en. sig. v rámci symbolového intervalu T_s

$$E_x = \int_{-\frac{T_s}{2}}^{T_s/2} x^2(t)\,dt$$

k en. signálom patria: neperiodické

Spektrum: spojité → FT

3.) Výkonová spektrálna hustota

PSD - Power Spectral Density

pre reálne signály

$$PSD = G_x(f) = \sum_{n=-\infty}^{\infty} |c_n|^2 \delta(f - n \cdot f_0)$$

n - index harmonickej zložky

c_n - komplexné Fourierove koeficienty

$$G_x(f) = |X(f)|^2 \quad \left[\frac{W}{Hz}\right]$$

$$x(t) \xleftrightarrow{FR} X(f)$$

vlastnosti: - nezáporná f.
- symetrická

$$P_x = \underbrace{\int_{-\infty}^{\infty} G_x(f)\,df}_{\text{2stranná hustota}} = 2 \underbrace{\int_{0}^{\infty} G_x(f)\,df}_{\text{1. stranná hustota}}$$

4.) Energetická spektrálna hustota

ESD - Energy Spectral Density

$$ESD = \Psi_x(f) = |X(f)|^2 \quad \left[\frac{J}{Hz}\right]$$

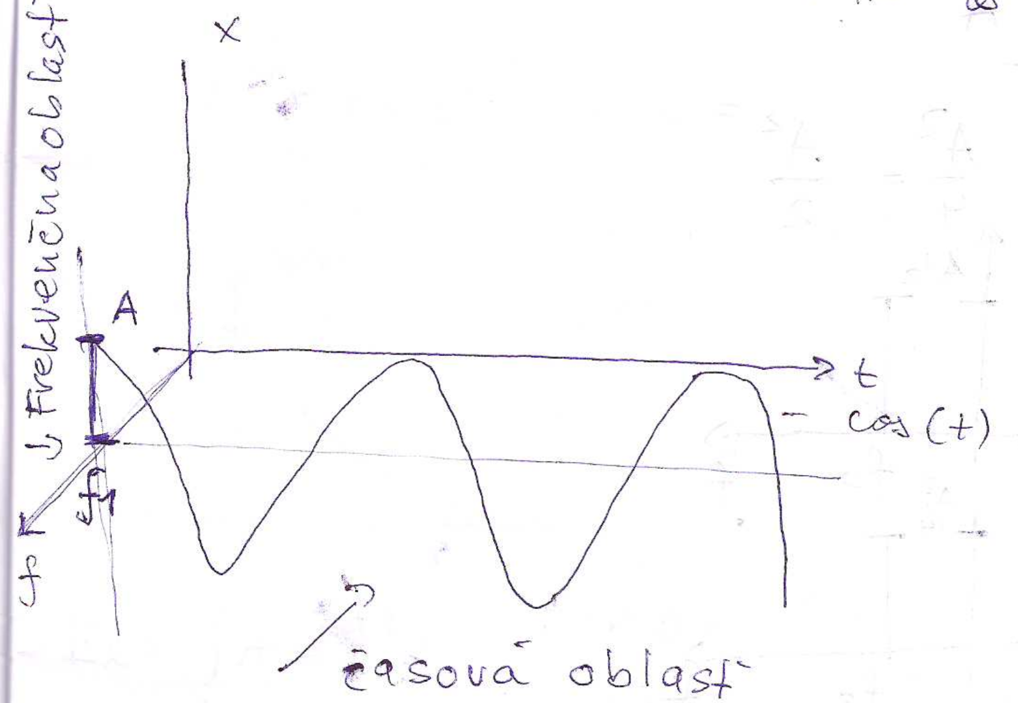
$$x(t) \xleftrightarrow{FT} X(f)$$

$$E_x = \underbrace{\int_{-\infty}^{\infty} \Psi_x(f)\,df}_{\text{2stranná ESD}} = 2 \underbrace{\int_{0}^{\infty} \Psi(f)\,df}_{\text{1.stranná ESD}}$$

5.) Parcevalové rovnosti:

Výk. sig. (reálne): $P_x = \sum_{n=-\infty}^{\infty} |c_n|^2 = \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt$

Energ. sig. (reálne): $E_x = \int_{-\infty}^{\infty} \Psi_x(f)\,df = \int_{-\infty}^{\infty} x^2(t)\,dt$



Pr. Vypočítajte str. normovaný výkon harm. signálu

$$k(t) = A\cos(2\pi f_0 t)$$

a.) v časovej oblasti

b.) vo frekvenčnej oblasti

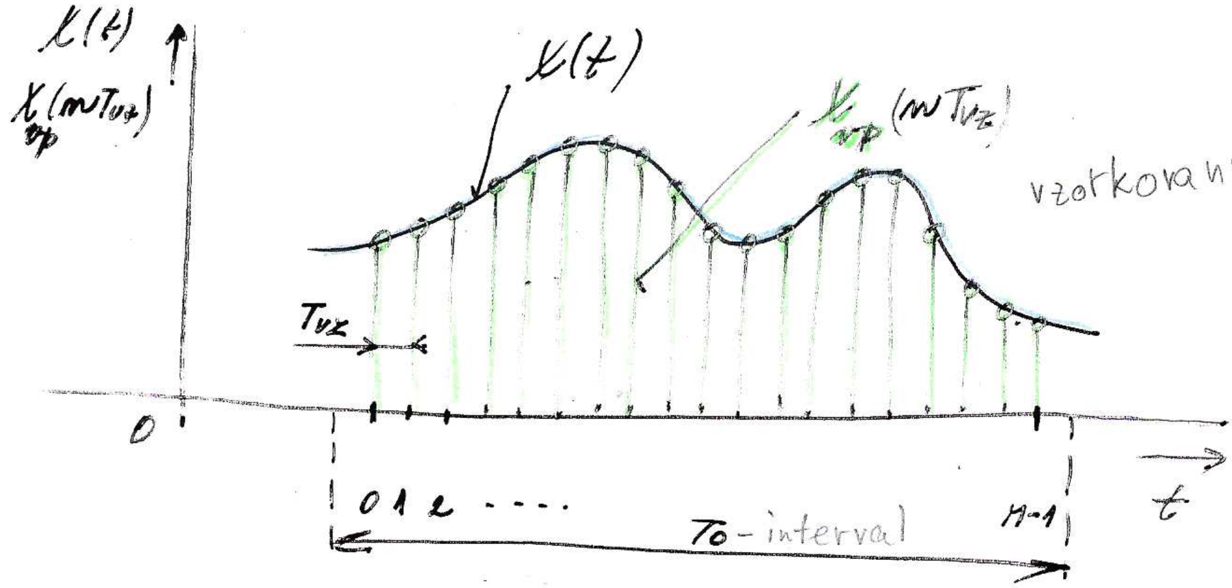
$$P_x = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} (A^2(\cos 2\pi f_0 t)^2 = \frac{A^2}{T_0}\int_{-T_0/2}^{T_0/2} \frac{1}{2}\left(1 + \cos 2(2\pi f_0 t)\right)dt =$$

$\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$$= \frac{A^2}{2T_0}\Big[t\Big]_{-T_0/2}^{T_0/2} + \underbrace{\left[\frac{\sin 4\pi f_0 t}{4\pi f_0}\right]_{-T_0/2}^{T_0/2}}_{0} = \frac{A^2}{2T_0}\Big[T_0 + 0\Big] = \frac{A^2}{2}$$

Výkon harm. signálu $\boxed{P_x = \frac{A^2}{2}}$

Diskrétna Fourierova transformácia



vzorkovanie sig.

$$\boxed{f_{vz} = \frac{1}{T_{vz}}}$$ vzorkovacia frek. a per. vzorkovania

DFT

počet vzoriek

vzdialenosť medzi vzorkami

$x(t)$ na $T_0 = M\, T_{vz}$

získanie vzoriek signálu:

prvá vzorka

$$x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$$

Dirák • sig. = hodnota sig. v mieste diraka

$$\sum_{n=-\infty}^{\infty} \delta(t-nT_{vz}) = \frac{1}{T_{vz}} \sum_{k=-\infty}^{\infty} e^{jk\omega_{vz}t}$$

Per. sled dirákov v čas. obl.

Rakůs B612 | 26.9.08

zápočet 30b - 15 min

Signály a ich charakteristiky

signály delíme na: a.) Deterministické < periodické, neperiod., kváziper.
(z výskytu)
b.) Stochastické

- periodické < harmonické, neharmonické

- stochastické - stacionárne
- nestacionárne
- ergodické

1.) Výkonové - $0 < P_x < \infty$

P_x - stredný normovaný výkon ($P = U \cdot I = \frac{U^2}{R} = I^2 \cdot R$

ak $R = 1\Omega \rightarrow P_x = x^2$

$x \rightarrow U(I)$

reálne signály: $P_x = \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$

k výkonovým signálom patria: periodické sig.
stochastické sig.

Spektrum: čiarové $\rightarrow$ FR

2.) Energetické signály

$0 < E_x < \infty$

E_x - str. normovaná energia

reálne signály $E_x = \int_{-\infty}^{\infty} x^2(t)\,dt$

b.) $P_x = \sum_{-\infty}^{\infty} |c_n|^2$

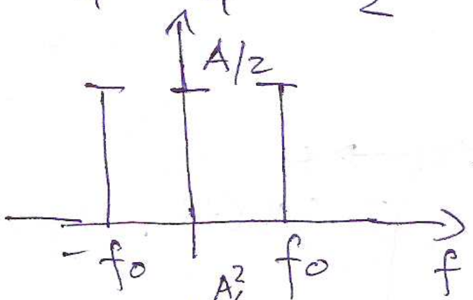
$a_1 = A$

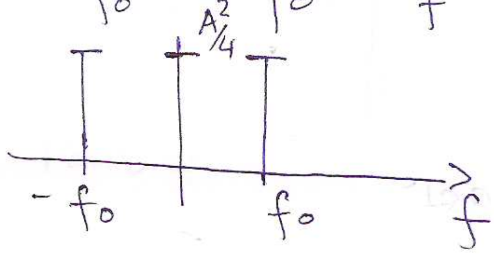
$b_1 = 0$

$a_0 = 0$

$|c_1| = \frac{1}{2}\sqrt{a_n^2 + b_n^2} = \frac{1}{2} A$

$$P_x = \sum_{n=-1}^{1} |c_n|^2 = \frac{A^2}{4} + \frac{A^2}{4} = \frac{A^2}{2}$$





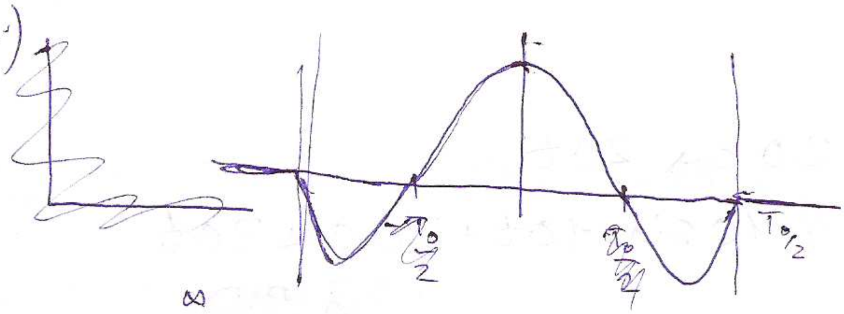
$\boxed{\text{Pr. 2.}}$ Určte kt. z nasledujúcich sig. je výkonový a kt. energetický

a.) $x(t) = \begin{cases} A\cos 2\pi f_0 t & -\frac{T_0}{2} \le t \le \frac{T_0}{2} \\ 0 & \text{inak} \end{cases}$

b.) $x(t) = \begin{cases} A \cdot e^{-at} & t \ge 0 \,;\ a \ge 0 \\ 0 & \end{cases}$ $\quad a = \text{konšt}$

c.) $x(t) = \cos(t) + 5\cos 2t$

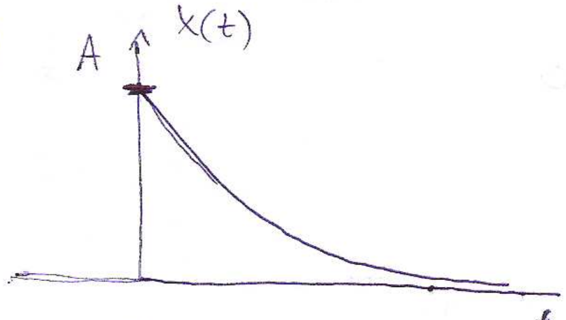
a.)



$$E_x = \int_{-\infty}^{\infty} (A \cos 2\pi f_0 t)^2 dt$$

$$E_x = A^2 \int_{-\frac{T_0}{2}}^{T_0/2} (\cos 2\pi f_0 t)^2 dt = \frac{A^2 T_0}{2}$$

b.)



$$E_x = \int_0^{\infty} A \cdot e^{-at} dt = A^2 \int_0^{\infty} (e^{-at})^2 dt = A^2 \int_0^{\infty} e^{-2at} dt =$$

$$A^2 \left[\frac{e^{-2at}}{-2a} \right]_0^{\infty} = \frac{A^2}{2a}$$

c.)

$$P_x = \sum_{n=-\infty}^{\infty} |c_n|^2 \qquad x(t) = \cos t + 5 \cos 2t$$

$a_1 = 1$

$b_1 = 0$

$a_2 = 5$ $\qquad c_1 = \frac{1}{2}$ $\qquad P_x = \sum_{-2}^{2} = 2 c_1^2 + c_2 = \cancel{\frac{1}{2}}$

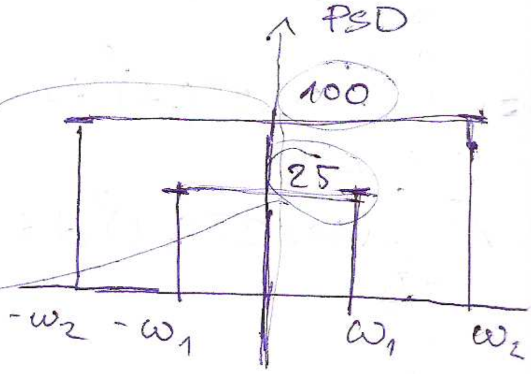
$a_2 = 5$

$b_2 = 0$ $\qquad c_2 = \frac{5}{2}$ $\qquad = 2(c_1^2 + c_2^2) = 2\left(\frac{1}{4} + \frac{25}{4}\right) = \underline{\underline{13}}$

$\boxed{\text{Pr. 3}}$ Vyp: P_x

a.) $x(t) = 10\cos 10t + 20\cos 20t$

b.) $x(t) = 10\cos 10t + i\,10\sin 10t + 20\cos 20t$



a.) $a_1 = 10 \qquad a_2 = 20$

$b_1 = 0 \qquad b_2 = 0$

$a_0 = 0$

$c_1 = 5 \qquad c_2 = 10$

$$P_x = \sum_{n=-2}^{2} |c_n|^2 = 2\left(c_1^2 + c_2^2\right) = 250$$

b.) $a_1 = 10 \qquad a_2 = 20$

$b_1 = 10 \qquad b_2 = 0$

$a_0 = 0$

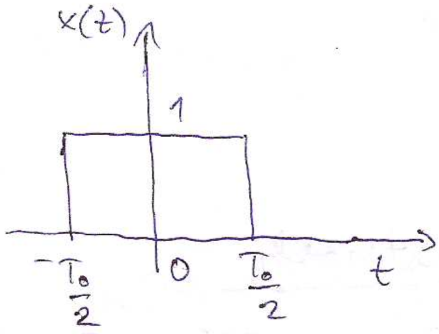
$c_1 = \frac{1}{2}\sqrt{10^2 + 10^2} = \sqrt{\frac{200}{2}} \qquad c_2 = 10$

$$P_x = \sum_{-2}^{2} \quad = 2\cdot\left(c_1^2 + c_2^2\right) = 300$$

13.10.08
cviko 2

$\boxed{\text{Pr.}}$ Pre signál $x(t) = \text{rect}\left|\frac{t}{T_0}\right|$

Zistite: a.) ESD
b.) E_x



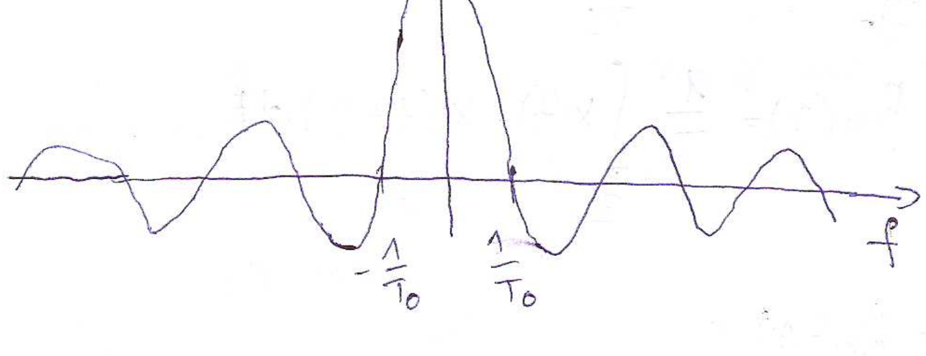
$$ESD = |X(f)|^2$$

$$x(t) \xleftrightarrow{FT} X(f)$$

$$X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi f t}\, dt = \int_{-T_0/2}^{T_0/2} e^{-j2\pi f t}\, dt = \left[\frac{e^{-j2\pi f t}}{-j2\pi f}\right]_{-T_0/2}^{T_0/2} =$$

$$= \frac{e^{-j2\pi f \frac{T_0}{2}} - e^{j2\pi f \frac{T_0}{2}}}{-j2\pi f} = T_0 \frac{\sin 2\pi f \frac{T_0}{2}}{\pi f \frac{T_0}{2}} = T_0 \sin(\pi f T_0) =$$

$$= T_0 \sin(f T_0)$$



ak $f = \frac{1}{T_0}$

$$ESD = T_0^2 \, \text{sinc}^2(f T_0)$$

b.) $$E_x = \int_{-\frac{T_0}{2}}^{T_0/2} x^2(t) = T_0$$

$$\boxed{\begin{aligned} si(x) &= \frac{\sin(x)}{x} \\ sinc(x) &= \frac{\sin(\pi x)}{\pi x} \end{aligned}}$$

Autokorelačná f-cia a jej vlastnosti

Autokorelácia - miera podobnosti signálu so sebou samým

a.) Energetické signály

pre reálne signály: $R_x(\tau) = \int_{-\infty}^{\infty} x(t) \cdot x(t \mp \tau)\, dt$

τ - časový posun

Vlastnosti: 1.) $R_x(\tau) = R_x(-\tau)$

2.) $R_x(0) \geq R_x(\tau)$

3.) $R_x(\tau) \xrightarrow{FT} \Psi_x(f)$ ESD

4.) $R_x(0) = E_x$

b.) Výkonové signály

pre reálne signály $R_x(\tau) = \frac{1}{T} \int_{-\frac{T}{2}}^{T/2} x(t) \cdot x(t+\tau)\, dt$

Vlastnosti: 1.) $R_x(\tau) = R_x(-\tau)$

2.) $R_x(0) \geq R_x(\tau)$

3.) $R_x(\tau) \xrightarrow{FT} G_x(f)$

4.) $R_x(0) = P_x$

Pozn. Autokorelačná f. periodických sig je tiež periodická f.

$\boxed{\text{Pr.}}$ Zistite, či z nasled. f-cií spĺňa podmienky autokorelačnej f-cie

a.) $R_x(\tau) = \begin{cases} 1 & \text{ak } -1 \leq \tau \leq 1 \\ 0 & \text{inak} \end{cases}$

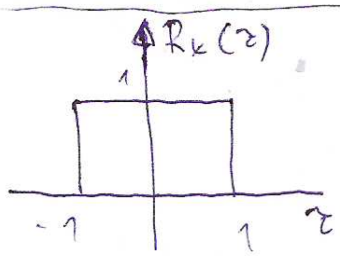
b.) $R_x(\tau) = \delta(\tau) + \sin 2\pi f \tau$

c.) $R_x(\tau) = e^{|\tau|}$

d.) $R_x(\tau) = \begin{cases} 1-|\tau| & -1 \leq \tau \leq 1 \\ 0 & \text{inak} \end{cases}$

---

a.)



1.) v

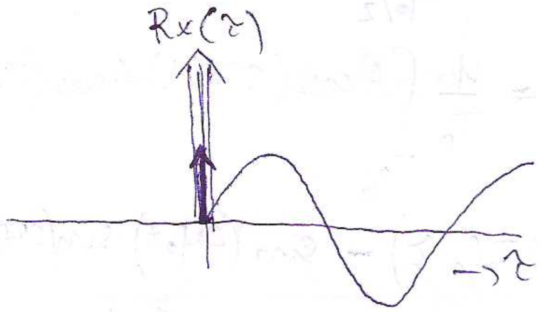
2.) v

3.) $R_x(\tau) \xrightarrow{FT} T \operatorname{sinc}(fT) \quad T=2$ //

$2 \operatorname{sinc}(2f) \neq ESD \Rightarrow$ nie

$R_X(\tau)$ nie je autokorelačná

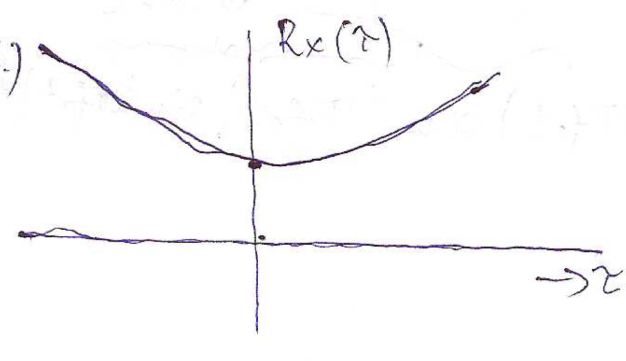
b.)



nie je autokorelačná nespĺňa

$R_x(\tau) \neq R_x(-\tau)$

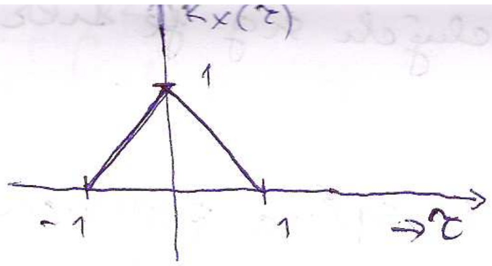
c.)



1.) V

2.) X

4.)



1.) v

2.) v

3.) $1-|\tau| \xleftrightarrow{FT} T\,\mathrm{sinc}^2(fT)$

4.) $R_x(0) = 1$

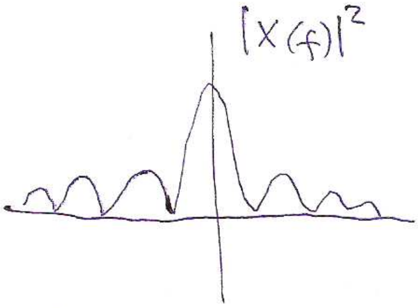
Pozn:

$$x(t) \longrightarrow X(f) \xrightarrow{\quad} |X(f)|^2 \quad \text{ESD}$$

$$\text{FT} \uparrow \downarrow \text{FT}$$

$$R_x(\tau)$$

z autokorelačnej f neurčím



$$x(t) = \begin{cases} 1 & -\frac{1}{2} \le t \le \frac{1}{2} \\ 0 & \end{cases} \Rightarrow E_x = 1 \qquad R_x(0) = E_x$$

$\boxed{\text{Pr.}}$ Nájdite autokorelačnú f-ciu pre

$$x(t) = A\cos(2\pi f_0 t)$$

b.) spočítajte výkon

a.)
$$R_x(\tau) = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\cdot x(t+\tau)\,dt = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} A\cos(2\pi f_0 t) A\cos(2\pi$$

$$= \frac{A^2}{T_0}\int_{-T_0/2}^{T_0/2} \cos(2\pi f_0 t)\cdot\left[\cos(2\pi f t_0)\cdot\cos(2\pi f_0 \tau) - \sin(2\pi f_0 t)\cdot\sin(2\pi f_0\right.$$

Ø A.2

$$= \frac{A^2}{T_0}\int_{-T_0/2}^{T_0/2} \cos^2(2\pi f_0 t)\cdot\cos(2\pi f_0 \tau) - \cos(2\pi f_0 t)\sin(2\pi f_0 t)\cdot\sin(2\pi f_0 t$$

$$= \cos(2\pi F_0 \tau) \frac{A^2}{T_0} \int_{-T_0/2}^{T_0/2} \cos^2(2\pi f_0 t)\, dt =$$

$$= \cos(2\pi f_0 \tau) \cdot \frac{A^2}{T_0} \int_{-T_0/2}^{T_0/2} \frac{1}{2}\Big[1 + \underbrace{\cos(4\pi f_0 t)}_{0}\Big] dt =$$

$$= \cos(2\pi f_0 \tau) \frac{A^2}{2T_0} \cdot T_0 = \frac{A^2}{2} \cos(2\pi f_0 \tau)$$

b) $P_x = R_x(0) = \frac{A^2}{2}$

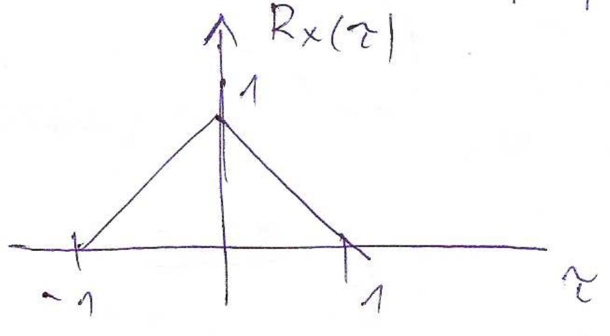
$\boxed{\text{Pr.}}$ Vieme že $X(f) = \operatorname{sinc}(f)$. Nájdite autokorel. funkciu signálu $R_x(\tau)$ signálu $x(t)$ cez

a.) ESD

b.) cez časovú oblasť

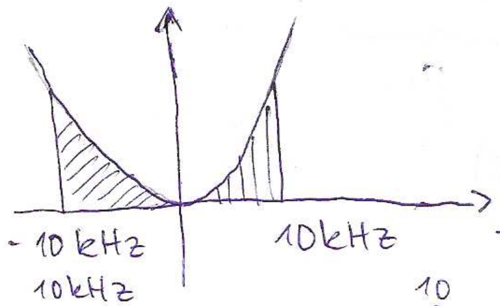
a.) $ESD = \Psi_x(f) = |X(f)|^2 = \operatorname{sinc}^2(f)$

$$\operatorname{sinc}^2(f) \longrightarrow \underbrace{\begin{cases} 1 - \frac{|\tau|}{T} & |\tau| \le T \\ 0 & |\tau| > T \end{cases}}_{R_x(\tau)} \qquad T = 1$$



Cviko 3
10.10.08

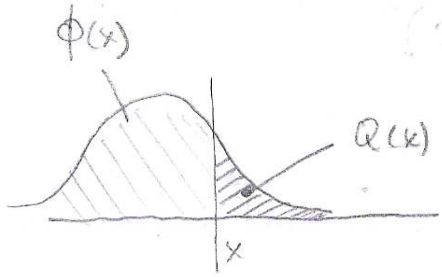
Pr. je daná 2-stranná ESD = $\Psi_x(f) = 10^{-6} f^2$.
Vypočítajte E_x v pásme $0 \div 10\,kHz$



$$E_x = \int_{-10kHz}^{10kHz} 10^{-6} f^2 df = 2\int_0^{10} 10^{-6} f^2 df = 2 \cdot 10^{-6}\left[\frac{f^3}{3}\right]_0^{10} = \frac{2}{3}10^{-6}$$

Pr. AWGN je daný disperziou $\sigma_0^2 = 10^{-7}$. Vypočítajte pravdepodobnosť, že napäťová úroveň šumu bude intervale: a.) $+100\,\mu V \div 500\,\mu V$

b.) $+1\,mV \div +3\,mV$



Tab. $N(0,1)$

$a = 0$

$\sigma_0^2 = 1$

ak $a \neq 0$, $\sigma^2 \neq 1$

$$\begin{matrix}\Phi'(x)\\ Q'(x)\end{matrix} = \begin{matrix}\Phi\\ Q\end{matrix}\left(\frac{x-a}{\sigma}\right)$$

Kalkulačka:
Excel

Pozn 1.

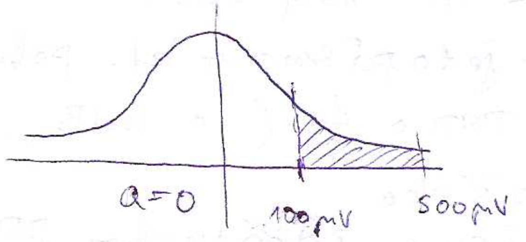
$$\mathrm{erfc}(x) = 2Q(x\sqrt{2})$$

$$Q(x) = \frac{1}{2}\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right)$$

Pozn 2

$$\Phi(x) + Q(x) = 1$$

$$\Phi(x) = 1 - \Phi(-x)$$



a.) $P(100\mu V < X < 500\mu V) =$

$= Q\left(\frac{100\cdot 10^{-6}}{\sqrt{10^{-7}}}\right) - Q\left(\frac{500\cdot 10^{-6}}{\sqrt{10^{-7}}}\right) = Q(0{,}316) - Q(1{,}58) =$

$= 0{,}3745 - 0{,}0571 = 0{,}3174$

b.) $P(1 < X < 3) = Q\left(\frac{1\cdot 10^{-3}}{\sqrt{10^{-7}}}\right) - Q\left(\frac{3\cdot 10^{-3}}{\sqrt{10^{-7}}}\right) =$

$= Q(3{,}16) - Q(9{,}48) = 0{,}00078 - \doteq 0{,}0008$

$Q(x) = \left(\frac{1}{9{,}48\sqrt{2\pi}}\right)\cdot e^{-\left(\frac{9{,}48^2}{2}\right)} = 0{,}042 \quad = 1{,}2827\cdot 10^{-21}$

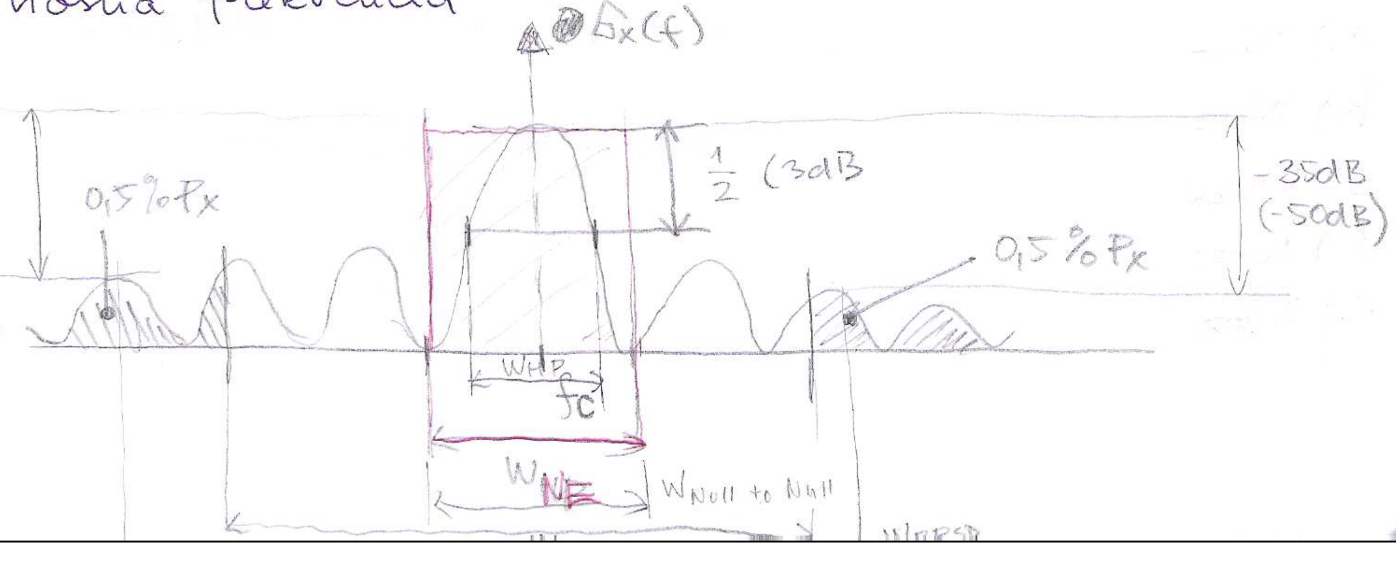
Frekvenčné pásma

obálka PSD digitálneho signálu má tvar: $si^2(x)$

$$PSD = T\left[\frac{\sin[\pi(f-f_c)\cdot T]}{\pi(f-f_c)\cdot T}\right]^2$$

T- doba impulzu

fc- nosná frekvencia



a.) W_{HP} - „polovičné" pásmo - HP - half power

- je to pásmo v kt. poklesne PSD o $\frac{1}{2}$ (o 3dB

b.) W_{NE} (Noise Equivalent) - je to šírka pásma ideálneho DP filtra (Nyquistovho), kt. má v pásme prepúšťania rovnaký ši... ako je rišk systému a na jeho výstupe je rovnaký výkon AWGN ako je výkon sig.

$$P_x = W_{NE} \cdot \max\{G_x(f)\} \Rightarrow W_{NE} = \frac{P_x}{\max\{G_x(f)\}}$$

c.) $W_{NULL\ TO\ NULL}$ - šírka pásma medzi hlavnými nulami okolo max. výkonu Najpoz.

d.) W_{FPC} - je to pásmo v kt. sa nachádza 99% P_x; symetricky t.j. 0,5% P_x je nad f_H a 0,5% P_x je pod f_D

e.) W_{BPSD} - definuje roslabenie mimo pásma (-35dB, -50dB)
(B = Bounded)

$\boxed{\text{Pr.}}$ Signál má daný PSD: $G_x(f) = 10^{-4}\left\{\frac{\sin[\pi(f-10^6)\cdot 10^{-4}]}{\pi(f-10^6)\cdot 10^{-4}}\right.$

max tam kde je Si (0)

a.) W_{HP}

b.) W_{NE}

c.) W_{N2N}

d.) W_{FPC}

e.) W_{BPSD}

a.)

$T = 10^{-4}$

$f_c = 10^6$

$G_x(f_c) = 10^{-4}$

$$G_x(f') = \frac{1}{2} G_x(f_c) = 10^{-4} \left\{ \frac{\sin[\pi(f' - 10^6) \cdot 10^{-4}]}{\pi(f' - 10^6) \cdot 10^{-4}} \right\}^2 =$$

$$= \frac{1}{2} 10^{-4}$$

$$\cancel{10^{-4}} \left\{ \frac{\sin[\pi(f' - 10^6) \cdot 10^{-4}]}{\pi(f' - 10^6) \cdot 10^{-4}} \right\}^2 = \frac{1}{2} \cdot \cancel{10^{-4}}$$

Sub: $\left(\frac{\sin(\alpha)}{\alpha} \right)^2 = \frac{1}{2}$ $\alpha = \pi(f' - 10^6) \cdot 10^{-4}$

$$\sin \alpha = \pm \frac{1}{\sqrt{2}} \cdot \alpha$$

$\alpha_i = \pm\sqrt{2} \cdot \sin \alpha_{i-1}$ $\alpha_0 = 1$

$\parallel$
1,3915

$$1{,}3915 = \pi(f' - 10^6) \cdot 10^{-4}$$

$$\frac{1{,}3915}{10^{-4}\pi} + 10^6 = f'$$

$$f' = 4{,}43\,kHz + 10^6$$

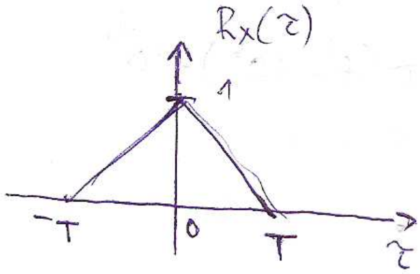
$$W_{HP} = 2 \cdot (\cancel{4{,}43\,kHz + 10^6}) \quad 8{,}86\,kHz$$

b.) $W_{NE} = \dfrac{P_x}{max\{G_x(f)\}}$

$P_x = R_x(0)$

PSD $\xrightarrow{FT}$ $R_x(\tau)$

~~PSD=~~ ~~PSD~~ $T\,\mathrm{sinc}^2 fT \longrightarrow \begin{cases} 1-\dfrac{|\tau|}{T} & ak\ |\tau| \le T \\ 0 & |\tau| > T \end{cases}$



$P_x = R_x(0) = 1$

$W_{NE} = \dfrac{1}{10^{-4}} = 10^4$