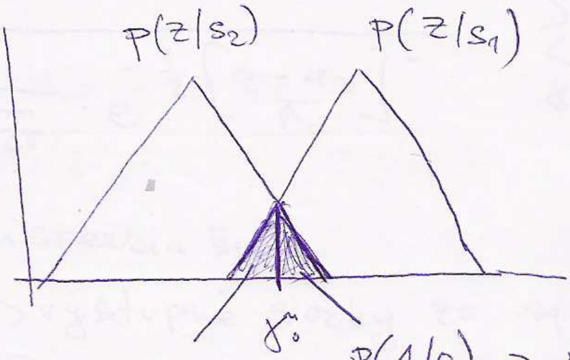
$a_2 = 0$

$$z \underset{H_2}{\overset{H_1}{\gtrless}}$$

$$\gamma \underset{H_2}{\overset{H_1}{\gtrless}} \ln \frac{0{,}7}{0{,}3} - \quad 0 -$$

$$\gamma = 12{,}466 \cdot 10^{-3}$$

b.) $\gamma = 12{,}5 \cdot 10^{-3}$



chybovosť detekcie

$$P_B = Q\left(\frac{a_1 - a_2}{2\sigma_0}\right)$$

pravdepodobnosť chyby na bit

$P(1/0) \rightarrow$ vyslem „0" a det „1"

$P(0/1) \rightarrow$ vyslem „1" det „0"

$(a_1 - a_2)^2 \rightarrow$ energia rozdielového signálu E_D

$a_1 - a_2 = \sqrt{E_D}$

$$E_d = \int_0^T (s_1(t) - s_2(t))^2 \, dt$$

$\frac{N_0}{2} = \sigma_0^2$ $N_0 \rightarrow$ PSD AWGN

$$P_B = Q\left(\sqrt{\frac{E_d}{2N_0}}\right)$$

pre UPNRZ: $P_B = Q\left(\sqrt{\frac{E_B}{N_0}}\right)$ → energia na bit

---

$P_B = ?$

$A = 5V$

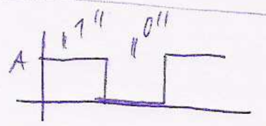
$\sigma_0^2 = 10^{-4}$

$P(0) = 0{,}3$

$P(1) = 0{,}7$

$s_1(t) = A$

$s_2(t) = 0$



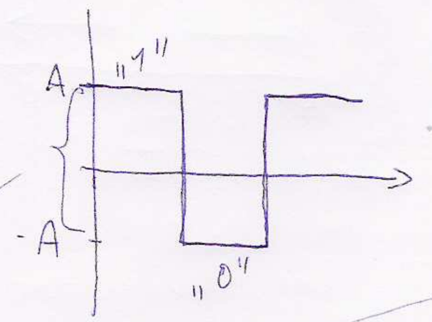
$$P_B = Q\left(\sqrt{\frac{E_D}{2N_0}}\right)$$

$$P_B = Q\left(\sqrt{\frac{E_D}{4\sigma_0^2}}\right)$$

$$E_D = \int_0^T \left(s_1(t) - s_2(t)\right)^2 dt = \int_0^T (A-0)^2 dt = A^2 T = 25 \cdot 10^{-3}$$

$$P_B = Q(7{,}906) = 1{,}353 \cdot 10^{-15}$$

$\boxed{\text{Pr:}}$



zmiši sa lebo je väčší rozdiel

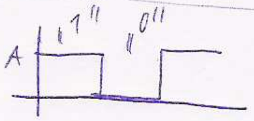
$$E_D = \int_0^T \left(s_1(t) - s_2(t)\right)^2 dt = \int_0^T \left(A - (-A)\right)^2 dt = 4A^2 T = 1{,}311 \cdot 10^{-56}$$

pre UPNRZ: $P_B = Q\left(\sqrt{\frac{E_B}{N_0}}\right)$ — energia na bit

$P_B = ?$

$A = 5V$
$\sigma_0^2 = 10^{-4}$

$P(0) = 0,3$
$P(1) = 0,7$
$s_1(t) = A$
$s_2(t) = 0$



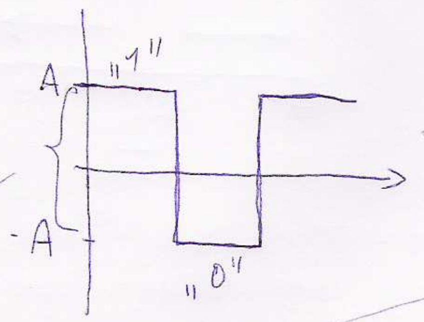
$$P_B = Q\left(\sqrt{\frac{E_D}{2N_0}}\right)$$

$$P_B = Q\left(\sqrt{\frac{E_D}{4\sigma_0^2}}\right)$$

$$E_D = \int_0^T \left(s_1(t) - s_2(t)\right)^2 dt = \int_0^T (A-0)^2 dt = A^2 T = 25 \cdot 10^{-3}$$

$$P_B = Q(7,906) = 1,353 \cdot 10^{-15}$$

$\boxed{\text{Pr.}}$



zmiší sa lebo je väčší rozdiel

$$E_D = \int_0^T \left(s_1(t) - s_2(t)\right)^2 dt = \int_0^T \left(A - (-A)\right)^2 dt = 4A^2T = 1,311 \cdot 10^{-56}$$