Cvičenie 2

30.9.10

Autokorelačná funkcia

1) Výkonové signály $R_x(\tau) = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\, x(t \pm \tau)\, dt$

(reálne per. sig.)

Vlastnosti:
1) $R_x(\tau) = R_x(-\tau)$ – párna
2) $R_x(0) \geq R_x(\tau) \Rightarrow$ v 0 max
3) $R_x(\tau) \xrightarrow{FT}$ PSD $G_x(f)$
4) $R_x(0) = P_x$ $\quad$ $R_x(\tau)$ je tiež periodická

2) Energetické signály $R_x(\tau) = \int_{-\infty}^{\infty} x(t)\, x(t+\tau)\, dt$

(reálne sig.)

Vlastnosti:
1) $R_x(\tau) = R_x(-\tau)$
2) $R_x(0) \geq R_x(\tau)$
3) $R_x(\tau) \xrightarrow{FT}$ ESD
4) $R_x(0) = E_x$

Vlastnosti ESD, PSD

a) ESD
1) $\Psi_x(f) \geq 0$ nezáporná f
2) $\Psi_x(f) = \Psi_x(-f)$ párna f.
3) $\Psi_x(f) \xrightarrow{FT^{-1}} R_x(\tau)$
4) $\int_{-\infty}^{\infty} \Psi_x(f)\, df = E_x$

b) PSD
1) $G_x(f) \geq 0$
2) $G_x(f) = G_x(-f)$
3) $G_x(f) \xrightarrow{FT^{-1}} R_x(\tau)$
4) $\int_{-\infty}^{\infty} G_x(f)\, df = P_x$

Pr 1. Zistite, ktoré funkcie môžu byť autokorelačné

1) $R_x(\tau) = \begin{cases} 1 & -1 \leq \tau \leq 1 \\ 0 & \text{inak} \end{cases}$

2) $R_x(\tau) = \delta(\tau) + \sin 2\pi f_0 \tau$

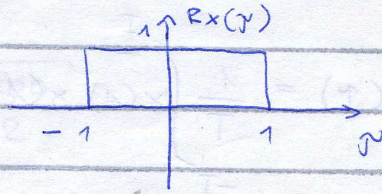
3) $R_x(\tau) = e^{|\tau|}$

4) $R_x(\tau) = \begin{cases} 1 - |\tau| & -1 \leq \tau \leq 1 \\ 0 & \text{inak} \end{cases}$

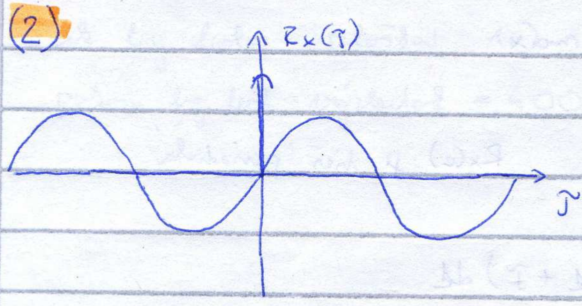
$R_x(\tau) \xrightarrow{FT} 2.\,\text{sinc}\,2f \neq ESD$

energetický sig.

1) párna
2) splnené
3) // nie je



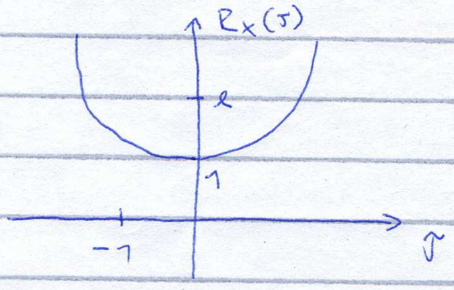
(2)



1) nie je párna

výkonový sig.

(3)

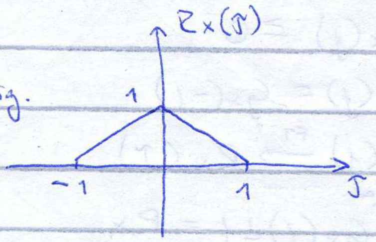


1) párna
2) v nule je min. – nie je autokorelačná

energetický sig.

(4)

energ. sig.



1) ✓
2) ✓
3) $R_x(\tau) \xrightarrow{FT} \text{sinc}^2 f = ESD$
je nezáporná
4) $R_x(0) = 1$

$$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$x(t) \xrightarrow{FT} X(f) \to |X(f)|^2 = ESD \xrightarrow{FT^{-1}} R_x(\tau)$$

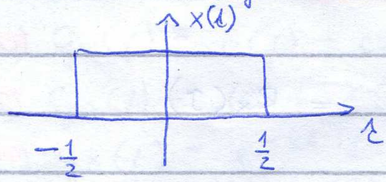
$$X(f) \neq \sqrt{ESD} \leftarrow |X(f)|^2 = ESD \xleftarrow{FT} R_x(\tau)$$

kvôli abs. hodnote

nedá sa z autokorelácie jednoznačne určiť pôvodný signál

z $R_x(\tau)$ určiť $x(t)$!

Je autokorelačná funkcia



$$\Rightarrow E_x = 1$$

Pr 2 a) Určite $R_x(\tau)$ pre sig. $x(t) = A\cos 2\pi f_0 t$

b) Určite P_x

(a) $$R_x(\tau) = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t)\,x(t+\tau)\,dt = \frac{A^2}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \cos 2\pi f_0 t \cdot \cos 2\pi f_0 (t+\tau)\,dt =$$

$$= \frac{A^2}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \cos 2\pi f_0 t \cdot \left[\cos 2\pi f_0 t \cos 2\pi f_0 \tau - \sin 2\pi f_0 t \sin 2\pi f_0 \tau\right] dt$$

$$= \frac{A^3}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \left(\cos^2 2\pi f_0 t \cos 2\pi f_0 \tau - \cos 2\pi f_0 t \sin 2\pi f_0 t \sin 2\pi f_0 \tau\right) dt$$

$$= \frac{A^3}{T_0}\left(\frac{T_0}{2}\cos 2\pi f_0 \tau - 0\right) = \frac{A^2 \cos 2\pi f_0 \tau}{2}$$

(b) $$P_x = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} A^2 \cos^2 2\pi f_0 t = \frac{A^2}{2}$$

$$R_x(0) = P_x$$

$$R_x(0) = \frac{A^2 \cos 2\pi \cdot f_0 \cdot \emptyset}{2} = \frac{A^2}{2}$$

Pr 3. Vieme spektrum, ktoré je $X(f) = \mathrm{sinc}(f)$. Nájdite $R_x(\tau)$ signálu $x(t)$ cez a) ESD

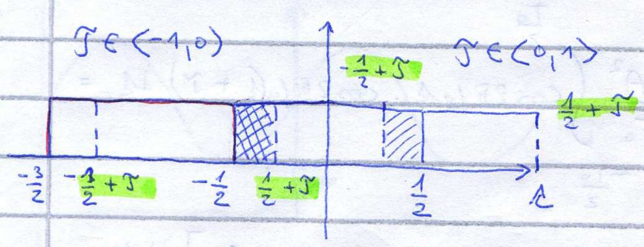
b) časovú oblasť

(a) $|X(f)|^2 = \mathrm{sinc}(f)^2 = ESD$

$$\mathrm{sinc}^2(f) \longrightarrow \begin{cases} 1-|\tau| & |\tau| \le 1 \\ 0 & \end{cases} = R_x(\tau)$$

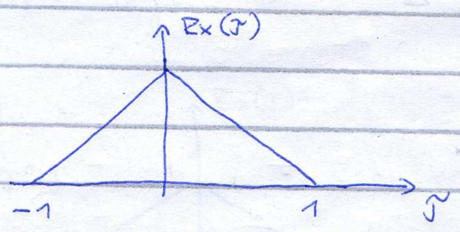
(b) $X(f) \xrightarrow{FT^{-1}} \mathrm{rect}\, t = x(t)$

$T = 1$



$$R_x(\tau) = \int_{-\frac{1}{2}}^{\frac{1}{2}+\tau} 1 \cdot 1 \, dt = \frac{1}{2} + \tau + \frac{1}{2} = \underline{1 + \tau}$$

$$R_x(\tau) = \int_{-\frac{1}{2}+\tau}^{\frac{1}{2}} 1 \, dt = \frac{1}{2} + \frac{1}{2} - \tau = \underline{1 - \tau}$$



Př. 4: Dvojstranná ESD má tvar: $\Psi_x(f) = 10^{-6} f^2$

Vyp Ex v pásmu $0 \div 10$ kHz

$$E_x = \int_{-\infty}^{\infty} 10^{-6} f^2 \, df = 2 \int_0^{10^4} 10^{-6} f^2 \, df = 2 \cdot 10^{-6} \left[\frac{f^3}{3} \right]_0^{10^4} =$$

$$= 2 \cdot 10^{-6} \left[\frac{10^{12}}{3} \right] = 2 \cdot \frac{10^6}{3}$$

Př. 5. Zjistěte kt. z následujících f. může být PSD (ESD) nějakého sig.

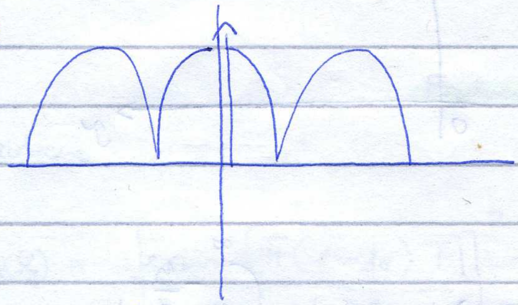
a) $G_x(f) = \delta(f) + \cos^2 2\pi f$

b) $G_x(f) = 10 + \delta(f-10)$

c) $G_x(f) = e^{-2\pi |f-10|}$

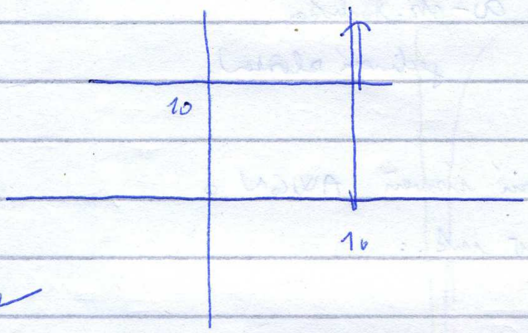
d) $G_x(f) = e^{-2\pi (f^2-10)}$

a)



$$P_x = \int_{-\infty}^{+\infty} [\delta(f) + \cos^2 2\pi f) \, df = 1 + \infty = \infty \neq P_x$$

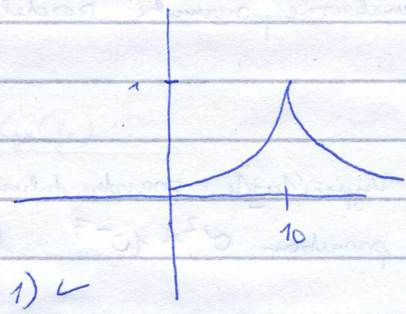
b)



1) ✓

2) ×

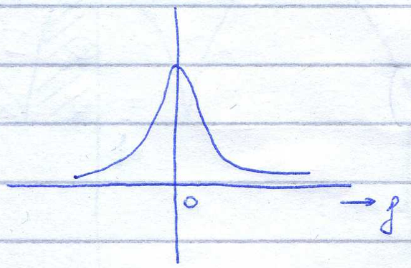
c)



1) ✓

2) ×

d)



je PSD