Fourierov rad

$$x(\lambda) = \tfrac{1}{2}a_0 + a_1 \cos \lambda + a_2 \cos 2\lambda + a_3 \cos 3\lambda + \cdots + b_1 \sin \lambda + b_2 \sin 2\lambda + b_3 \sin 3\lambda + \cdots \qquad \text{(A.1)}$$

exponenciálny tvar:

$$x(t) = \frac{a_0}{2} + \frac{1}{2}\sum_{n=1}^{\infty} \left[(a_n - jb_n)e^{j2\pi nf_0 t} + (a_n + jb_n)e^{-j2\pi nf_0 t}\right] \qquad \text{(A.11)}$$

komplexné koeficienty:

$$c_n = \begin{cases} \tfrac{1}{2}(a_n - jb_n) & \text{for } n > 0 \\ \dfrac{a_0}{2} & \text{for } n = 0 \\ \tfrac{1}{2}(a_n + jb_n) & \text{for } n < 0 \end{cases} \qquad \text{(A.12)}$$

$$c_n = |c_n| e^{j\theta_n} \qquad \text{(A.17)}$$

$$c_{-n} = |c_n| e^{-j\theta_n} \qquad \text{(A.18)}$$

potom:

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi nf_0 t} \qquad \text{(A.13)}$$

$$c_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)e^{-j2\pi nf_0 t}\,dt \qquad \text{(A.14)}$$

nakoľko:

$$\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} e^{j(n-m)2\pi f_0 t}\,dt = \delta_{nm} = \begin{cases} 1 & \text{for } n = m \\ 0 & \text{for } n \neq m \end{cases} \qquad \text{(A.15)}$$

$$\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)e^{-j2\pi mf_0 t}\,dt = \sum_{n=-\infty}^{\infty} c_n \delta_{nm} = c_m \qquad \text{(A.16)}$$

Fourierova transformácia

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\,dt \qquad \text{(A.26)}$$

$$x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}\,dt \qquad \text{(A.27)}$$

$x(t)$	$X(f)$
1. $\delta(t)$	1
2. 1	$\delta(f)$
3. $\cos 2\pi f_0 t$	$\frac{1}{2}[\delta(f-f_0) + \delta(f+f_0)]$
4. $\sin 2\pi f_0 t$	$\frac{1}{2j}[\delta(f-f_0) - \delta(f+f_0)]$
5. $\delta(t-t_0)$	$\exp(-j2\pi f t_0)$
6. $\exp(j2\pi f_0 t)$	$\delta(f-f_0)$
7. $\exp(-a\lvert t\rvert)$, $a>0$	$\dfrac{2a}{a^2 + (2\pi f)^2}$
8. $\exp\left[-\pi\left(\frac{t}{T}\right)^2\right]$	$T\exp[-\pi(fT)^2]$
9. $u(t) = \begin{cases} 1 & \text{for } t>0 \\ 0 & \text{for } t<0 \end{cases}$	$\frac{1}{2}\delta(f) + \dfrac{1}{j2\pi f}$
10. $\exp(-at)\,u(t)$, $a>0$	$\dfrac{1}{a + j2\pi f}$
11. $t\exp(-at)\,u(t)$, $a>0$	$\dfrac{1}{(a + j2\pi f)^2}$
12. $\mathrm{rect}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}\,fT$
13. $\cos 2\pi f_0 t\left[\mathrm{rect}\left(\frac{t}{T}\right)\right]$	$\frac{T}{2}[\mathrm{sinc}(f-f_0)T + \mathrm{sinc}(f+f_0)T]$
14. $W\,\mathrm{sinc}\,Wt$	$\mathrm{rect}\left(\frac{f}{W}\right)$
15. $\begin{cases} 1 - \frac{\lvert t\rvert}{T} & \text{for } \lvert t\rvert \le T \\ 0 & \text{for } \lvert t\rvert > T \end{cases}$	$T\,\mathrm{sinc}^2 fT$
16. $\sum_{m=-\infty}^{\infty} \delta(t - mT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$
$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{f}{a}\right)$
$x(t-t_0)$	$X(f)\exp(-j2\pi f t_0)$
$x(t)\exp(j2\pi f_0 t)$	$X(f-f_0)$
$\dfrac{d^n x}{dt^n}$	$(j2\pi f)^n X(f)$
$(-j2\pi t)^n(x(t))$	$\dfrac{d^n X}{df^n}$
$\int_{-\infty}^{t} x(\tau)d\tau$	$\dfrac{1}{j2\pi f}X(f) + \frac{1}{2}X(0)\delta(f)$
$x_1(t) * x_2(t)$	$X_1(f)X_2(f)$
$x_1(t)x_2(t)$	$X_1(f) * X_2(f)$

často používané vzťahy:

$\mathrm{rect}\,(f/2W) = 1$ for $-W < f < W$, 0 for $\lvert f\rvert > W$, and $\mathrm{sinc}\,x = (\sin \pi x)/\pi x$.

$$\cos x \cos y = \tfrac{1}{2}\cos(x+y) + \tfrac{1}{2}\cos(x-y) \qquad \text{(D.1)}$$

$$\sin x \sin y = -\tfrac{1}{2}\cos(x+y) + \tfrac{1}{2}\cos(x-y) \qquad \text{(D.2)}$$

$$\sin x \cos y = \tfrac{1}{2}\sin(x+y) + \tfrac{1}{2}\sin(x-y) \qquad \text{(D.3)}$$

$$\cos x \sin y = \tfrac{1}{2}\sin(x+y) - \tfrac{1}{2}\sin(x-y) \qquad \text{(D.4)}$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y \qquad \text{(D.5)}$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y \qquad \text{(D.6)}$$

$$\cos^2 x = \tfrac{1}{2}(1 + \cos 2x) \qquad \text{(D.7)}$$

$$\sin^2 x = \tfrac{1}{2}(1 - \cos 2x) \qquad \text{(D.8)}$$

$$\sin x \cos x = \tfrac{1}{2}\sin 2x \qquad \text{(D.9)}$$

$$\sin x + \sin y = 2\sin\tfrac{1}{2}(x+y)\cos\tfrac{1}{2}(x-y) \qquad \text{(D.10)}$$

$$\sin x - \sin y = 2\cos\tfrac{1}{2}(x+y)\sin\tfrac{1}{2}(x-y) \qquad \text{(D.11)}$$

$$\cos x + \cos y = 2\cos\tfrac{1}{2}(x+y)\cos\tfrac{1}{2}(x-y) \qquad \text{(D.12)}$$

$$\cos x - \cos y = -2\sin\tfrac{1}{2}(x+y)\sin\tfrac{1}{2}(x-y) \qquad \text{(D.13)}$$

$$\sin x = \frac{e^{jx} - e^{-jx}}{2j} \qquad \text{(D.14)}$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2} \qquad \text{(D.15)}$$

$$\left.\begin{aligned} \int_{-\pi}^{\pi} \sin m\lambda\, d\lambda &= 0 \\ \int_{-\pi}^{\pi} \cos m\lambda\, d\lambda &= 0 \\ \int_{-\pi}^{\pi} \sin m\lambda \cos n\lambda\, d\lambda &= 0 \end{aligned}\right\} \text{where } m \text{ and } n \text{ are any integers} \qquad \text{(A.2)}$$

$$\left.\begin{aligned} \int_{-\pi}^{\pi} \sin m\lambda \sin n\lambda\, d\lambda &= 0 \\ \int_{-\pi}^{\pi} \cos m\lambda \cos n\lambda\, d\lambda &= 0 \end{aligned}\right\} \text{for } m \neq n \qquad \text{(A.3)}$$

$$\left.\begin{aligned} \int_{-\pi}^{\pi} (\sin m\lambda)^2\, d\lambda &= \pi \\ \int_{-\pi}^{\pi} (\cos m\lambda)^2\, d\lambda &= \pi \end{aligned}\right\} \text{for } m = n \qquad \text{(A.4)}$$

MARTIN

P_B: $Q\left(\sqrt{\frac{2E_B}{N_0}}\right)$

$Q\left(\sqrt{\frac{E_B}{N_0}}\right)$

$\frac{1}{2}\exp\left(-\frac{1}{2}\frac{E_B}{N_0}\right)$

$\frac{1}{2}\exp\left(-\frac{E_B}{N_0}\right)$

$2Q\left(\sqrt{\frac{2E_B}{N_0}}\left[1-Q\left(\sqrt{\frac{2E_B}{N_0}}\right)\right]\right)$

$P_E(M)$: $2Q\left(\sqrt{\frac{2E_S}{N_0}}\sin\frac{\pi}{M}\right)$

$(M-1)\,Q\left(\sqrt{\frac{E_S}{N_0}}\right)$

$\frac{1}{M}\exp\left(-\frac{E_S}{N_0}\right)\sum_{j=2}^{M}(-1)^j\binom{M}{j}\exp\left(\frac{E_S}{jN_0}\right)$

$x>3: Q(x)=\left(\frac{1}{x\sqrt{2\pi}}\right)\exp\left(-\frac{x^2}{2}\right)$

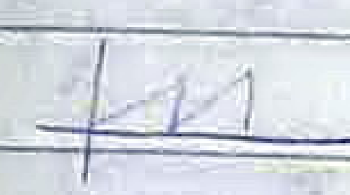
$Q(x) = \int_x^\infty (1/\sqrt{2\pi})\exp(-u^2/2)\,du$

	$Q(x)$									
x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2168	0.2148
0.8	0.2169	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002

Martin Rakús BG12 · cvičí v BG15

c MSK 21.2.

započítaná 12. týždeň za 30 bodov → veci na cvičku

Delenie signálov
- → stochastické
- → Deterministické
  - Periodické (FR – spektrum čiarové) — Harmonické: cos x, sin x; neharmonické 
  - Neperiodické (FT – spektrum spojité)
  - Kvázi periodické ($\omega_k \neq k\Omega$) · existuje aspoň ω bez $\frac{\omega}{k\omega}$ je iracionál. číslo

Delenie signálov:

1. podľa vývoja signálov v čase (alebo ich hodnôt)

stochastické signály
- nestacionárne
- stacionárne
- ergodické

2. podľa sústavy normovaný
- energie E_x
- výkonu P_x

Pozn: ak normy P_x

$$P_x = UI = I^2R = \frac{U^2}{R} = x^2/R = x^2(t)$$

ak $R = 1\Omega$

2.1 výkonové signály $1 < P_x < \infty$ [W]

komplex. sig $\bar{x}(t)$

reálny sig. $P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |\bar{x}(t)|^2 dt$

Patria sem
- periodické signály
- stochastické sig.
- kvázi. periodické sig.

2. Energetické sig. $0 < E_x < \infty$ [J]

komplex. sig. $E_x = \lim_{T\to\infty} \int_{-T/2}^{T/2} |x(t)|^2 \, dx$

reálny sig. $= \int_{-\infty}^{\infty} x^2(t) \, dt$

Parsevalova rovnica (reálny sig.)

a) $E_x = \int_{-\infty}^{\infty} x^2(t)\, dt = \int |X|^2 df$; $X(f)$ je vzťahom signálu

$x(t) \overset{F}{\rightleftarrows} X(f)$ $\qquad |X(f)|^2 = \Psi_x(f)$

$E_x = \int_{-\infty}^{\infty} \Psi_x(f) df = 2\int_{0}^{\infty} \Psi_x(f) df$

b) $P_x = \frac{1}{T}\int_{-T/2}^{T/2} x^2(t) dt = \sum_{n=-\infty}^{\infty} |c_n|^2$

G_x komplex. Fourierova koef. (výkonové spek.) $\qquad x(t) \to X(f)$

$G(x)(f) = 4\sum_{n=-\infty}^{\infty} |c_n|^2 \delta(f - f_0) \qquad G_x(f) = \sum_{n=-\infty}^{\infty} |c_n|^2 \delta(f - nf_0)$

$P_x = \int_{-\infty}^{\infty} G(x)(f) = 2\int_{0}^{\infty} G(x)(f) df \qquad G_x(f) = |X(f)|^2 = PSD \; [\frac{W}{Hz}]$

dvojstranné $\qquad$ jednostranné

7. $x(t) = A\cos 2\pi f t \qquad P_x = ?$

a) v čas. oblasti

b) v frekvenčnej obl.

$\cancel{c_n = \frac{1}{T}\int_{-\infty}^{\infty} (A\cos(\omega t)) e^{-jn\omega t} dt = A\int_{-\infty}^{\infty}}$ $\qquad T = 2\pi$

$\frac{1}{T}\int_{-\infty}^{\infty} \quad = P_x = \frac{1}{T}\int_{-\infty}^{\infty} A^2\cos^2 2\pi f t = \frac{A^2}{2\pi}\int_{-\infty}^{\infty} \frac{1}{2}(1 + \cos 4\pi f t) = \frac{A}{4\pi}\int_{-\infty}^{\infty} 1 dt + \int_{-\infty}^{\infty} \cos 4\pi f t$

$= \frac{A^2}{4\pi}(2\pi) = \frac{A^2}{2}$

$$\boxed{P_x = \frac{A^2}{2}}$$

- výkon harmonického signálu periodického signálu m

cez frekvenční oblast — c MSK0 21.2.

$x(t) = A\cos 2\pi f t$

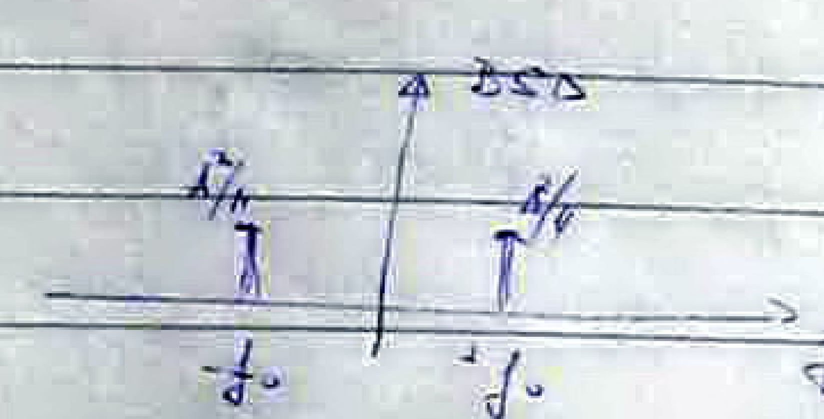
$$P_x = \sum_{-\infty}^{\infty} |c_n|^2$$

$$\Rightarrow c_n = \begin{cases} \frac{1}{2}(a_n - jb_n) \\ \frac{a_0}{2} \\ \frac{1}{2}(a_n + jb_n) \end{cases}$$

keďže signál je párny, čiže b_n sú nulové, koeficient a_0 je v tomto prípade 0

$$= a_1 = A \Rightarrow c_1 = c_{-1} = \frac{A}{2}$$

$$P_x = (c_1)^2 + (c_{-1})^2 = \frac{A^2}{2^2} + \frac{A^2}{2^2} = \frac{2A^2}{4} = \frac{A^2}{2}$$



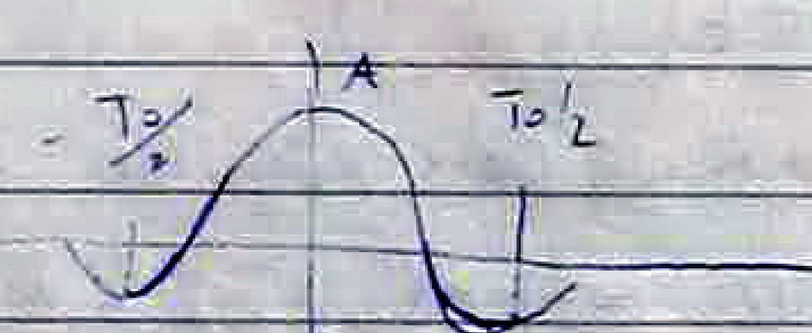
- Pr 2 Určite, či z nasledujúcich signálov je výkonový, al. energetický, teda vypočítať príslušnú veličinu

a) $x(t) = \begin{cases} A\cos 2\pi f_0 t \\ \text{inak} \end{cases}$

b) $x(t) = \begin{cases} A\exp(-at) & t>0,\ a>0 \\ 0 \text{ inak} \end{cases}$

c) $x(t) = \cos t + 5\cos 2t$

- energetické signály sú periodické signály

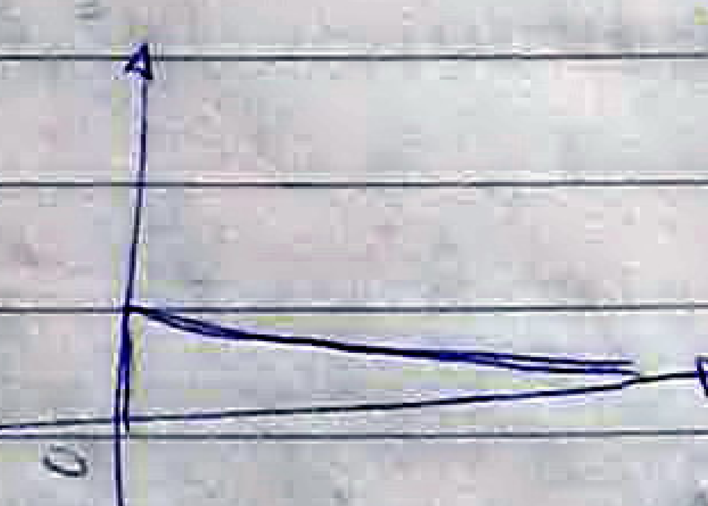


a) energet. signál

$$E_x = \int_{-\infty}^{\infty} [A\cos(2\pi f_0 t)]^2 dt = \int_{-T/2}^{T/2} [A\cos(2\pi f_0 t)]^2 dt = \frac{A^2 T_0}{2}$$

b)

$$E_x = \int_0^{\infty} [A\exp(-at)]^2 dt = A^2 \int_0^{\infty} e^{-2at} dt$$



$$= A^2 \left[\frac{e^{-2at}}{-2a} \right]_0^{\infty} =$$

$$\frac{A^2}{-2a}[0-1] = \frac{A^2}{2a}$$

c) $\cos t + 5\cos 2t$ periodický → výkonový signál →

$$P_x = \sum_{n=-\infty}^{\infty} |c_n|^2$$

$$c_n = \begin{cases} \frac{1}{2}(a_n - jb_n) \\ \frac{a_0}{2} \\ \frac{1}{2}(a_n + jb_n) \end{cases}$$

Kováč -3-

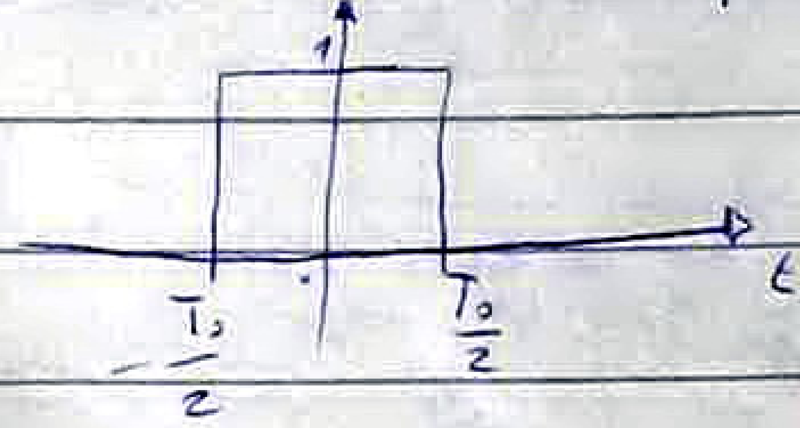
MSK 28.2.

$\rightarrow$ $b_n = 0$ $\quad a_1 = 1$ $\quad c_1 = c_{-1} = \frac{1}{2}$ $\quad P_x = \left(\frac{5}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{5}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

$A_0 = 0$ $\quad a_2 = 5$ $\quad c_2 = c_{-2} = \frac{5}{2}$ $\quad = \frac{25}{4} + \frac{1}{4} = \frac{26}{4} \cdot \frac{13}{2} + \frac{13}{2}$

$= \underline{\underline{13}}$

• Pr. 3 $\quad x(t) = \mathrm{rect}\left[\frac{t}{T_0}\right]$



Vyp. a) E_x

b) ESD

$$E_x = \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} 1\,dt = \left[t\right]_{-\frac{T_0}{2}}^{\frac{T_0}{2}} = \underline{\underline{T_0}}$$

$$\mathrm{rect}\,\frac{t}{T} \Leftrightarrow T\,\mathrm{sinc}\, fT$$

$\mathrm{si}\, x = \frac{\sin x}{x}$

$\mathrm{sinc}\, x = \frac{\sin(\pi x)}{(\pi x)}$

$$ESD = |X(f)|^2 = (T\,\mathrm{sinc}\, fT)^2$$

cMSK 28.2.

Pr. $\quad P(x) = ?$

$x(t) = 10\cos 10t + 10\sin 10t + 20\cos 20t$

$$P(x) = \frac{1}{T}\int_{-\infty}^{\infty} x^2\,dt = \frac{1}{T}\int 100\cos^2 10t + 100\sin^2 10t + 400\cos^2 20t\,dt = \frac{100}{T}\int_{-\infty}^{\infty} \cos^2 10t + \sin^2 10t + 4\cos^2 20t$$

$=$

možno počítať cez $\quad P(x) = \sum |c_n|^2$

alebo $\quad P_x = \frac{A_1^2}{2} + \frac{A_2^2}{2} + \frac{A_3^2}{2} = 50 + 50 + 200 = 300\,W$

Autokorelačné funkcie

Energet. sig.

$$R_x(\tau) = \int_{-\infty}^{\infty} x(t)\,x(t+\tau)\,dt$$

Výkonové signály

$$R_x(\tau) = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\,x(t-\tau)\,dt$$

Vlastnosti

1) $R_x(\tau) = R_x(-\tau)$ $\quad$ párna fcia $\quad R_x(\tau) = R_x(-\tau)$

2) $R_x(\tau) \leq R_x(0)$ $\quad R_x(\tau) \leq R_x(0)$

3) $R_x(\tau) \xleftrightarrow{FT} \Psi_x(f)$ $\quad$ Wiener-Chinčinove vzťahy $\quad R_x(\tau) \xleftrightarrow{FR} G_x$

4) $R_x(0) = E_x$ $\quad R_x(0) = P_x$

• Pr. Zistite, kt. funkcia je autokorelačná

a) $R_x(\tau) < \begin{matrix} 1 & -1 \le \tau \le 1 \\ 0 & \text{inak} \end{matrix}$

b) $R_x(\tau) = \delta(\tau) + \sin 2\pi f_0 \tau$

c) $R_x(\tau) = e^{|\tau|}$

d) $R_x < \begin{matrix} 1 - |\tau| & -1 \le \tau \le 1 \\ 0 \ \text{inak} & \text{inak} \end{matrix}$

d) 

test na párnosť $\nearrow R_x(\tau) \neq R_x(-\tau)$

<u>a) párna</u> b) nepárna c) ~~nep~~ párna <u>d) párna</u>

test na 2. vlastnosť

a) platí d) platí c) neplatí

test na 3. vlastnosť

a) neplatí

$\xrightarrow{FT}$ $T\,si(f)$ + ... do nachotácie aj záp. hodnoty

d) $\xleftrightarrow{FT} \left\{ \begin{matrix} 1 - \frac{|t|}{T} \le T \\ 0 \quad |t| > T \end{matrix} \right. = \operatorname{sinc}^2 fT$

4. vlastnosť

• $x(t) \xrightarrow{FT} X(f)$

$\downarrow$

$|X(f)|^2 = \Psi(f) \cdot 1 \xrightarrow{FT^{-1}} R_x(\tau)$

~~$R_x(\tau) \xrightarrow{FT} \Psi(f)$~~

$R_x(\tau) \xrightarrow{FT} \Psi(f) = |X(f)|^2$

$\downarrow$

~~$|X(f)|$~~ $\quad \sqrt{|X(f)|^2} = X(f) = ~~X~~(f) \rightarrow x(t)$

• Pr. $x(t) = A\cos 2\pi f_0 t$

a) $R_x(\tau) = ?$

b) $P_x = ?$

$$R_x(\tau) = \int_{-T/2}^{T/2} A\cos 2\pi f_0 t \cdot A\cos(2\pi f_0 t + \tau)\, dt =$$

$$= \frac{A^2}{T} \int_{-T/2}^{T/2} \frac{1}{2}\cos(2\pi f_0 + 2\pi f_0 + \tau) + \frac{1}{2}\cos(2\pi f_0 - 2\pi f_0 - \tau) =$$

$$= \frac{1}{2}\frac{A^2}{2T} \int_{-T/2}^{T/2} \cos(4\pi f_0 + \tau) + \cos(-\tau) =$$

$$= \frac{A^2}{2\pi} \left[\frac{\sin(4\pi f_0 t + \tau)}{4\pi f_0} + \cos(-\tau) \right]_{-T/2}^{T/2}$$

DOKONČIT

výsl. $\left[\frac{A^2}{2} \cos 2\pi f \tau \right]$

- Př. Víem spektrum $X(f) = \mathrm{sinc}(f)$

  Vyp: $R_x(\tau)$ cez:

  a) ESD

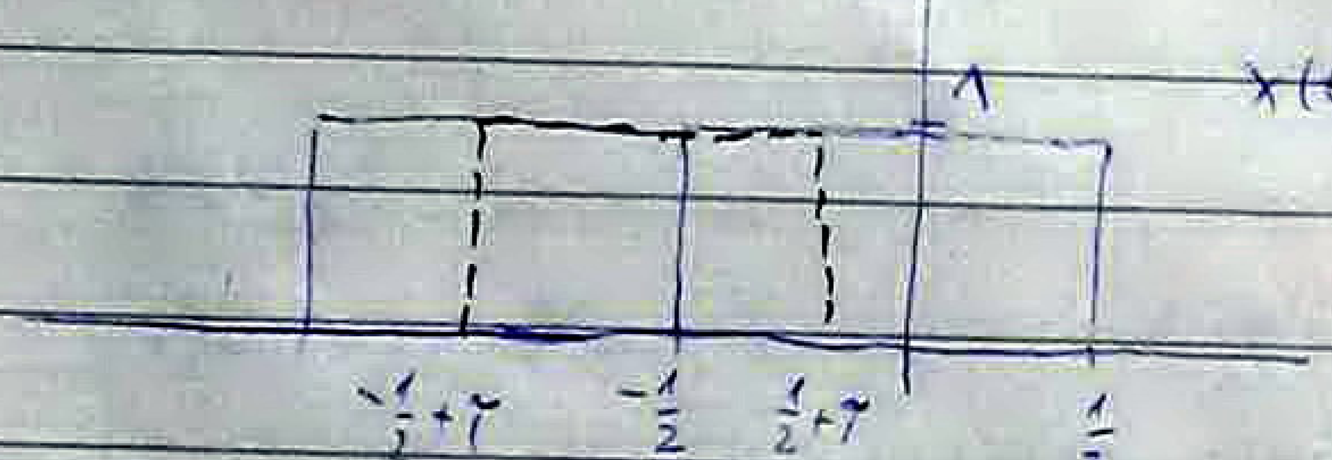
  b) čas. obl.

a) $|X(f)|^2 = \mathrm{sinc}^2(f)$

$$\updownarrow FT^{-1}$$

$$R_x(\tau) = \begin{cases} 1 - \frac{|\tau|}{1} & \tau \le 1 \\ 0 & \text{inak} \end{cases}$$

b) $X(f) \xleftrightarrow{FT^{-1}} \mathrm{rect}\left(\frac{t}{T}\right)$ $T = 1$



$$R_x(\tau) = \frac{1}{T} \int_{-\infty}^{\infty} x(t)\, x(t-\tau)\, dt =$$

$$= \int_{-\infty}^{0} x(t)\, x(t-\tau)\, dt + \int_{0}^{\infty} x(t) x(t-\tau)\, dt$$

~~pro~~ $\tau = \langle -1, 0 \rangle$

$$R_x(\tau) = \int_{-\frac{1}{2}}^{\frac{1}{2}+\tau} 1 \cdot 1 \, dt = \frac{1}{2} + \tau + \frac{1}{2} = 1 + \tau$$

$\tau = \langle 0, 1 \rangle$

$$R_x(\tau) = \int_{0}^{\frac{1}{2}} 1 \cdot 1 \, dt = 1 - \tau$$

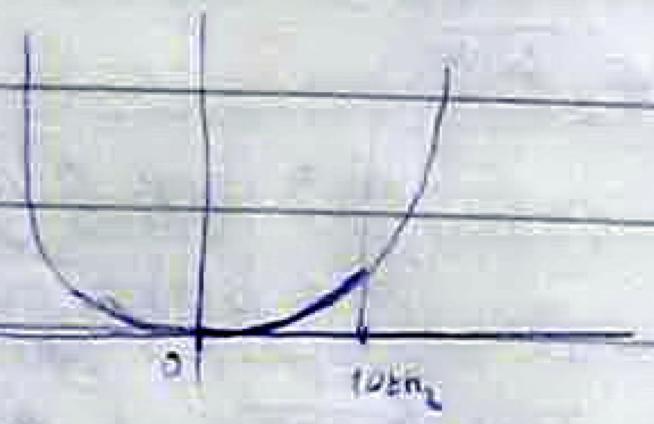
Vlastnosti ESD, PSD

ESD	PSD
1, $\Psi_x(f) \ge 0$	$G_x(f) \ge 0$
2, $\Psi_x(f) = \Psi_x(f)$	$G_x(f) = G_x(-f)$
3, $\int_{-\infty}^{\infty} \Psi_x(f)\, df = E_x$	$\int_{-\infty}^{\infty} G_x(f)\, df = P_x$
4, $\Psi_x(f) \xrightarrow{FT^{-1}} R_x(\tau)$	$G_x(f) \xrightarrow{FT^{-1}} R_x(\tau)$

• Pr. $\Psi_x(f) = 10^{-6} f^2$

$E_x = ?$ v pásme $0 \div 10 kHz$

ak $\Psi_x(f)$ je dvojstranná



$$E_x = 2\int_0^{10k} \Psi(f)\,df = 2\int_0^{10k} 10^{-6} f^2\,df = 2\cdot 10^{-6}\left[\frac{f^3}{3}\right] =$$

dvojstranná

$$= \frac{2}{3}\cdot 10^{6}\ \mathrm{J}$$

• Pr. Zistiť, či nasled. f. môže byť PSD náhodného sig.

a) $G_x(f) = \delta(f) + \cos^2 2\pi f$

b) $G_x(f) = e^{(-2\pi|f-10|)}$

c) $G_x(f) = 10 + \delta(f-10)$

d) $G_x(f) = e^{[-2\pi(f^2-10)]}$

nie je PSD [3 vlastnosti]

AWGN

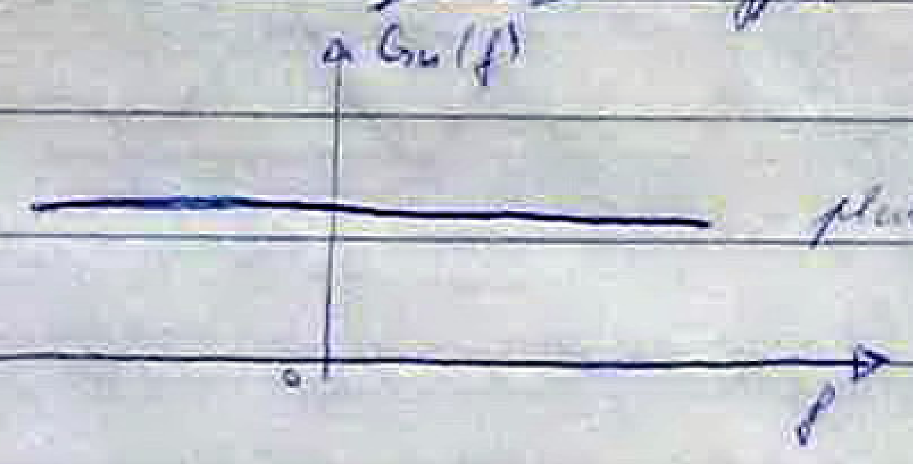
(Aditive White Gaussian Noise)

tepelný šum – šum spôsobený pohybom elektrónov

– dá sa popísať Gaussovským šumom s $a = 0$ σ^2

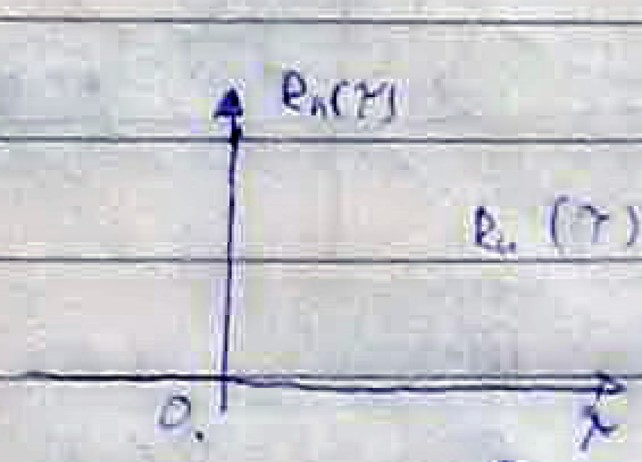
Centrálna limitná veta

– Rozdelenie pravdepodobnosti súčtu j štatisticky nezávislých náhodných procesov sa blíži ku Gaussovskému až ak $j \to \infty$ bez ohľadu na to, akého ~~[illegible]~~ typu sú jednotlivé rozdelenia



$$G_n(f) = \frac{N_0}{2}\left[\frac{W}{Hz}\right]$$

platí pre $f \in \langle 0{,}3 - 10^{12} \rangle Hz$



$$R_n(\tau) = \frac{N_0}{2}\delta(\tau)$$

$$P_n = \int_{-\infty}^{\infty} G_n(f)\,df \to \infty$$

– v reálnych systémoch $W_n \gg W_s$ → potom reálny šum považujeme za AWGN

↑ skutočná šírka p. reálneho systému

AWGN má normálne rozdelenie pravdep.

Náhodná premenná X má normálne rozdelenie pravdepodobnosti $N(a, \sigma^2)$ → s parametrami a, σ^2 ak jej hustota rozd. pravdepodobnosti :

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-a)^2}{2\sigma^2}}$$

$a = E(x)$ – stredná h.

$\sigma^2 = D(x)$ – disperzia

distribučná f. $F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t-a}{2\sigma^2}}\, dt$

vyjadruje pravdep., že $X \le x$

Normované normálne rozdelenie $N(0,1)$ $a = 0$

$\sigma = 1$

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$\Phi(x) = \frac{1}{2\pi} \int_{-\infty}^{x} e^{-\frac{t^2}{2}}\, dt$$ → tabuľková

$$F(x) = \Phi\left(\frac{x-a}{\sigma}\right)$$

$N(a, \sigma^2)$

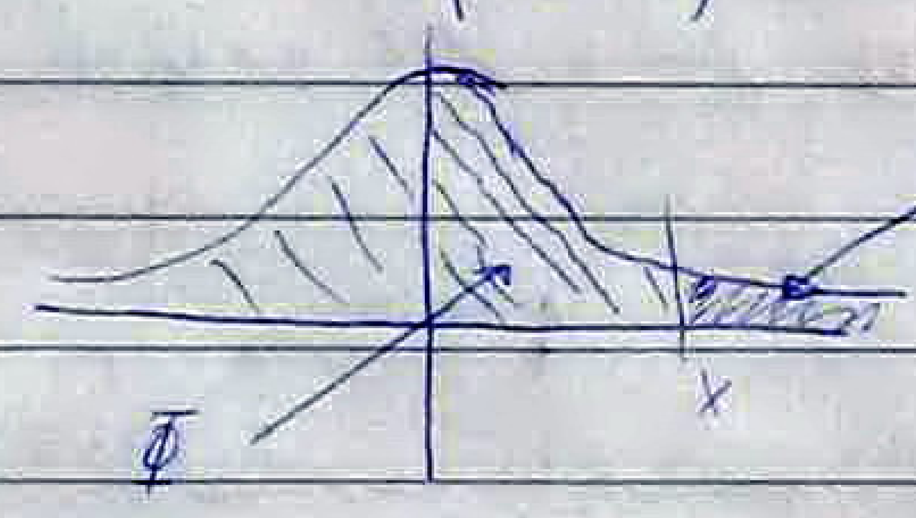
Komplementárna chybová f. Chybová funkcia: Error funct.

pre $N(0,1)$

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-\frac{t^2}{2}}\, dt$$

Vyjadruje pravd. že $X \ge x$

$$Q'(x) = Q\left(\frac{x-a}{\sigma}\right)$$



$\Phi(x) + Q(x) = 1$

$\Phi(x) = 1 - \Phi(-x)$

- Pr: AWGN má disp. $\sigma_0^2 = 10^{7}$

Vyp. pravd. že napätie úrovni šumu bude v int.:

a) $+100\,\mu V \div 500\,\mu V$

b) $1\,mV \div 3\,V$

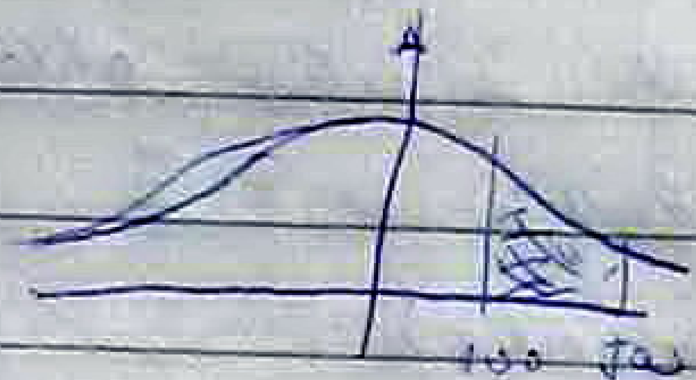
pozn.: ďalšia forma:

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-t^2}\, dt$$ pre $N(0,1)$

CMSK

$erfc(x) = 2Q(x\sqrt{2})$

$Q(x) = \frac{1}{2} erfc\left(\frac{x}{\sqrt{2}}\right)$



$Q(x) = \frac{1}{2} erfc\left(\frac{x}{\sqrt{2}}\right)$

$erfc(x) = 2Q(x\sqrt{2})$

$P(100\,mV \leq x \leq 500\,mV)$

$P(100\,mV \leq x \leq 500\,mV) = Q'(100) - Q'(500) =$

$Q' = \frac{100 \cdot 10^{-6}}{\sqrt{10^{-7}}} - Q\frac{500 \cdot 10^{-6}}{\sqrt{10 \cdot 10^{-7}}} = Q_c(0{,}316) - Q_c(1{,}58)$

$= 0{,}3745 - 0{,}057$

$= \underline{0{,}3174}$

b) $P(x \geq 1\,mV) = Q(3{,}16)$

$= 0{,}00086$

$P(x \geq 3\,mV) = Q(9{,}486) -$

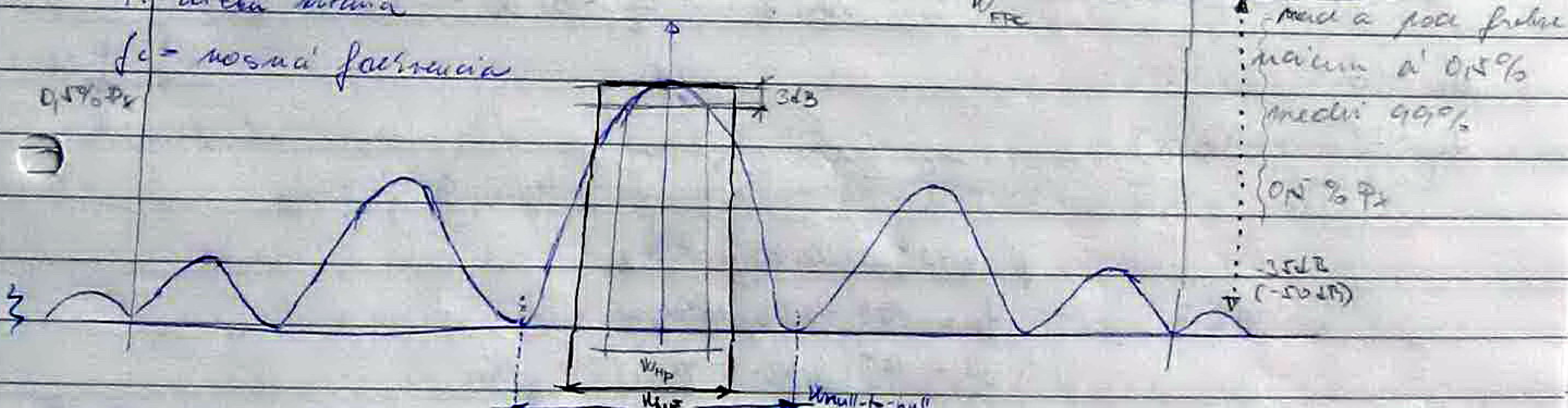
$\left(\frac{3 \cdot 10^{-3}}{\sqrt{10^{-7}}}\right)$

Frekvenčné pásma

PSD reálneho dig. signálu $\quad G_x(f) = T\left[\frac{\sin \pi (f - f_c) \cdot T}{\pi (f - f_c) \cdot T}\right]^2$

T - dĺžka hrania

f_c - nosná frekvencia



1, W_{HP} (half power) - pásmo, v kt. poklesne PSD o 3dB

2, W_{NE} (noise equivalent) - šírka pásma pravouhlého filtra, ktorý má v pásme prepúšťania rovnaký zisk ako reálny systém, a prepustí rovnaký výkon šumu, ako je reálny výkon na výstupe systému.

$P_x = W_{NE} \cdot G_x(f)_{max} \Rightarrow W_{NE} = \frac{P_x}{G_x(f)_{max}}$

3. $W_{null\text{-}to\text{-}null}$ – pásmo medzi 2 nulami okolo hlavného laloku

$$W_{null\text{-}to\text{-}null} \doteq \frac{1}{T} \quad \text{v ZP}$$

4. W_{FPC} (fractional power containment)

(USA) – pásmo v kt. sa nachádza 99% ~~s~~ výkonu signálu

5. W_{BPSD} (bounded PSD)

– mimo pásma (toho) PSD nesmie prekročiť −35 ; −50 dB

6. Absolútne pásmo – pásmo, v kt. sme ešte schopní merať nejaký signál (mimo tohto pásma $G_x(f) = 0$)

• Pr

$$G_x(f) = 10^{-4}\, si^2\left(\pi (f - 10^6) \cdot 10^{-4}\right)$$

① pásmo

$$10 \log \frac{G_x(f)}{G_x(f')} = 3\,dB$$

$$f_c = 10^6\,Hz$$

$$G_x(f') = \frac{1}{2}\, \cancel{G_x(f)} =$$

$$= 10^{-4} \left[\frac{\sin \pi (f' - 10^6) \cdot 10^{-4}}{\pi (f' - 10^6) \cdot 10^{-4}}\right] = \frac{1}{2} \cdot 10^{-4}$$

subst.

$$\alpha = \pi (f' - 10^6) \cdot 10^{-4}$$

$$\sin \alpha = \pm \frac{\alpha}{\sqrt{2}} \rightarrow \alpha = \pm \sqrt{2} \sin \alpha$$

$\alpha_0 = 1{,}0$

$\alpha_i = \pm \sqrt{2} \sin(\alpha_{i-1})$

$\alpha_1 = 1{,}1900$ $\quad \alpha_3 = 1{,}36955$ $\quad \alpha_5 = 1{,}3858$

$\alpha_2 = 1{,}3129$ $\quad \alpha_4 = 1{,}3810$ $\quad \boxed{\alpha_6 = 1{,}3911}$ rad

Atučí 6 iterácií

$$f' \doteq 10^6 \pm 4{.}43 \cdot 10^3\,Hz$$

$$f_H = 10^6 + 4{.}43 \cdot 10^3\,Hz$$

$$f_d = 10^6 - 4{,}43 \cdot 10^3\,Hz$$

$$W_{HP} = f_H - f_d = 8{,}86\,kHz$$

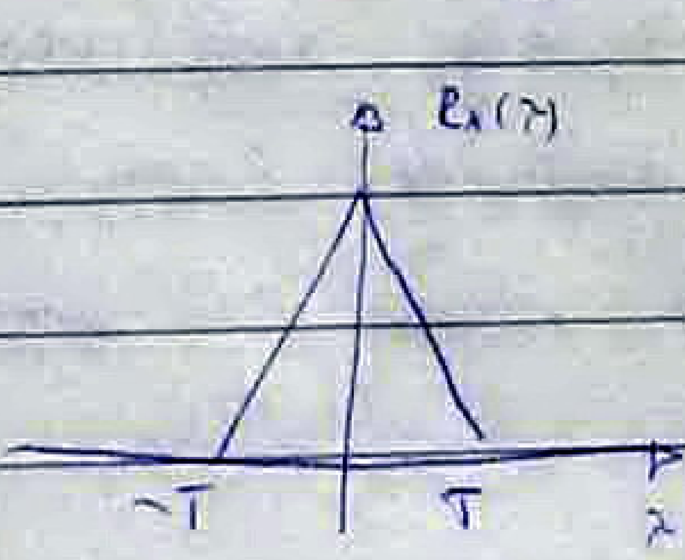
② 2. pásmo

$$W_{NE} = \frac{P_x}{G_x(f)_{max}}$$

$$R_x(\tau) \xleftrightarrow{FT} G_x(f)$$

$$R(\tau) = \mathcal{F}^{-1}(G_x(f))$$

$$T\, sinc^2\, fT \xleftrightarrow{\mathcal{F}^{-1}} \begin{cases} 1 - \frac{|\tau|}{T} & |\tau| \leq T \\ 0 & |\tau| > T \end{cases}$$



$$R_x(0) = 1 = P_x$$

$$W_{NE} = \frac{1}{10^{-4}} = 10\,kHz$$

CMSK
7.3.

3. $W_{null\text{-}to\text{-}null}$

$G_x(f) = 0$

$0 = 10^{-4} \sin \alpha \Rightarrow \alpha = 0 + k\pi$

$k = 1 \qquad \alpha = \pi$

$f' = 10^6 \pm 10^4\,Hz$ $\qquad W_{null\text{-}to\text{-}null} = 20\,kHz$

(4.) $0{,}99 = \int_{f_c - \Delta f}^{f_c + \Delta f} 10^{-4} \sin^2(\pi(f' - f_0) \cdot 10^{-4})\, df$

$[F]_{f_c - \Delta f}^{f_c + \Delta f} = 0{,}99$ $\qquad W_{FPC} = 205{,}720\,kHz$

(5.) $W_{3PSD} = 370\,kHz$

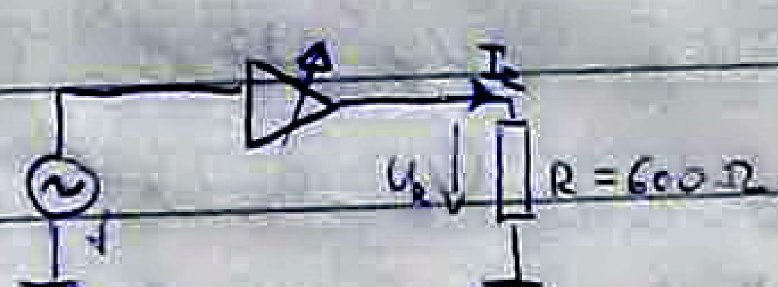
- vypočítat.

CMSK
14.3.

1. úloha: Ověřte Parsevalovu teorému pro:

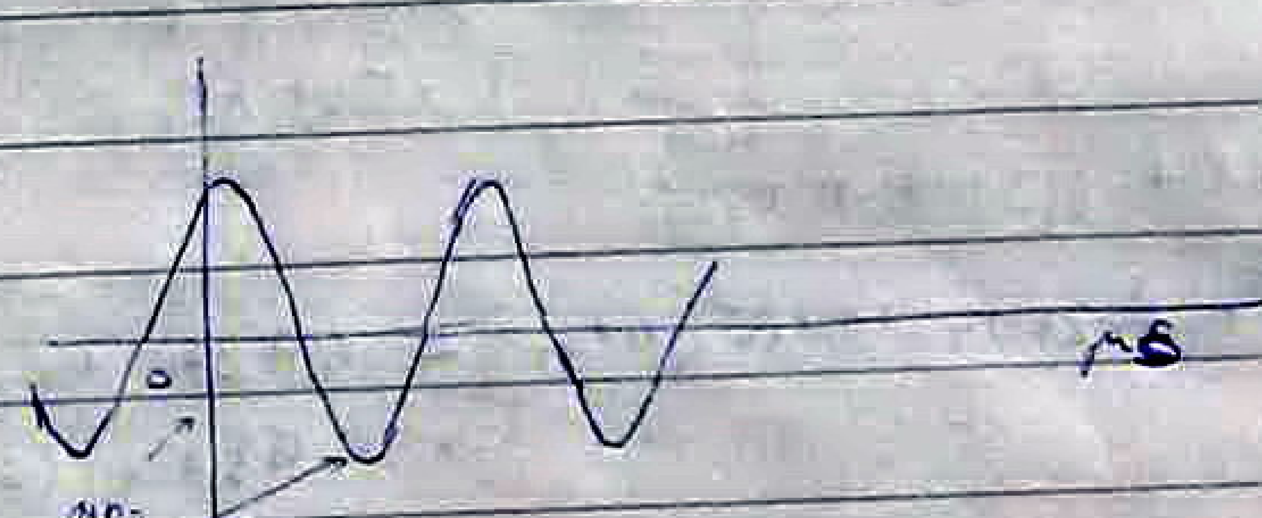
a, harmonický signál $x(t) = 2\sin 2\pi\, 2{,}083 \cdot 10^3 t$

b, neharmonický signál inv. TTL $A = -4V$



$f = \frac{f_0}{12} \cdot \frac{1}{4} \doteq 2{,}083\,kHz$

$f_0 = 100\,kHz$



frequency: 2 083 Hz

voltage: 1,433 V

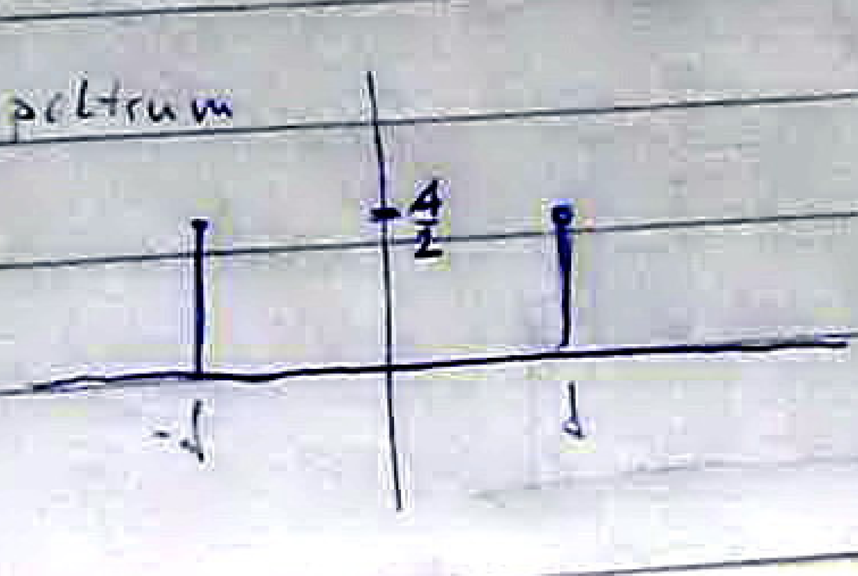
Peak to peak: 4071 V

$\boxed{A = \frac{4071}{2}}$

$P_x = \frac{A^2}{2} \doteq 2W = \frac{2^2}{2}$

$= \frac{\frac{\left(\frac{4071}{2}\right)^2}{2}}{600} = \ldots P_2 = \frac{A^2}{2} : 600 = 3{,}3\,mW \doteq P_2 = \frac{U_e^2}{R} = \frac{1{,}433^2}{600} = 3{,}42\,mW$

spektrum



lí. kl. hodnoty
ču polovična

frequency 2082 Hz
amplitude f. 6,95 Hz

maximální: $\frac{X_{ef}}{2} \; 1{,}694\,V \quad \rightarrow A = \frac{2 A_{ef}}{2}\sqrt{2} = 1{,}46\,V$

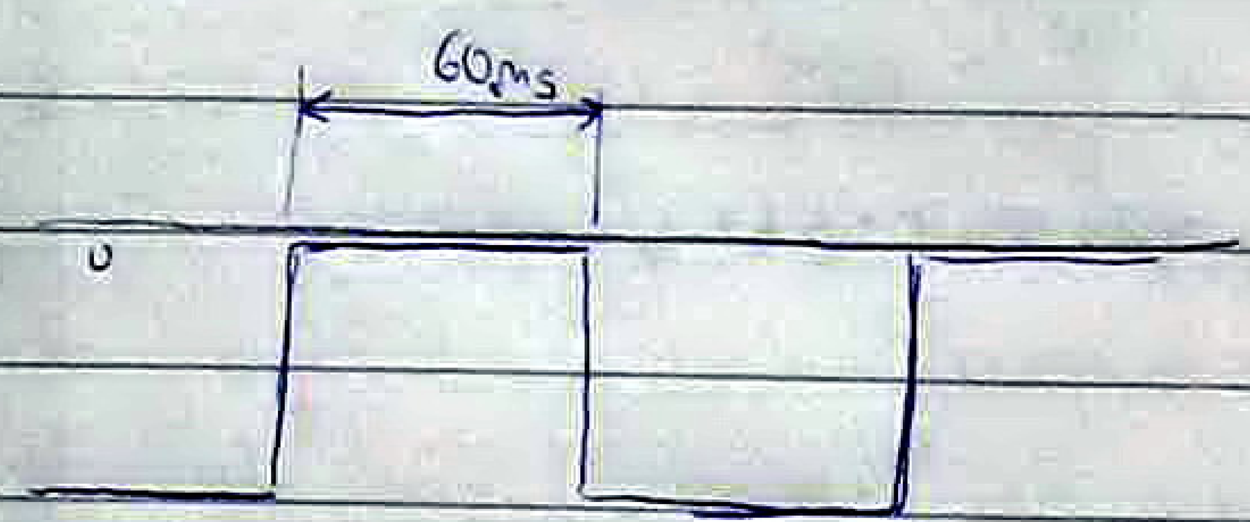
$P_x = \frac{1}{T} \sum_{n} |c_n|^2 \qquad c_1 = S_1 \cdot \frac{A}{2} = 0{,}98$

$P(1) = 2 \cdot 0{,}98^2 = 3{,}29\,W$

$P_x = \sum |c_n|^2$

$c_1 = c_{-1} = \frac{A}{2} = 0{,}98$

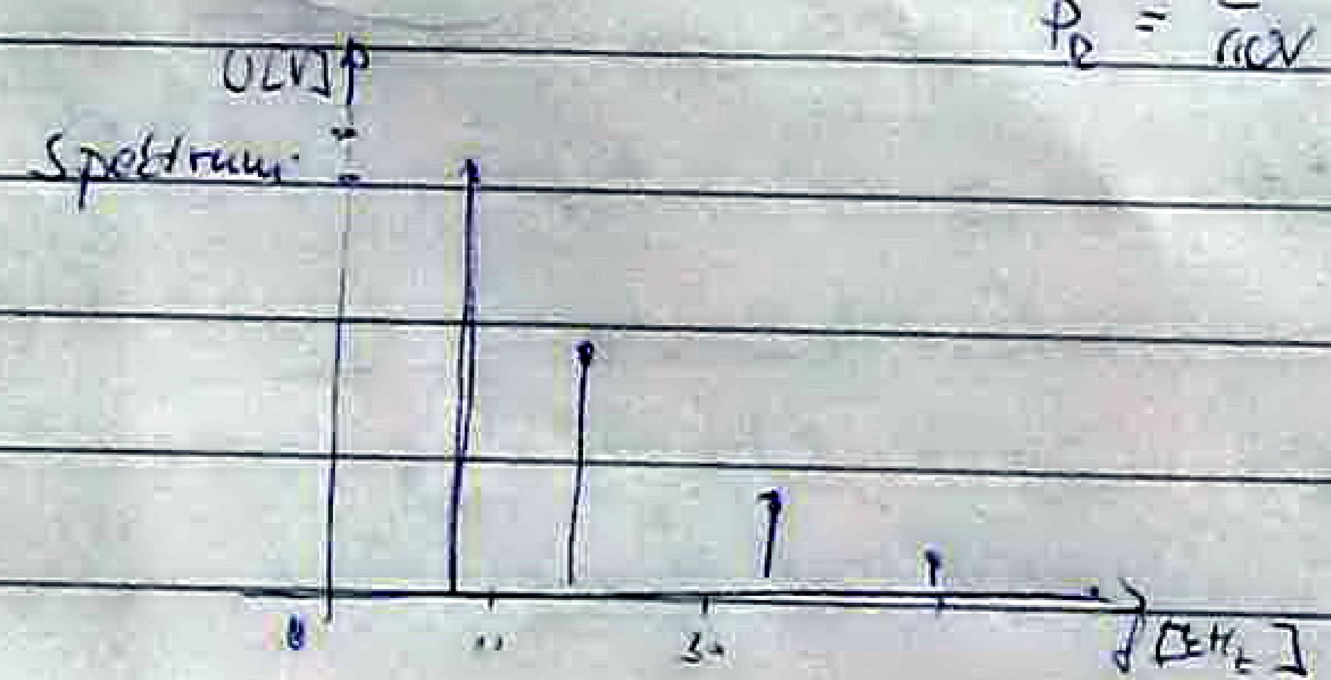
signál 2:



perioda	120 ms
$f = 8{,}33\,kHz$	8330 Hz
AC efekt.	1970
U peak to peak	3983

$P_x = \frac{1}{T}\int_{-T/2}^{T/2} x^2\,dt = \int \frac{A^2}{2} \cdot 8we^{N}$

$P_e = \frac{V}{RCV} = 8{,}333\,mW$



žádné frekvence: 8,396 kHz	.... V_{ef} 812 mV		1. h.
25,08	V_{ef} 0,27 V		3. h
41,66 kHz	V_{ef} 0,17 V		5. h.
58,37 kHz	V_{ef}	0,11 V	7. h
75,01 kHz		0,009	9. h.

$2 \cdot \sqrt{2} = 2{,}82$

$P = ? \qquad P = 2{,}297 + 0{,}76 + 0{,}48 + 0{,}3111 + 0{,}25$

$P = \sum \frac{2\sqrt{2} \cdot V_{ef}}{2} = 1{,}1485 + 0{,}352 + 0{,}2404 + 0{,}155 + 0{,}0127 = 1{,}937 + (\text{jednosm. složka})^2$

$= 1{,}937 + (-2)^2 =$

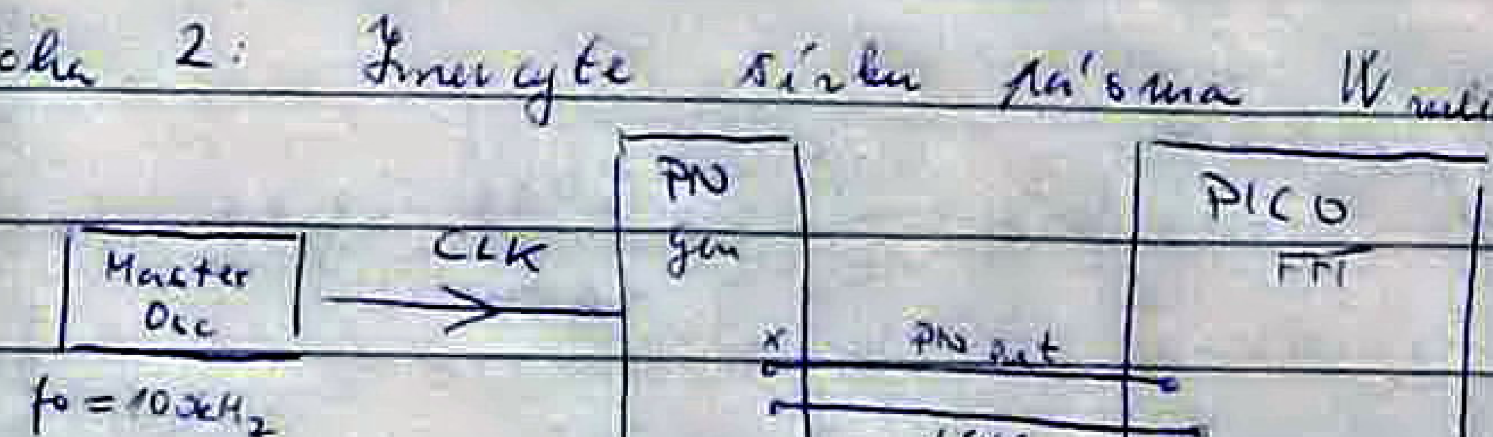
$A_1 = 2\sqrt{2} \cdot 0{,}812\,V$

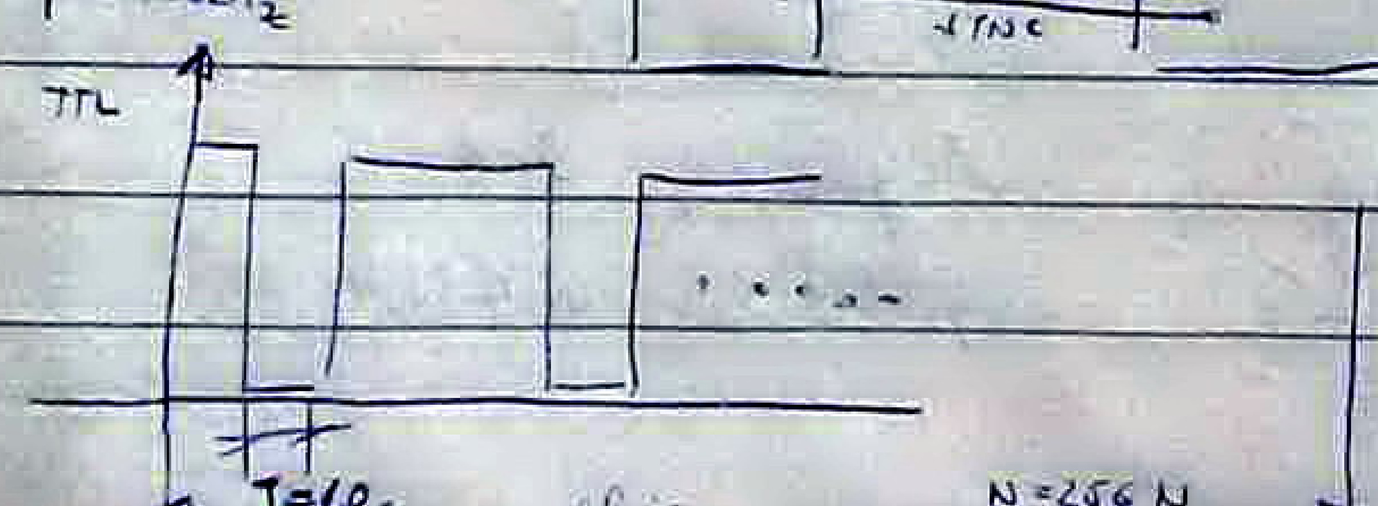
$c_1 = c_{-1} = \frac{A_1}{2} = \sqrt{2} \cdot 0{,}814$

$P_x = 2(|c_1|^2 + |c_3|^2 + |c_5|^2 + |c_7|^2 + |c_9|^2) + A_0^2 \qquad A_0 = -2V$

$P_x = 2(\qquad) + 4 = 7{,}1252$

Úloha 2: Změřte šířku pásma W ... diskrétního PN sig. v záhl. pásmu





CMSK 14.3.

- B = ten istý príklad

$A = 5V$

$$P_B = Q\left(\frac{a_1 - a_2}{2\sigma_0}\right)$$

$R_B = 1kbps \qquad P_B = ?$

$\sigma_0^2 = 10^{-6}$

a_i - sl. n na výstupe kor. ak korelujeme sig. s normovanými prototypmi

$$\psi_1(t) = \frac{s_1(t)}{\sqrt{E_B}} = \frac{1}{\sqrt{T}} = \frac{A}{A\sqrt{T}} =$$

E_B - integrál

$$= \int_0^T A^2 dt = A^2 T \qquad \sigma_0^2 = \frac{N_0}{2}$$

$a_1(T) = A\sqrt{T}$

$$E_{B_2} = \int_0^T -A^2 T = -A^2 T \qquad 2\sigma_0 = \sqrt{2N_0}$$

$a_2(T) = 0$

$$E_d = \int_0^T [s_1(t) - s_2(t)]^2 dt \qquad \bar{E}_B \text{ - stredná energia na bit } \quad E_B = \frac{A^2 T}{2}$$

rozdielová energia

$$P_B = Q\left(\frac{A\sqrt{T}}{\sqrt{2N_0}}\right) = Q\left(\sqrt{\frac{A^2 T}{2N_0}}\right) = Q\left(\sqrt{\frac{E_d}{2N_0}}\right) = Q\left(\sqrt{\frac{E_B}{N_0}}\right)$$

$$= Q(7{,}906) = 1{,}353 \cdot 10^{-15}$$

$$E_d = \int_0^T [s_1(t) - s_2(t)]^2 dt = 4A^2 T$$

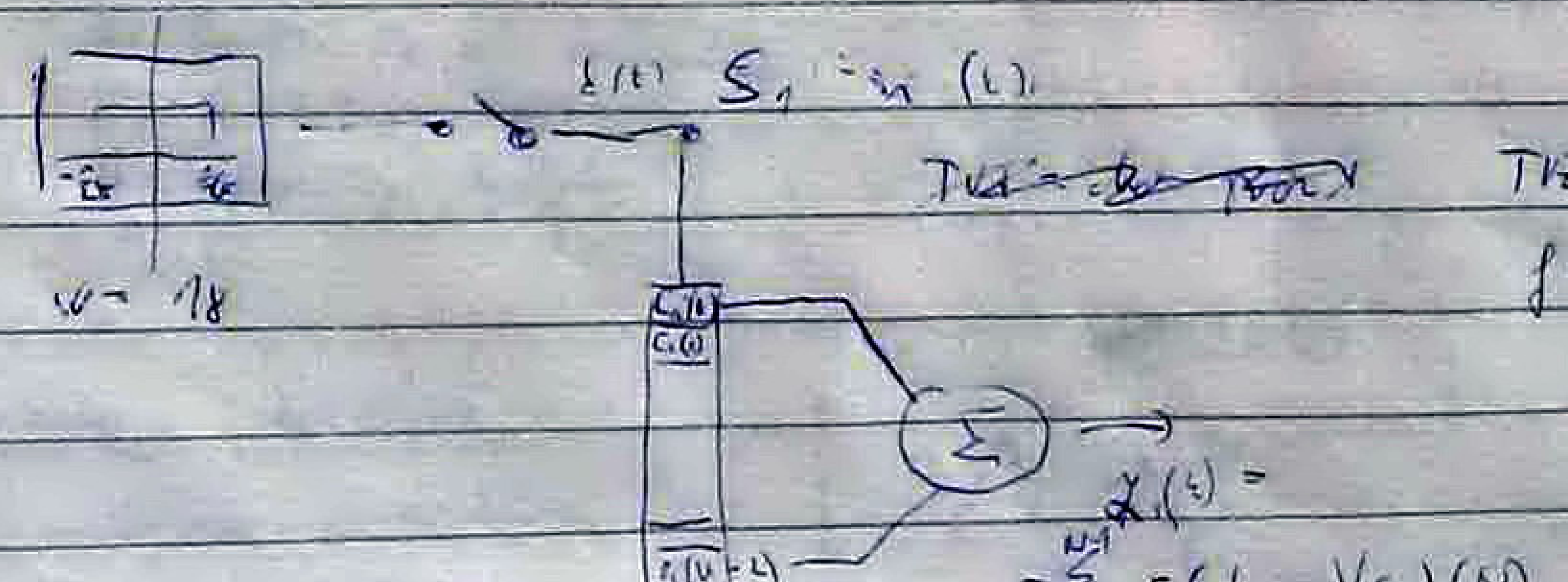
$$P_B = Q\left(\frac{A\sqrt{T} - (-A\sqrt{T})}{\sqrt{2N_0}}\right) = Q\left(\sqrt{\frac{4A^2T}{2N_0}}\right) = Q\left(\sqrt{\frac{E_d}{2N_0}}\right) = Q\left(\sqrt{\frac{2E_B}{N_0}}\right) = Q(11{,}81) = 1{,}31 \cdot 10^{-32}$$

$R_B \to 1 Mbps$

$$E_B = \frac{25 \cdot \frac{1}{100}}{2} \qquad Q(7{,}95) = 0{,}3085$$

Realizácia spriahnutých filtrov

Digitálne spriahnutých filtrov



TVZ - doba vzorkovania

$f_0 = 2W$ - nyquistova freq.

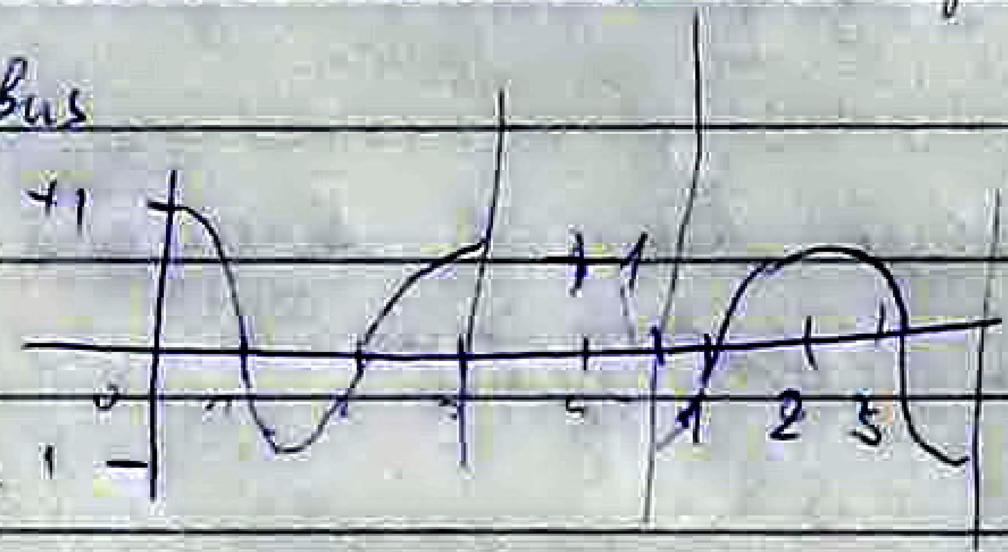
$f \geq$

$f_{vz} = \quad 4 f_0$

$$x(t) = \sum_{n=0}^{N-1} r(t-n) \cdot s_1(N)$$

- Jaký bude výstup z čísl. přizp. filtru, až uvažujeme např. BPSK namodulovaného sig. s 3 harmonickými nosnými.

celý průběh



po namodulování:

$s_1(0) = 1$

$s_1(1) = 0$

$s_2(2) = -1$

$s_2(3) = 0$

$s_1(0) = -1$

$s_1(1) = 0$

$s_2(2) = 1$

$c_1(n) = k \cdot s[(N-1)-n]$

$c_1(0) = 0$

$c_2(1) = -1$

$c_2(2) = 0$

$c_3(1)\ 1$

$c_2(0) = 0$

$c_2(1) = -1$

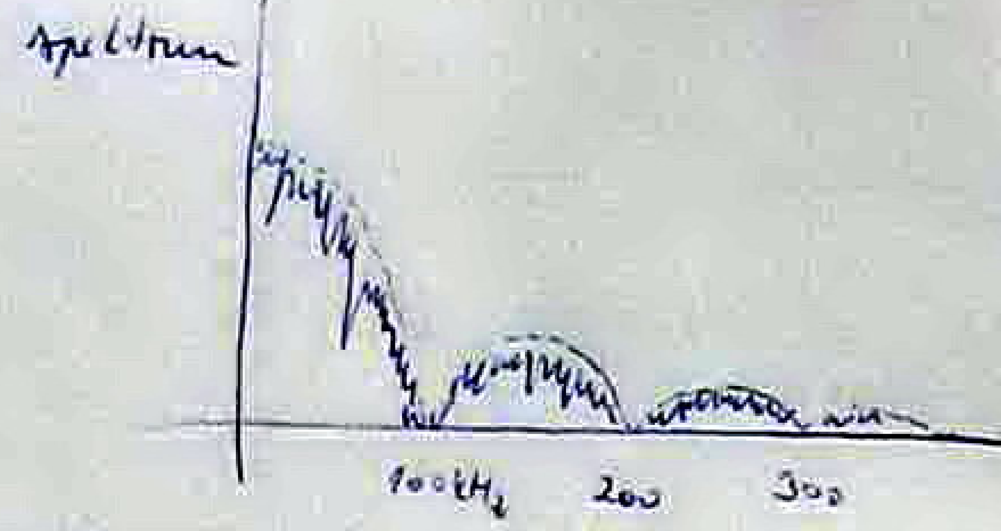
$c_2(2) = 0$

$c_2(3) = -1$

$$z_1\Big|_{t_3=3} = \sum_{n=0}^{3} s_1(3-n)\, c_1 n = 2\,(4-2) = +2$$

$$z_2\Big|_{t_3} = \sum_{n=0}^{3} s_2(3-n)\, c_2(n) = z_2(t_3 - 3) = -2$$

Null-to-null = $\frac{1}{T}$



– šírka signálu

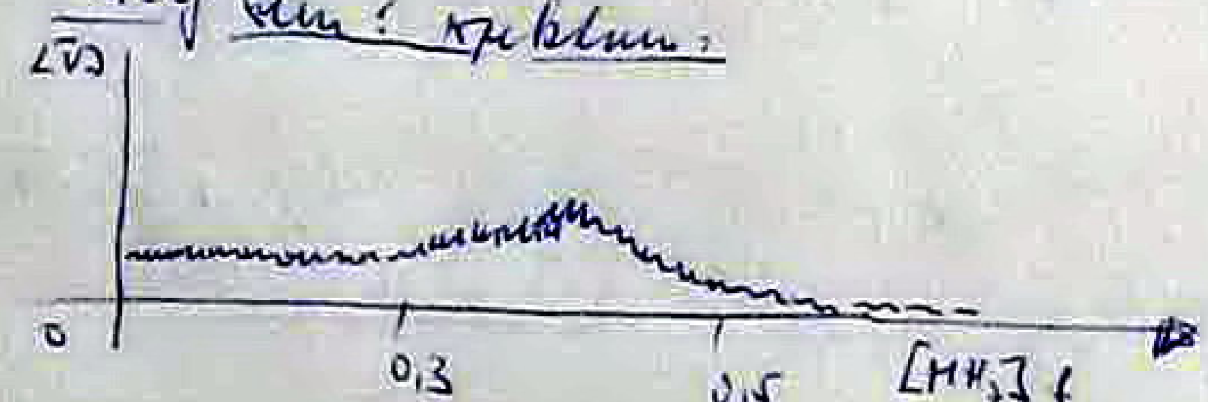
ÚLOHA 3

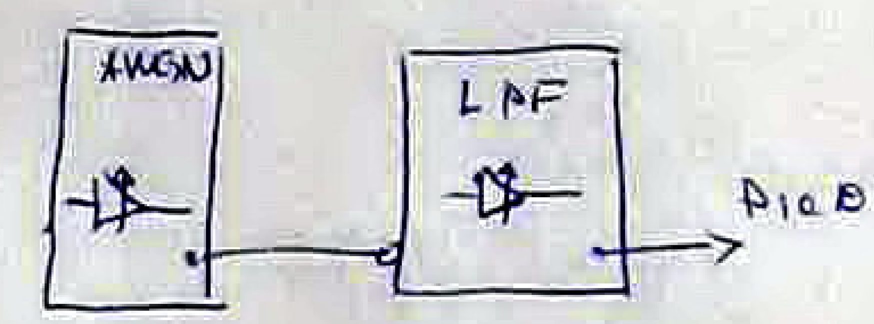
Zmerajte výk. spektrum AWGN generátora a vyp.

a) $\frac{N_0}{2}$ v pásme $0 \div 30kHz$

b) P_N — ‖ —

biely šum: spektrum:





v tomto prípade šum do 300kHz môžeme považovať za biely.

namerané: $|X_n(f)| = -30dB$ v pásme $\langle 0, 30kHz \rangle$

a) $\frac{N_0}{2} = |X_n(f)|^2 = 10^{-6} \frac{W}{Hz}$

PSD

$-30 = \log \frac{U}{U_0}$

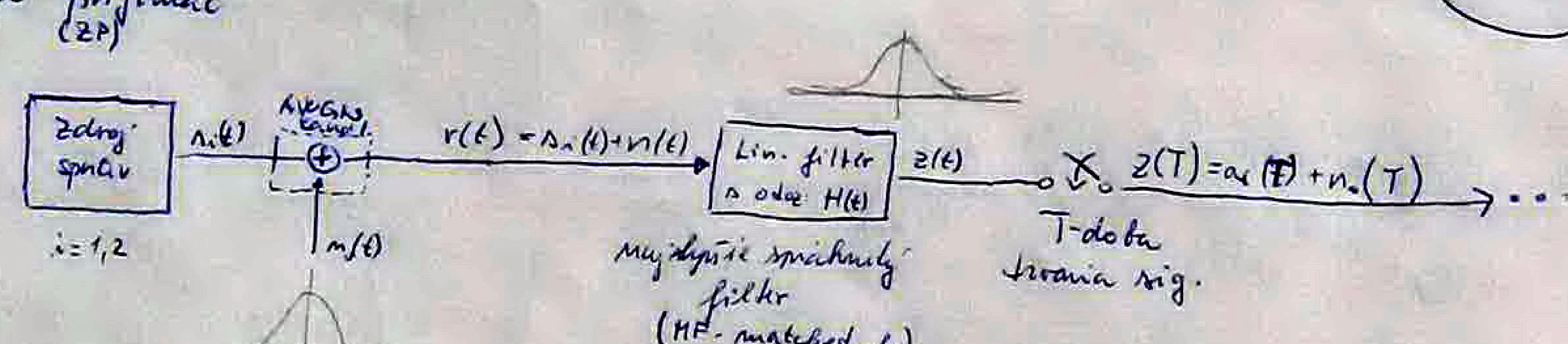
$-30 = \frac{U_x}{U_0}$

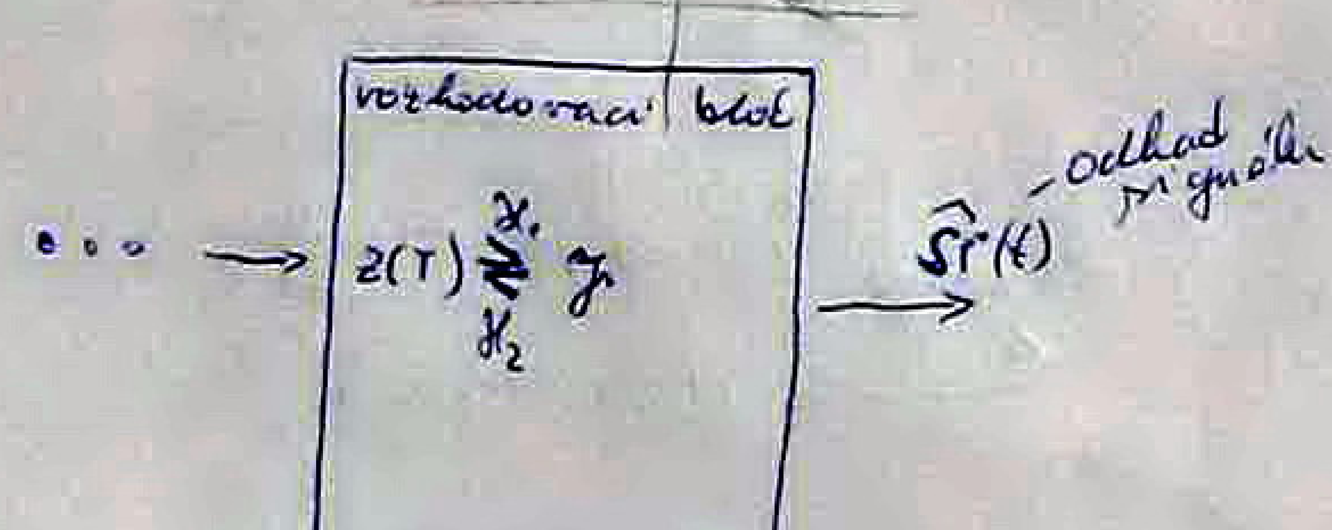
$$P_n = \int_0^{30k} \frac{N_0}{2}\, df = \int_0^{30k} 10^{-6} = 10^{-6} \cdot 3 \cdot 10^{4} = \underline{0{,}03\,W}$$

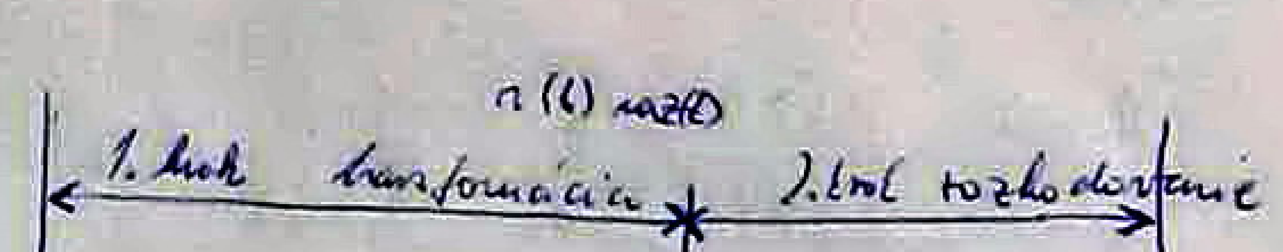
Detekcia binárnych sig. za prítomnosti AWGN

CMSK 21.3

opt. prijímač (ZP)







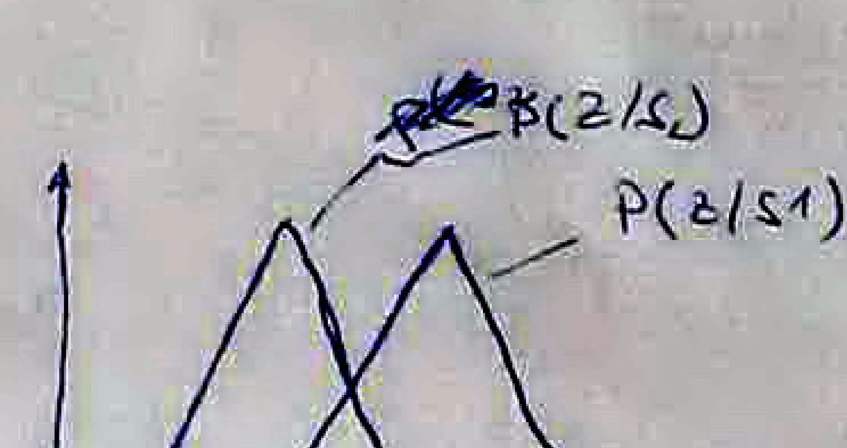
H_1 - hypotéza že $s_1(t)$ bol vyslaný

H_2 - hypotéza $s_2(t)$

γ - referenčná hodnota

Rozhodovacie kritériá

výstup z detektora je väčšinou spojitá náhodná premenná



$$P(s_i/z) = \frac{P(z/s_i)\, P(s_i)}{P(z)}$$

$P(s_i)$ - apriórna pravdep. vyslania s_i

$P(z)$ - úplná pravdep. $P(z) = \sum_i P_i\, P(z/s_i)$

-LICE-

1. **MAP** – maximal aposteriory prob.

$$P(s_1/z) \underset{\mathcal{H}_2}{\overset{\mathcal{H}_1}{\gtrless}} P(s_2/z)$$

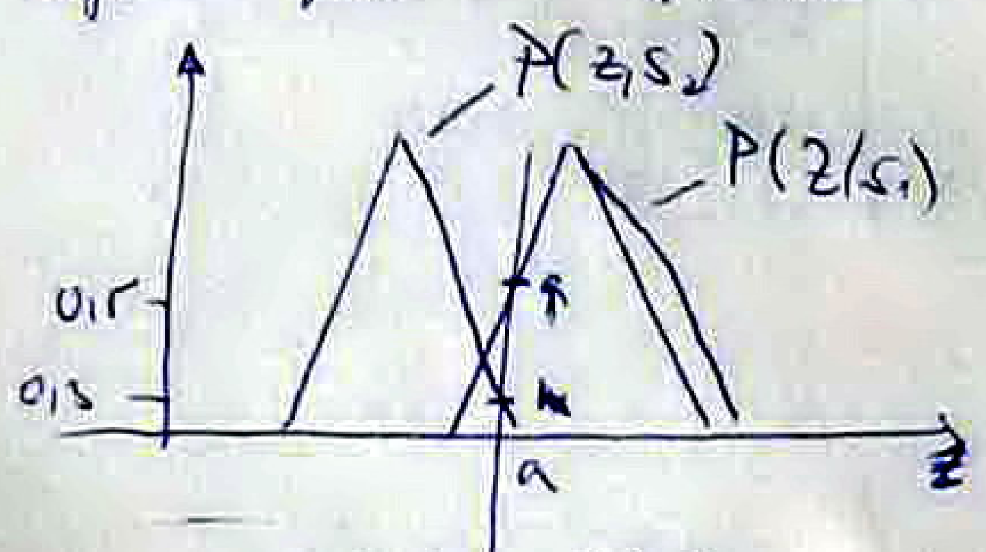
$$\boxed{\frac{P(s_1/z)}{P(s_2/z)} \underset{\mathcal{H}_2}{\overset{\mathcal{H}_1}{\gtrless}} \frac{P(s_2)}{P(s_1)}}$$ max. (väčšia) pravdep. po detekcii

2. ML – Maximal Likelihood

$P(s_2) = P(s_1) = \frac{1}{2}$

$$\boxed{P(z/s_1) \underset{\mathcal{H}_2}{\overset{\mathcal{H}_1}{\gtrless}} P(z/s_2)}$$

• Pr Systém používa binárn. sig. výstup z detektora s pripustením f z.



pre $z = a$

$P(s_1) = 0{,}3$

$P(s_2) = 0{,}7$

Pomocou:

a) Map určte bl. sig bol vysl.

b) ML – "

$P(s_1 \cap s_2) = P(s_1) \cdot P(s_2) = P(s_1 + s_1)$

a) $\frac{P(s_1/z)}{P(s_2/z)} \underset{\mathcal{H}_2}{\overset{\mathcal{H}_1}{\lessgtr}} \frac{P(s_2)}{P(s_1)}$

$P(s_1/z) = \frac{P(z/s_1)\,P(s_1)}{P(z)} =$

$P(z) = \sum_i P(z/s_i)\,P(s_i)$

$P(s_1/z) \underset{\mathcal{H}_2}{\overset{\mathcal{H}_1}{\gtrless}} P(s_2/z) \rightarrow s_2$

$0{,}3 \quad 0{,}7 = z_2$

b) $P(s_1/z) \underset{\mathcal{H}_2}{\overset{\mathcal{H}_1}{\gtrless}} P(s_2/z)$

$$\boxed{\begin{aligned} P(z/s_2) &= 0{,}3 \\ P(z/s_1) &= 0{,}5 \end{aligned}}$$

$\rightarrow s_1$

$P(z) = P(s_1)\,P(z/s_1) + P(s_2) \cdot P(z/s_2)$

$0{,}3 \cdot 0{,}5 + 0{,}7 \cdot 0{,}3 = 0{,}15 + 0{,}21$

$\boxed{0{,}36}$

$\frac{P(z/s_1)\,P(s_1)}{P(z)} \lessgtr \frac{P(z/s_2)\,P(s_2)}{P(z)}$

$(0{,}5 \cdot 0{,}3)/0{,}36 \lessgtr (0{,}3 \cdot 0{,}7)/0{,}36$

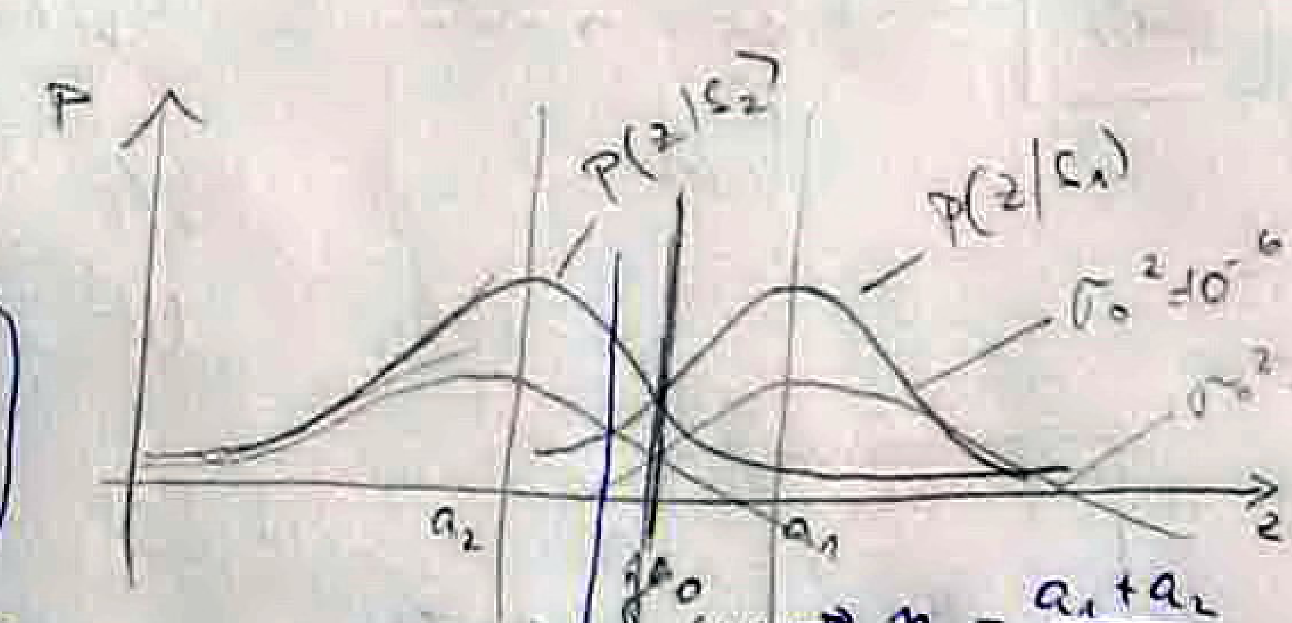
a) s_2

MAP + AWGN:

$$\frac{\frac{1}{\sigma_0\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(z-a_1)^2}{\sigma_0^2}}}{\frac{1}{\sigma_0\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(z-a_2)^2}{\sigma_0^2}}} \underset{\mathcal{H}_2}{\overset{\mathcal{H}_1}{\gtrless}} \frac{P(s_2)}{P(s_1)}$$

$$z \underset{\mathcal{H}_2}{\overset{\mathcal{H}_1}{\gtrless}} \underbrace{\left\{ \ln \frac{P(s_2)}{P(s_1)} - \frac{a_2^2 - a_1^2}{2\sigma_0^2} \right\} \frac{\sigma_0^2}{a_1 - a_2}}_{\gamma_{MAP}}$$

(MAP) $\frac{P(z/s_1)}{P(z/s_2)} \underset{\mathcal{H}_2}{\overset{\mathcal{H}_1}{\gtrless}} \frac{P(s_2)}{P(s_1)}$



(ML) $\rightarrow \gamma_0 = \frac{a_1 + a_2}{2}$

(MAP) γ_0 ← ak $P(s_1) > P(s_2)$

• Pr Na signalizáciu je použitý UNRZ

$A = 5V$

$s_1(t) = A \quad 0 \le t \le T$

$s_2(t) = 0 \quad 0 \le t \le T$

$\sigma_0^2 = 10^{-6}$

$R_b = 1kbps$

$P(s_1) = 0{,}7$

$P(s_2) = 0{,}3$

$a_1 = A^2 T$

$a_2 = 0$

$$\gamma_{MAP} = \left\{ \ln \frac{P(s_2)}{P(s_1)} - \frac{a_2^2 - a_1^2}{2\sigma_0^2} \right\} \frac{\sigma_0^2}{a_1 - a_2}$$

$$\left\{ \ln \left\{ \frac{0{,}3}{0{,}7} \right\} + \frac{A^4 T^2}{2 \cdot 10^{-6}} \right\} \frac{10^{-6}}{A^2 T - 0}$$

$$\ln \{0{,}428\} + \frac{0{,}625}{2 \cdot 10^{-6}} \Big\} \frac{10^{-6}}{25 \cdot \frac{1}{1000}}$$

$\gamma_{(MAP)} = 12{,}49 \cdot 10^{-3}$

$-0{,}84729 +$

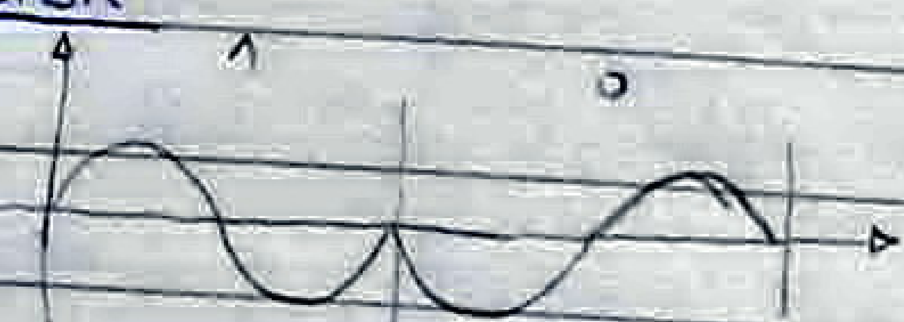
CMSK 21.2.

Na signalizáciu je použitý BPSK sig. s parametrom $\frac{E_B}{N_0} = 6{,}8\,dB$. Na detekciu sú použité spriahnuté filtre.

a) uvažujeme optimálnu rozhodovaciu úroveň $\gamma_0 = 0$. Urč. P_B

b) ako sa zmení P_B ak sa γ zmení na $\gamma = 0{,}1\sqrt{E_B}$

BPSK



$$P_B = Q\left(\sqrt{\frac{2E_B}{N_0}}\right)$$

– do Q funkcie sa nedajú dosadzovať dB

a) $6{,}8\,dB \triangleq 10^{0{,}68} = 4{,}7863$ $\qquad \triangleq 10 \log 6{,}8$

$P_B = Q(3{,}099) \cong 10^{-3}$

b) $\gamma = 0{,}1\sqrt{E_B}$

$x = \gamma$

$a = \pm\sqrt{E_B}$

$\sigma_0 = \sqrt{\frac{N_0}{2}}$

$$P_B(0 \to 1) = Q\left(\frac{0{,}1\sqrt{E_B} - (-\sqrt{E_B})}{\sqrt{\frac{N_0}{2}}}\right) = Q\left(1{,}1\sqrt{\frac{2E_B}{N_0}}\right) = 0{,}0003$$

$$P_B(1 \to 0) = Q\left(\frac{0{,}1\sqrt{E_B} - \sqrt{E_B}}{\sqrt{\frac{N_0}{2}}}\right) = Q\left(0{,}9\sqrt{\frac{2E_B}{N_0}}\right)$$

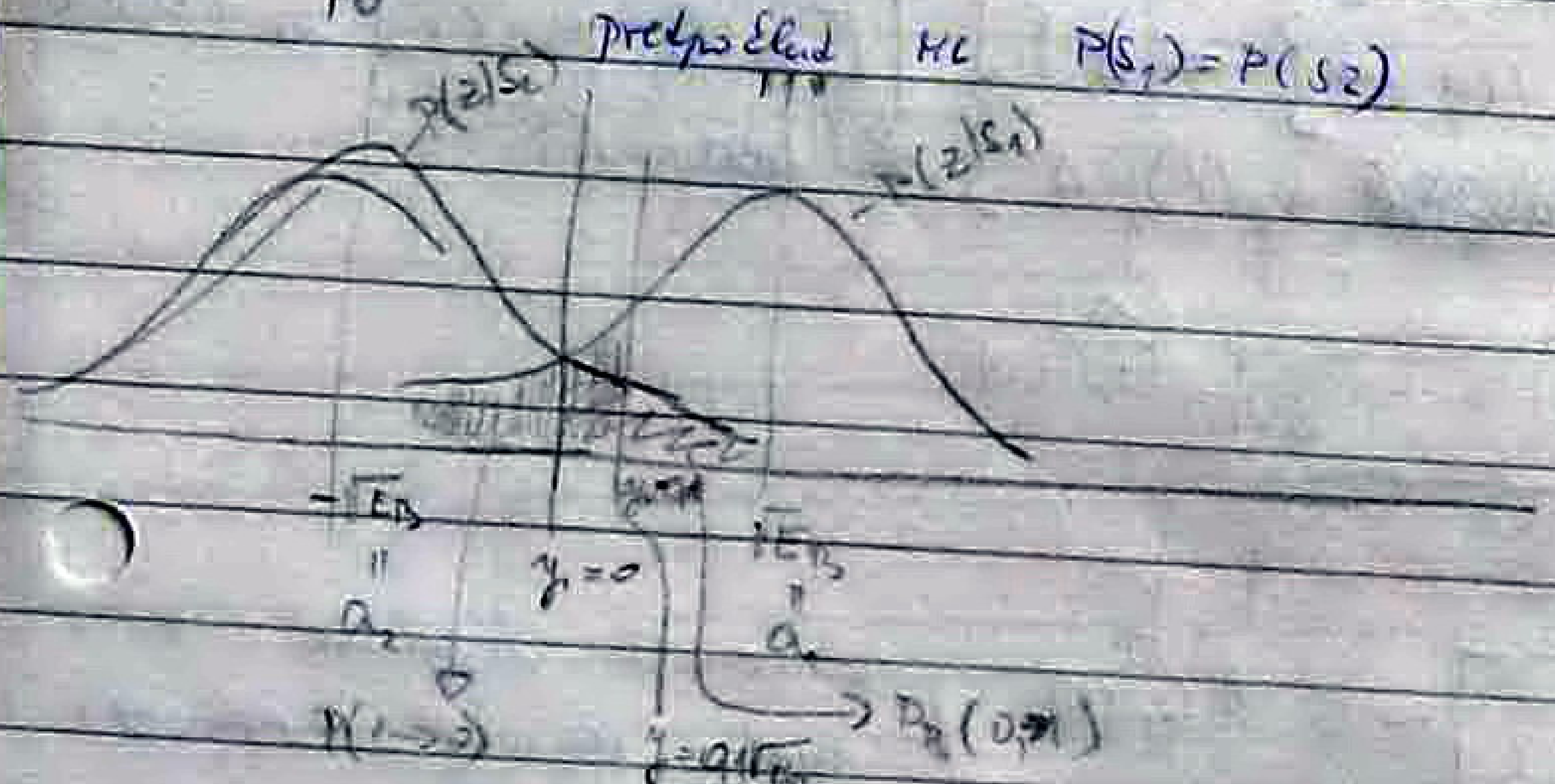
$\Phi(-x) = Q(x)$

$= 0{,}0027$

celková chybovosť: (veta o úplnej pravdepodobnosti)

$$P_B = P_B(0 \to 1) \cdot P(s_2) + P_B(1 \to 0) \cdot P(s_1) = 1{,}5 \cdot 10^{-3}$$

10" predpoklad HC $P(s_1) = P(s_2)$



Spriahnuté filtre

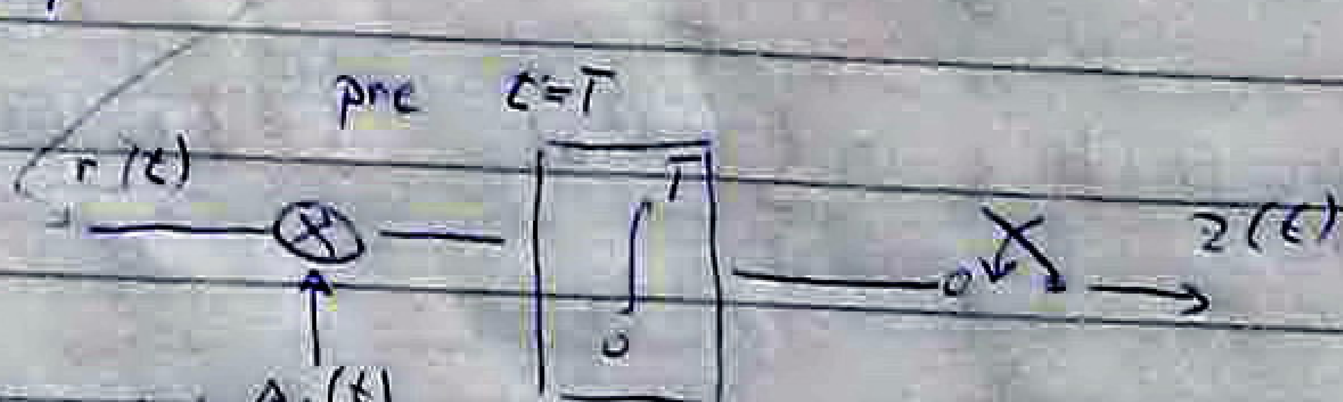
$$H(f) = k\, S^*(f)\, e^{-j2\pi f T}$$

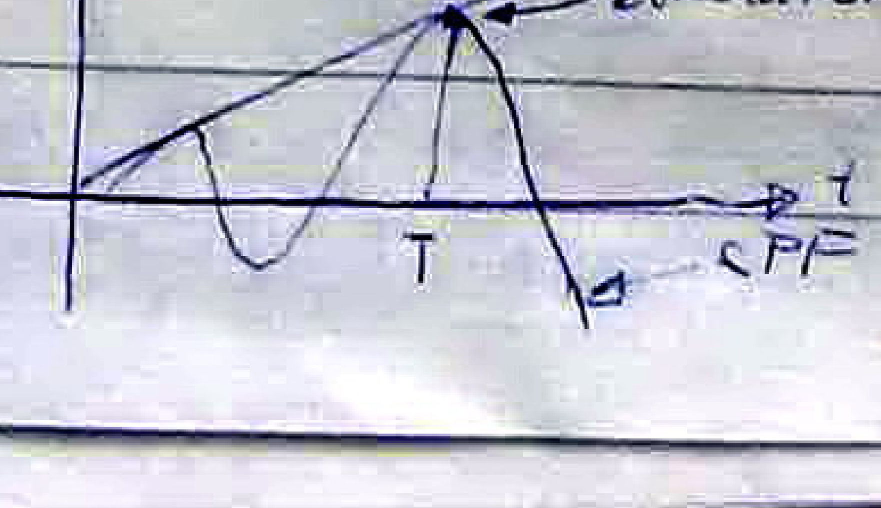
$\updownarrow$

$$h(t) = \begin{cases} ks(T-t) & 0 \le t \le T \\ 0 & t > T \end{cases} \qquad \equiv [V.3]$$

CMSK 28.2.

$r(t)$ — $\boxed{SPF}$ — $z(t)$

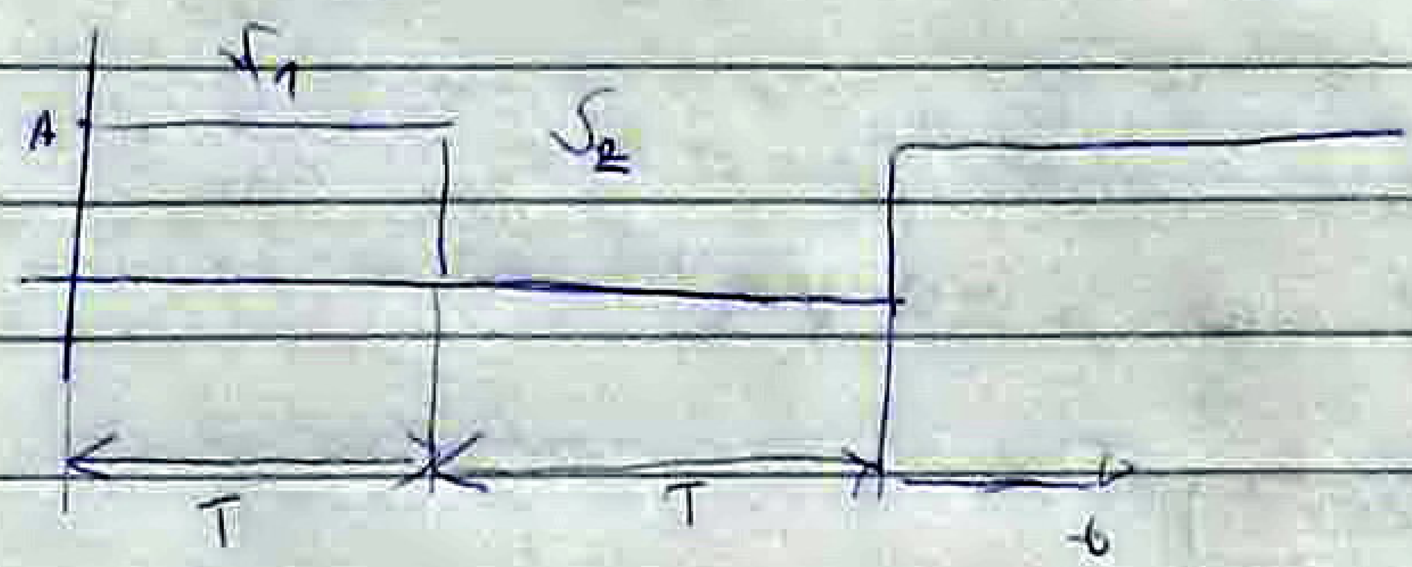




musí byť synchronizovaný

$\left[\frac{1}{V_c}\right]$

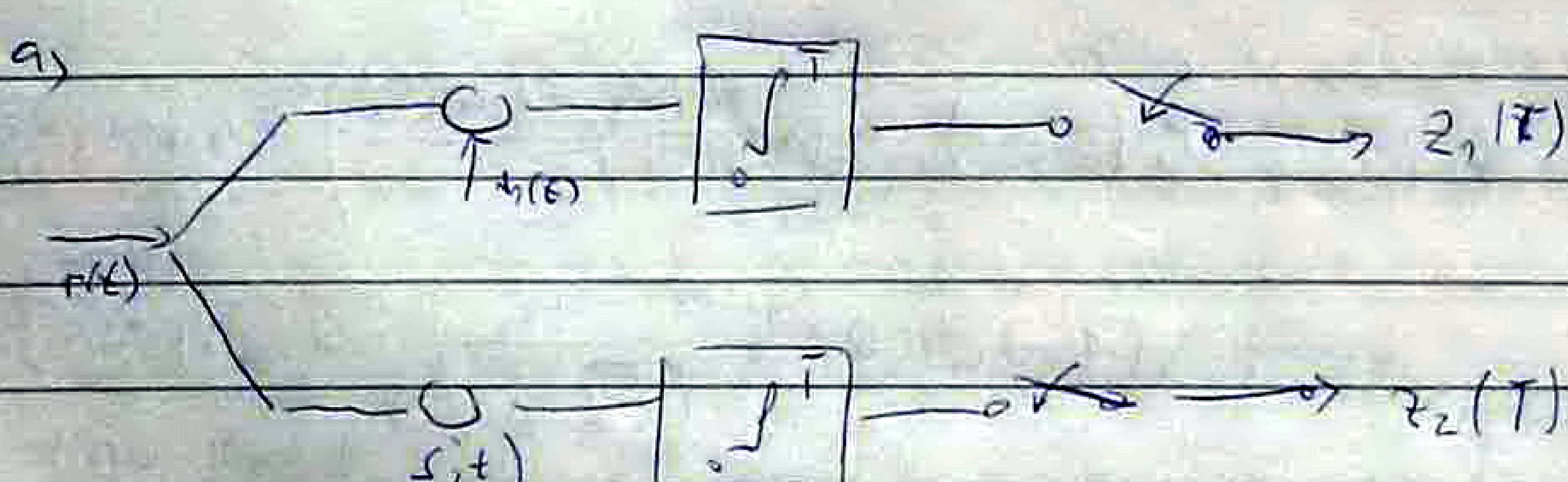
7. Aký bude výstup SPF ak použijeme a) UNRZ b) RNRZ



$s_1(t) = 0 \quad 0 \le t < T$ „1"

$s_2(t) = A \quad$ — " — „0"

a)



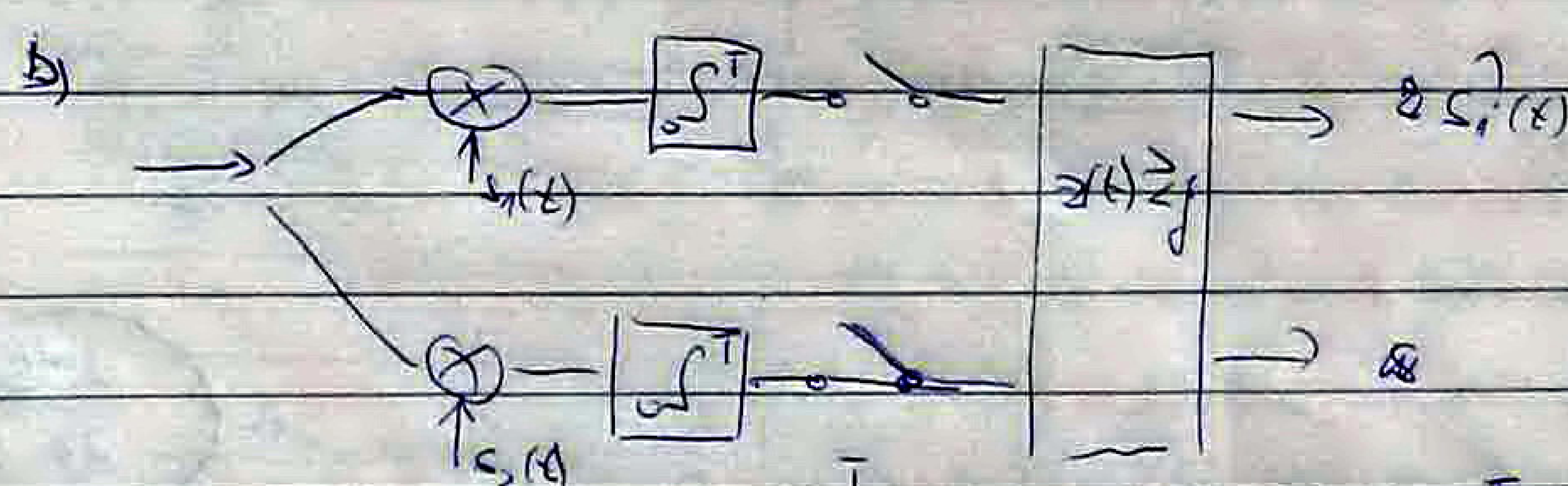
$r(t) = s_1(t) + n_0(t)$

(AWGN)

$a_1(T) = E\{z_1/s_1(t)\} = E\{\int_0^T r(t)\cdot s_1(t)\,dt\} = E\{\int_0^T [s_1(t)+n_0(t)]\, s_1(t)\,dt\} =$

$= E\{A^2 + A\,n_0(t)\,dt\} = \int_0^T E\{n_0(t)\cdot A^2\} = AT$

b)



v tomto prípade môžu vzniknúť 4 možnosti lebo máme dva sig. a dve možné detekcie, tieto vzduch hodnoty BNRZ pre $s_2 = -1$

$a_1(T) = E(z_1(T)/s_1) = E\{\int_0^T r(t)\, s_1(t)\,dt\} = E\{\int_0^T [s_1(t) + n_0(t)]\, s_1(t)\,dt\} =$

$= E\{A^2 + A n_0(t)\,dt\} = \int_0^T A^2\,dt + \int \underbrace{E\{n_0(t)\,A\}}_{0} = A^2T$

$a_2 \ldots E(z_1(T)/s_2) = -A^2T$

$a_3 \ldots E(z_2(T)/s_1) = -A^2T$

$a_4 \ldots E(z_2(T)/s_2) = AT$

CMSK 8.4.

GFT

- báza priestoru musí byť ortogonálna

$$\int_0^T \varphi_j(t)\cdot\varphi_k(t)\,dt = K_j\cdot\delta_{jk}$$

K_j - konšt. $\quad K_j = 1$ ortogonálna báza

$\{\varphi_j\}$ - báza $\quad j = 1 \dots N \quad \delta_{jk} = \begin{cases} 1 & j=k \\ 0 & j\neq k \end{cases}$

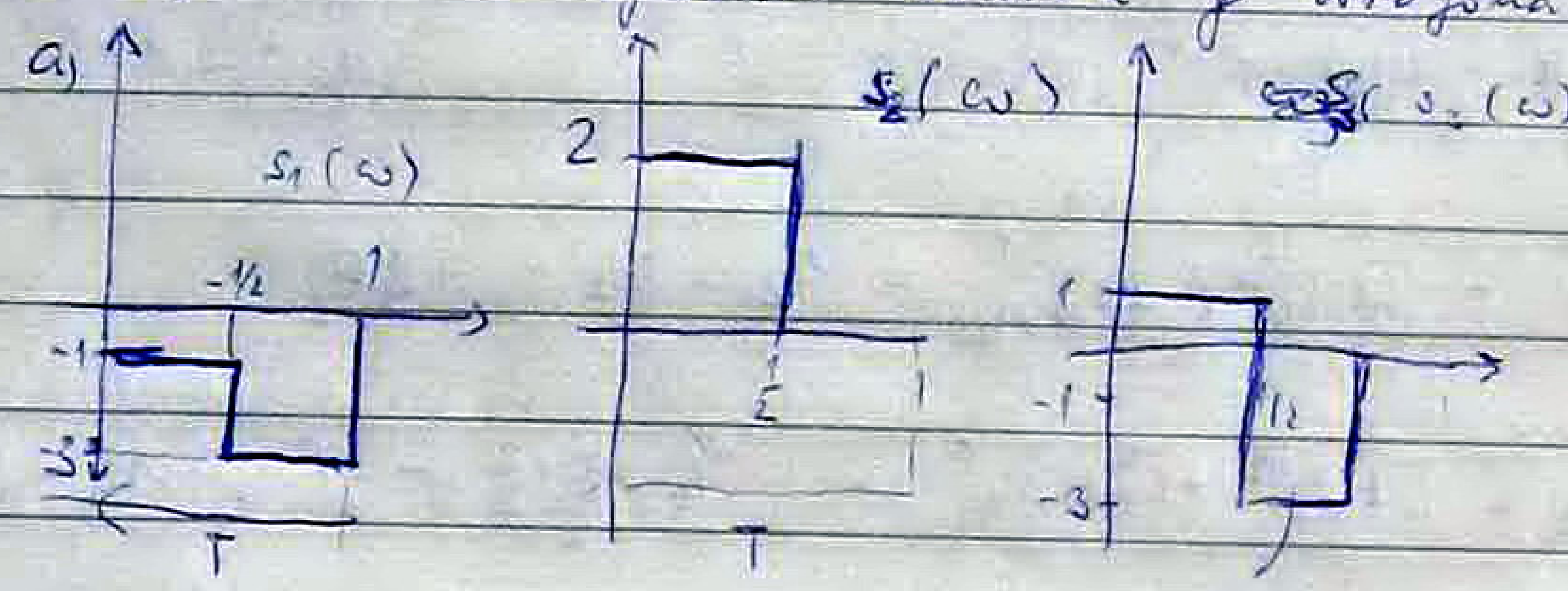
signál. abeceda $\quad s_i(t) = \sum_{j=1}^{N} a_{ij}\,\varphi_j(t) \quad i = 1 \dots M$

súradnice

$$a_{ij} = \frac{1}{K_j}\int_0^T s_i(t)\,\varphi_j(t)\,dt$$

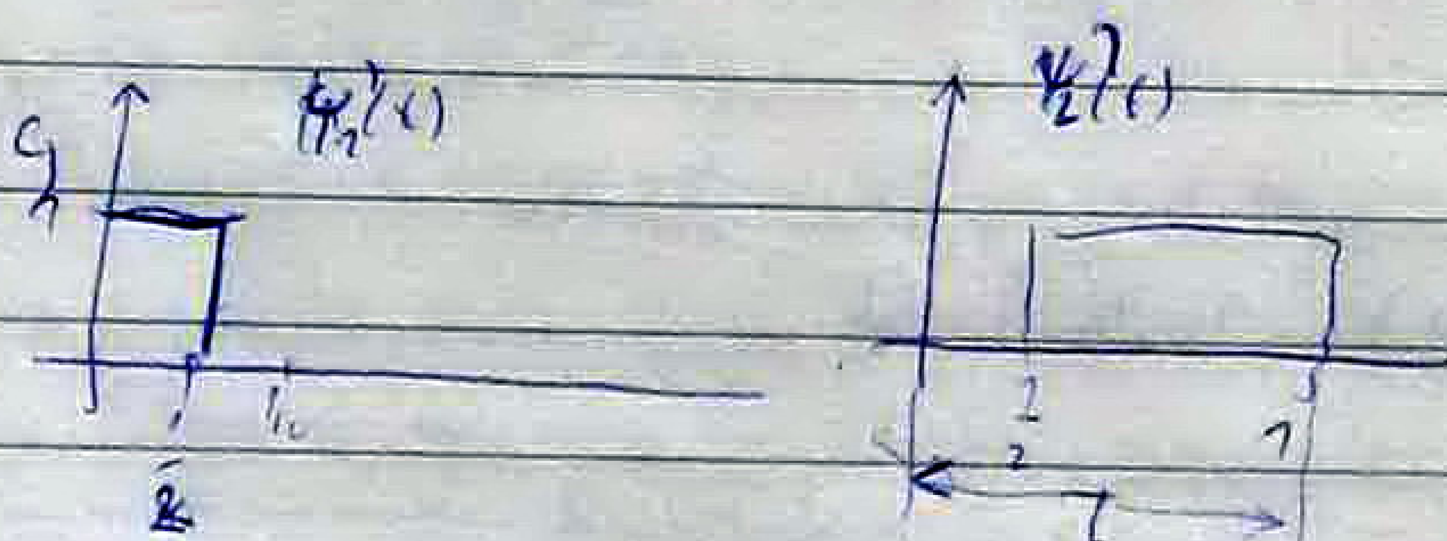
• Pr. Zistite, či z nasledujúcich množín je ortogonálna

a)



b)

c)



- ak bola množina ortogonálna, ~~ktoré ...~~

a) $s_1(t)\cdot s_2(t) = \int_0^1 s_1(t)\cdot s_2(t)\,dt = \int_0^{1/2} -1\cdot 2\,dt + \int_{1/2}^{1} -3\cdot 0\,dt = -2\cdot\frac{1}{2} = -1$

b) $\int_0^{1/2} 1\cdot 1\,dt + \int_{1/2}^{1} -1\cdot 1\,dt = [t]_0^{1/2} + [-t]_{1/2}^{1} = 0$ - báza

c) $\int_0^{1/2} 0\,dt + \int_{1/2}^{1} 0\,dt = 0$ - báza ~~ortonormálna~~

- ak sú signály závislé (pri a) sú) dajú sa vyjadriť pomocou alebo menej ~~signa~~ báz. signálmi

$k_1 = \int_0^T \varphi_1^2(t)\,dt = 1 \Rightarrow$ ortonormálna

$k_2 = \int_0^T \varphi_2(t)\,dt = 1$

$$a_{11} = \frac{1}{1}\int_0^T s_1(t)\cdot\varphi(t)\,dt = \int_0^{1/2}(-1)1\,dt + \int_{1/2}^1(-3)(-1)\,dt = -1/2 + \frac{3}{2} = \underline{1}$$

$$a_{12} = \int_0^{1/2}(-1)\,dt + \int_{1/2}^1(-3)\,dt = -\frac{1}{2} - \frac{3}{2} = -2 = \underline{-2}$$

$s_1 = [-1, -2]$

$$a_{21} = \frac{1}{1}\int_0^{1/2} 2\cdot 1\,dt + \int_{1/2}^1 0\cdot(-1)\,dt = 1+0 = 1$$

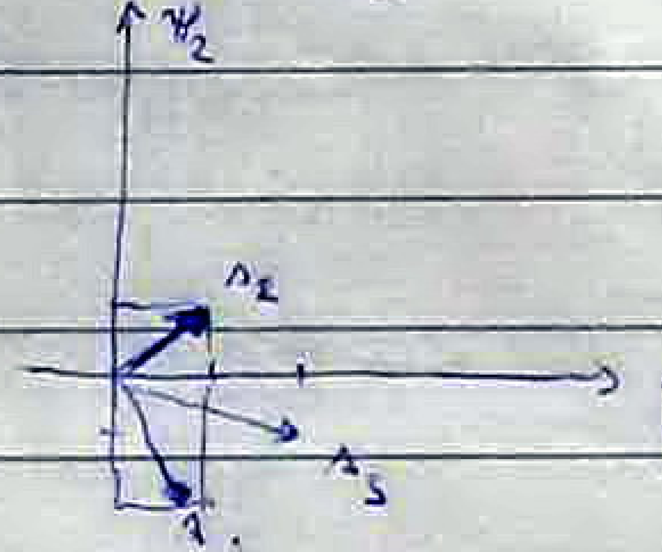
$$a_{22} = \int_0^{1/2} 2\cdot 1\,dt + \int_{1/2}^1 0\cdot 1\,dt = 1+0 = 1$$

$s_2 = [1, 1]$

$$a_{31} = \int_0^{1/2} 1\,dt + \int_{1/2}^1 -3\cdot 1\,dt = [t]_0^{1/2} + [-3t]_{1/2}^1 = 1/2 - 3/2 = +2,0$$

$$a_{32} = \int_0^{1/2} 1\cdot 1 + \int_{1/2}^1 -3\,dt = [t]_0^{1/2} + [3t]_{1/2}^1 = 1/2 - 3 + \frac{3}{2} = -1$$

$s_3 = [2, -1]$



Výsledky pri poslednej báze:

$k'_1 = 1/2 = k'_2$; $s_1 = [-1, -3]$, $s_2 = [2, 0]$, $s_3 = [1, -5]$

Gramm-Schmidtova metóda hľadania bázových funkcií

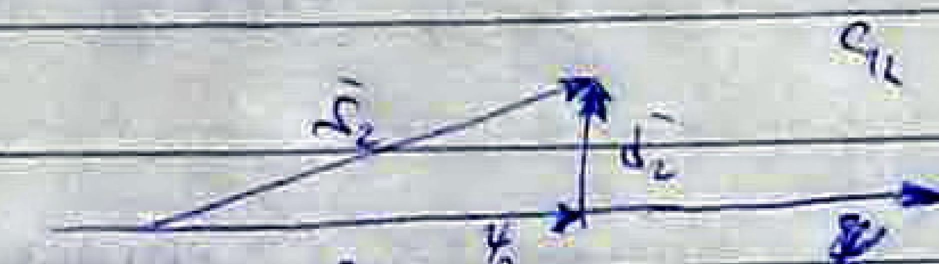
$S = \{s_1(t);\ s_2(t);\ \ldots\ s_M(t)\}$

predpoklad. $s_i(t)$ sú reálne sig. $i = 1 \ldots M$

$\varphi_1(t) = \frac{s_1}{\sqrt{E_1}}$ $\quad E_1 = \int_0^T s_1^2(t)\,dt$

$\varphi_2(t) = \frac{d_2(t)}{\sqrt{E_2}}$ $\quad E_2 = \int_0^T d_2^2(t)\,dt$ $\quad d_2(t) = s_2(t) - c_{12}\varphi_1(t)$

$c_{12} = \int_0^T s_2(t)\cdot\varphi_1(t)\,dt$



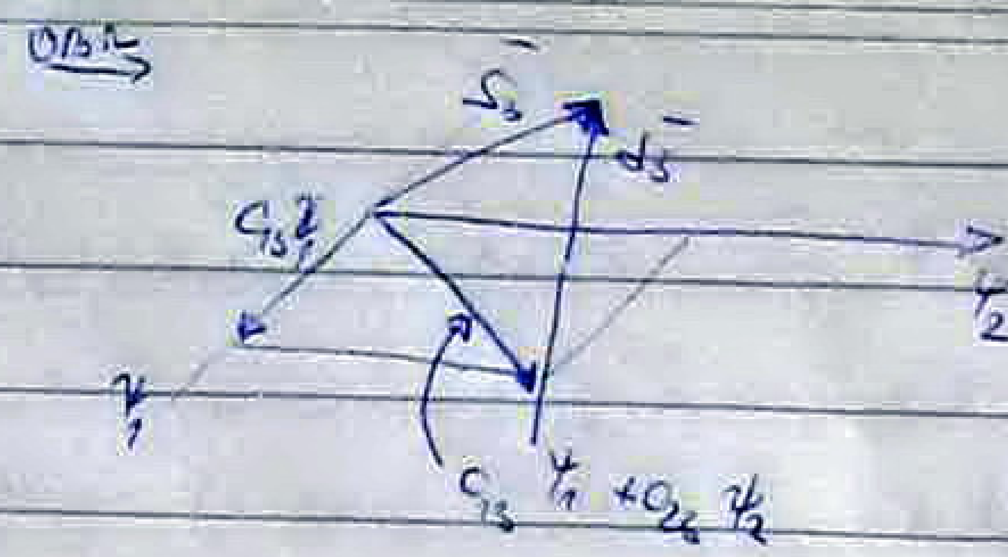
$\varphi_3(t) = \frac{d_3(t)}{\sqrt{E_3}}$ $\quad E_3 = \int_0^T d_3^2(t)\,dt$ $\quad d_3(t) = s_3(t) - c_{13}\varphi_1(t) - c_{23}\varphi_2(t)$

$c_{13} = \int_0^T s_3(t)\varphi_1(t)\,dt$

obr. →

$c_{23} = \int_0^T s_3(t)\varphi_2(t)\,dt$

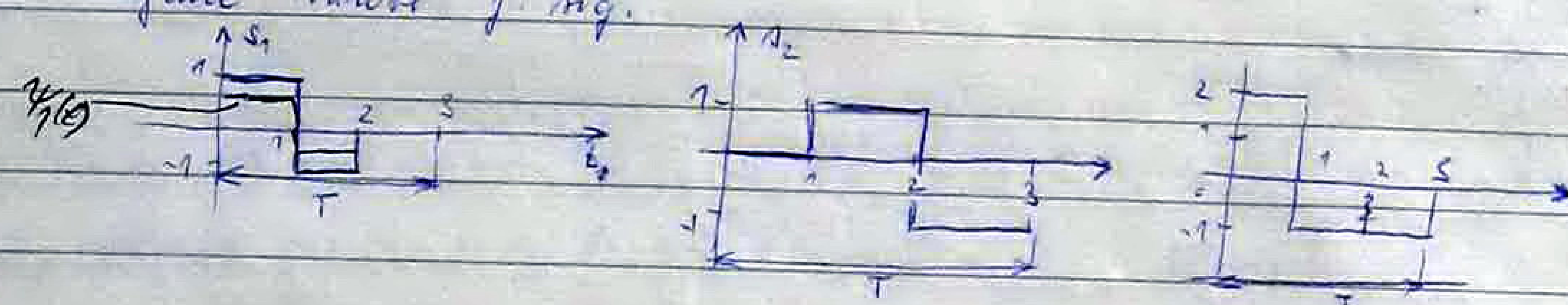
OBR



⋮

$$\psi_e(t) = \frac{d_e(t)}{\sqrt{E_e}} \qquad E_e = \int^T d_e^2(t)\,dt \qquad d_e(t) = s_e(t) - \sum_{j=1}^{e-1} c_{je}\psi_j$$

- Př: Najděte bázové f. sig.



$$E_1 = \int_0^T s_1^2(t)\,dt = 2$$

$$\psi_1 = \frac{s_1}{\sqrt{2}}$$

2. b. f.

$$\psi_2(t) = \frac{s_2(t)}{\sqrt{E_2}}$$

$$c_{12} = \int_0^T s_2(t)\cdot\psi_1(t)\,dt$$

$$c_{12} = \int_0^1 0\,\frac{1}{\sqrt{2}} + \int_1^2 (-1)\left(\frac{-1}{\sqrt{2}}\right) + \int_2^3 (-1)\,\frac{1}{\sqrt{2}}(0)$$

$$c_{12} = \frac{-2}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$$

$$d_2(t) = s_2(t) - c_{12}\psi_1(t) = s_2(t) + \frac{1}{\sqrt{2}}\cdot\frac{s_1(t)}{\sqrt{2}}$$

$$d_2(t) = s_2(t) + \frac{s_1(t)}{2}$$

$$E_2 = \int_0^T d_2^2(t) = \int_0^1 \left(\frac{1}{2}+0\right)^2 dt + \int_1^2 \left(1-\frac{1}{2}\right)^2 dt + \int_2^3 (-1)^2\,dt =$$

$$= 1/4 + 1/2 \cdot 1/4 + 3 - 2 = \frac{6}{4} = \frac{3}{2}$$

$$\psi_2(t) = \frac{s_2 + \frac{s_1(t)}{2}}{\sqrt{\frac{3}{2}}}$$

$$= \frac{1}{\sqrt{6}}\cdot s_1(t) + \frac{2}{\sqrt{6}}\,s_2(t)$$

$$c_{13} = \int^T s_3\cdot\psi_1 = \int_2^1 2\cdot\frac{1}{\sqrt{2}} + \int_1^2 (-1)\left(-\frac{1}{\sqrt{2}}\right) + \int_2^3 (-1)(0) =$$

$$= \frac{2}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{3}{\sqrt{2}}$$

$$c_{23} = \int^T s_3(t)\,\psi_2(t) = \int^1 2\cdot\frac{1}{\sqrt{6}} + \int_1^2 (-1)\cdot\frac{1}{\sqrt{6}}\,dt + \int_2^3 (-1)\cdot\left(-\frac{2}{\sqrt{6}}\right) dt$$

$$= \frac{2}{\sqrt{6}} - \frac{1}{\sqrt{6}} + \frac{2}{\sqrt{6}} = \frac{3}{\sqrt{6}}$$

$$d_3(t) = s_3(t) - c_{13}\psi_1(t) - c_{23}\psi_2(t) =$$

$$= s_3(t) - \frac{3}{\sqrt{2}}\cdot\frac{s_1(t)}{\sqrt{2}} - \left(\frac{3}{\sqrt{6}}\right)\left(\frac{s_1(t)}{\sqrt{6}} + \frac{2}{\sqrt{6}}\right) =$$

$$d_3(t) = s_3(t) - \frac{3}{2}s_1(t) - \frac{1}{2}\left(s_1(t) + s_2(t)\right) =$$

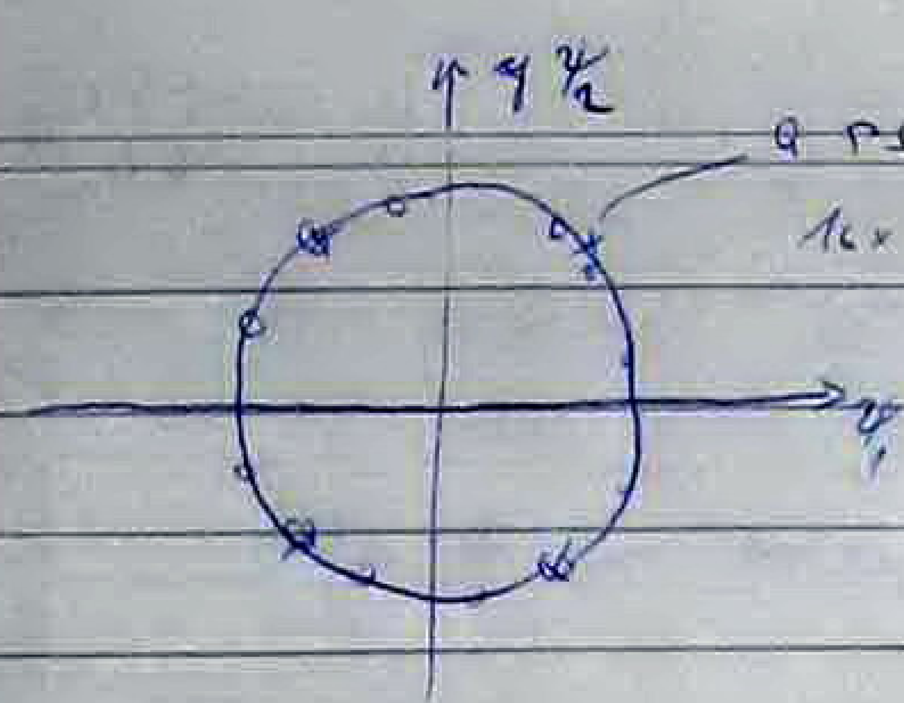
$$= s_3(t) - 2s_1(t) - s_2(t) = 0$$

$$\psi_3(t) = 0$$

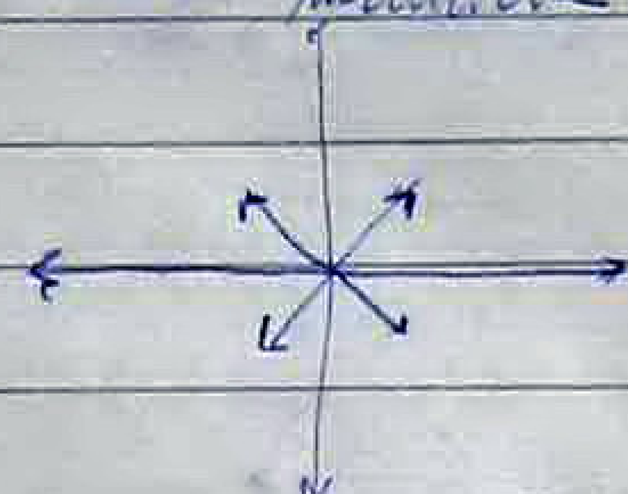
$$\bar{s}_1 = [\sqrt{2}, 0]$$

$$\bar{s}_2 = \left[-\frac{1}{\sqrt{2}}; \frac{3}{\sqrt{6}}\right]$$

$$\bar{s}_3 = \left[\frac{3}{2}; \frac{3}{\sqrt{6}}\right]$$



8QUAM modulácia



prenosová vs symbolová rýchlosť

CHSK
11.4.

Nebinárne modulácie

počet stavov - M;

M = 2; - binárne mod.

M > 2; nebinárne mod.

det. - ... / ...

det. prekl.

M>2.

1. ked BPSK - antipodálne stavy

$P_B = Q\{\sqrt{\frac{2E_B}{N_0}}$

$-\sqrt{E_B}$ 0 $\sqrt{E_B}$

$d = 2\sqrt{E_B}$

2. koh. BFSK → ortog. sig.

$P_B = Q\{\sqrt{\frac{E}{N_0}}$

$\Delta f = \frac{1}{2T}$

Δ $\frac{1}{2T}$ - BFSK pre ... rovnakú P_B ako BPSK potrebuje asi 3 dB väčší $\frac{E_b}{N_0}$

3) nekoh. príjem.

nekoh. BDPSK $P_B = \frac{1}{2}e^{-\frac{E_B}{N_0}}$ BDPSK potrebuje o 1 dB väčší $\frac{E_B}{N_0}$ ako BDPSK

(Wi-Fi)

MSK 11.4.

4. nekoh. BFSK $P_B = \frac{1}{2} e^{-\frac{E_B}{2N_0}}$

výh. .. jednoduchý prijímač

$M>2$ J. koh MPSK $P_S(M) \simeq 2Q\left(\sqrt{\frac{2E_S}{N_0}} \sin\frac{\pi}{M}\right)$

6. nekoh. MPSK $P_S(M) \simeq 2Q\left(\frac{2E_S}{N_0} \sin\frac{\pi}{\sqrt{2}M}\right)$

7. koh. MFSK $P_S \leq (M-1)Q\left(\sqrt{\frac{E_S}{N_0}}\right)$

8. nekoh MFSK $P_S = \frac{1}{M} \exp\left[-\frac{E_S}{N_0}\right] \sum_{j=2}^{M} (-1)^j \binom{M}{j} \exp\left[\frac{E_S}{j N_0}\right]$

ortog. MFSK $P_B = \frac{\frac{M}{2}}{M-1} P_S = \frac{2^{k-1}}{2^k-1} P_S$

neortog. MFSK $P_B = \frac{P_S}{\log_2 M} = \frac{P_S}{k}$ lebo sa neponúžija Grayov kód

- Pr Systém s BPSK mod. urobí 100 chýb za deň, $R_0 = 1$ kbps, $N_0 = 10^{-10} \frac{W}{Hz}$
  a) $P_b = ?$
  b) Bude stačiť na zabezpečenie danej P_B (z a)) sl. výkon sig. $S = 10^{-6}$ W?

a) $P_S = Q\left(\sqrt{\frac{2E_B}{N_0}}\right)$ $R_0 = 1$kbps $\simeq \frac{1}{1000} = T$ $\Delta f = \frac{1}{2000}$

$P_{sek} =$ ~~...~~ $1{,}15740 \cdot 10^{-3} \Leftarrow \frac{100}{24 \cdot 60 \cdot 60}$

$P_B = P_{chyb\,sek} \cdot T = 1{,}157$

jedno výkon, ale označuje S

b) $S\cancel{B} = E/T$ $E_B = S \cdot T = 10^{-9}$

$P_{B_1} \geq Q\left(\sqrt{\frac{2E_B}{N_0}}\right) = \sqrt{\frac{2 \cdot 10^{-9}}{1 \cdot 10^{-10}}}$

$Q(4{,}47) = \underline{4{,}09 \cdot 10^{-6}}$ $\Rightarrow$ NESTAČÍ

Pr. Kt. systém má nižšiu P_B?
a) ortog. koh BFSK $\frac{E_B}{N_0} = 12$dB $P_B =$ ~~...~~
b) ortog. nekoh BFSK $\frac{E_B}{N_0} = 14$dB $P_B =$ ~~...~~

a) $P_B = Q\left(\sqrt{\frac{E_B}{N_0}}\right) = Q(3{,}981) = 3{,}72 \cdot 10^{-5}$

~~...~~ $R = 10 \cdot \log_{10} X$

$X = 10^{1{,}2}$

b) $P_B = \frac{1}{2} e^{-\frac{E_B}{2N_0}} = \frac{1}{2} e^{-\frac{25{,}12}{2}} =$

$= 1{,}7558 \cdot 10^{-6}$

25.4.

Pr. Kt. systém je lepší vzhľadom k P_B. Predp. AWGN a použitie Grayovho kódu

a) koh. 8FSK $\frac{E_B}{N_0} = 8dB$

b) koh. 8PSK $\frac{E_B}{N_0} = 13dB$

a) $P_S = (8-1)\,Q\left(\sqrt{\frac{E_S}{N_0}}\right)$

$k = 3$ (lebo $2^3 = 8$)

$\frac{E_S}{N_0} = k\,\frac{E_B}{N_0}$

$8dB = 10 \log x$

$x = 10^{0,8}$

$\underline{x = 6,3095}$

$\underline{18,928 = 3 \cdot 6,3095} = \frac{E_S}{N_0}$

$P_S = 7 \cdot Q(4,3507) = 4,98 \cdot 10^{-5}$

$P_B = P_S \cdot \frac{4}{7}$

b) $10 \log_{10} \frac{E_B}{N_0} \Rightarrow x = \frac{E_B}{N_0} = \underline{19,95}$

$\frac{E_B}{N_0} \cdot k = \frac{E_S}{N_0} \Rightarrow \frac{E_S}{N_0} = \underline{59,85}$

$P_S(8) = 2Q\left(\sqrt{\frac{2E_S}{N_0}}\,\sin\frac{\pi}{M}\right)$

$= 2Q\left(\sqrt{2 \cdot 59,85}\,\sin\frac{\pi}{8}\right) = 2Q(4,187) = 2,972 \cdot 10^{-5}$

$\underline{P_B} = \frac{2,972 \cdot 10^{-5}}{\log_2 8} = 9,906 \cdot 10^{-6}$

Pr. Syst. používa koh. 8FSK. Všetky signály sú rovnako pravdepodobné.

$S_i(t) = A \cdot \cos 2\pi f_i T$ $\quad i = 1 \dots 8$ $\quad$ pre $0 \leq t \leq T_S$ $\quad T_S = 0,2\,ms$

Prijatá amplitúda nosnej je 1mV. $\frac{N_0}{2} = 10^{-11} \frac{W}{Hz}$

$P_B = ?$

$P_S = (M-1)\,Q\left(\sqrt{\frac{E_S}{N_0}}\right)$

$E_S = \frac{A^2 T_S}{2} = \frac{1 \cdot 10^{-6} \cdot 0,2 \cdot 10^{-3}}{2} = 10^{-10}\ J$

$= 7 \cdot Q\left(\sqrt{\frac{10^{-10}}{2 \cdot 10^{-11}}}\right) = 7 \cdot Q(\sqrt{5}) = 7 \cdot Q(2,236) = 0,0903$

$\frac{4}{7}\,0,0875 = P_B = 0,05$

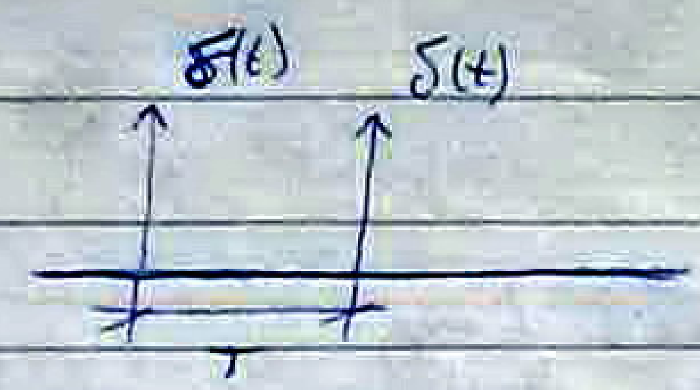
»PÍSOMKA ZREJME V 12. TÝŽDNI !

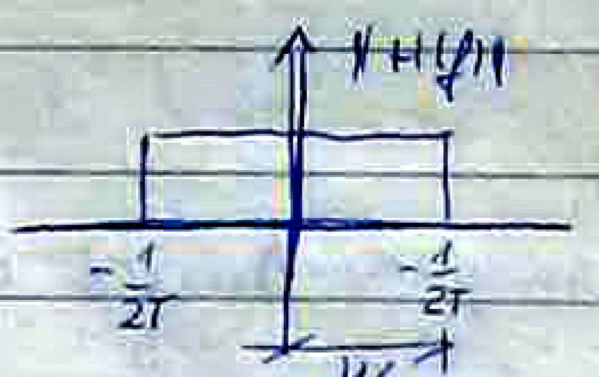
MSK 25.4

ISI (Inter Symbol Interference) – **Medzisymbolová interferencia**

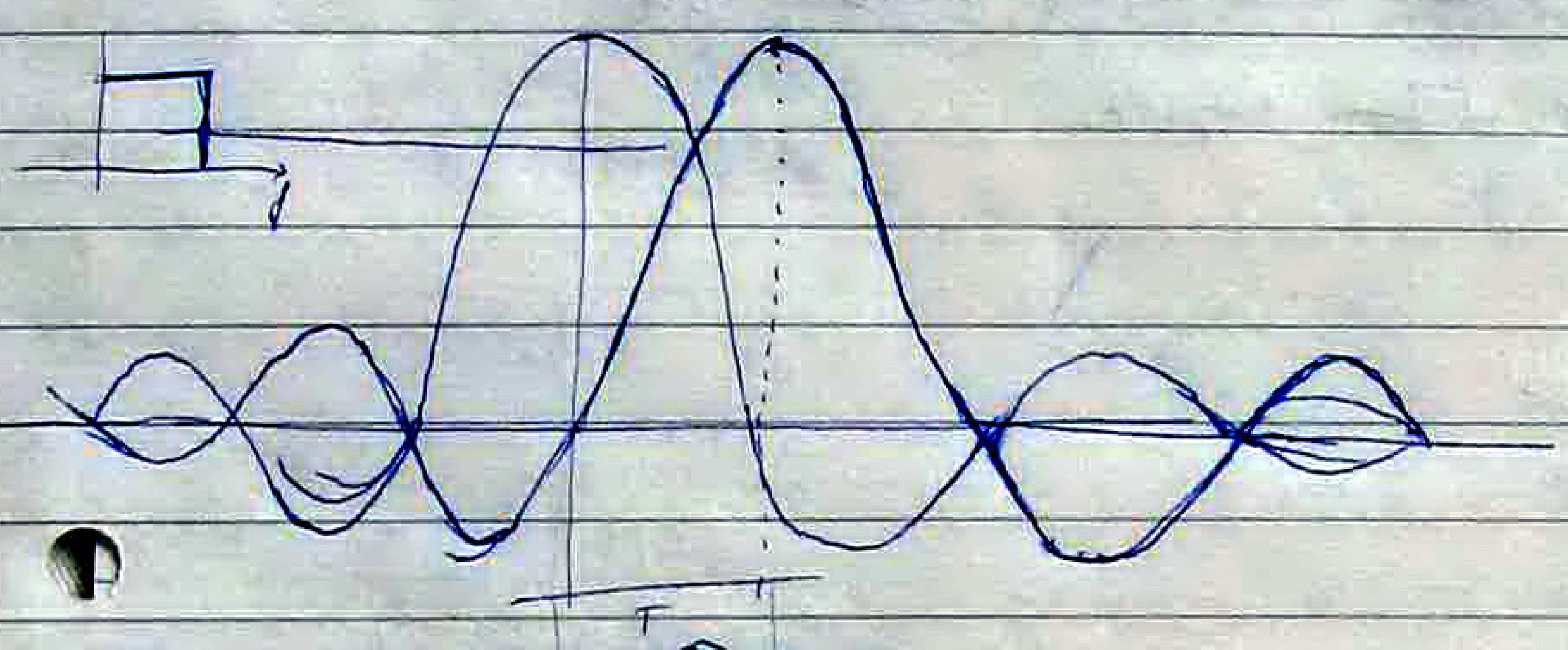
Kapacita kanála – max. teoretická prenosová rýchlosť, keď sa chybovosť P_B priblíži k nule

$C = 2W_0 \log_2 M$ [bps]





$\rightarrow ISI \overset{!}{\Rightarrow} 0$

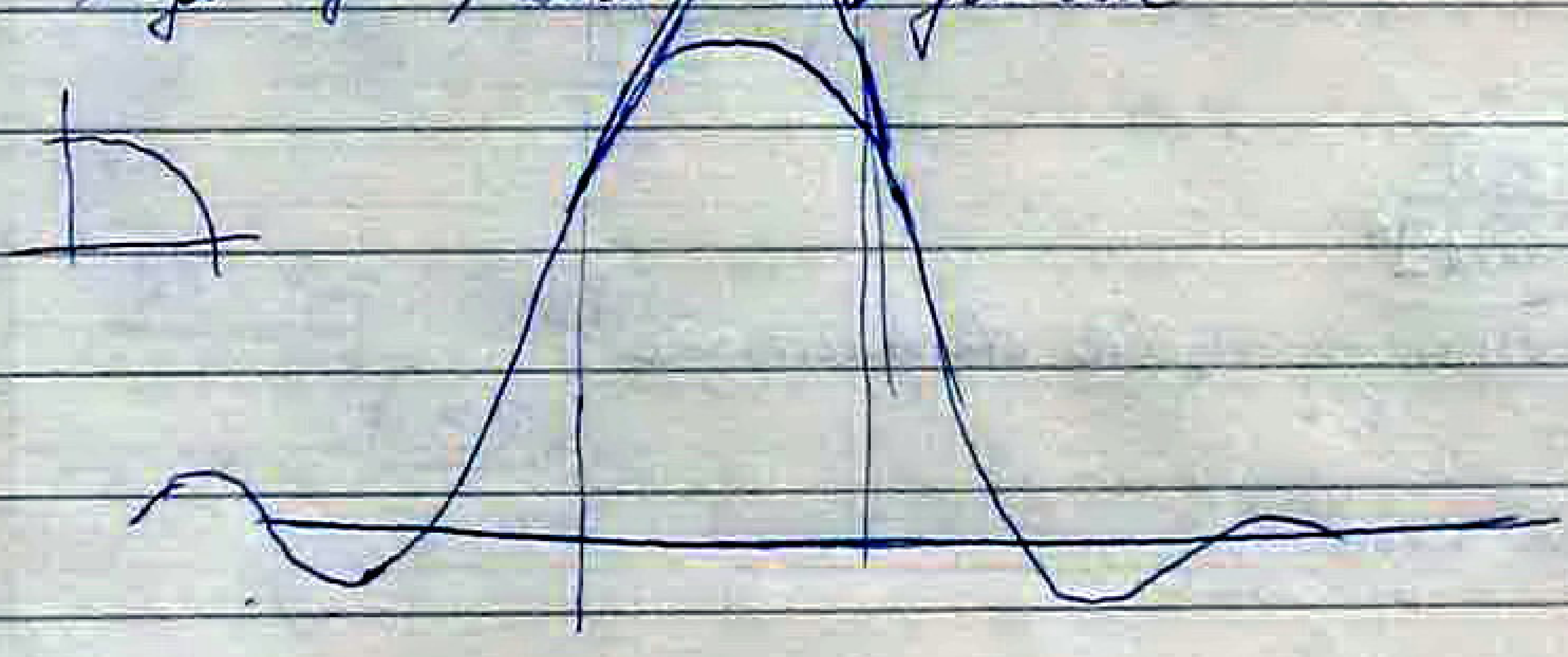


$$\boxed{W_0 = \frac{1}{2T}}$$ Nyquistova šírka pásma

minimálna nosná šírka pásma

$R_S = 2W_0$ max. modulačná rýchlosť bez ISI

↗ majú byť posunuté ... funkcie



Nyquistova hranica sa nedá realizovať lebo:

1.) Nyquistov filter je nerealizovateľný (nekauzálny)

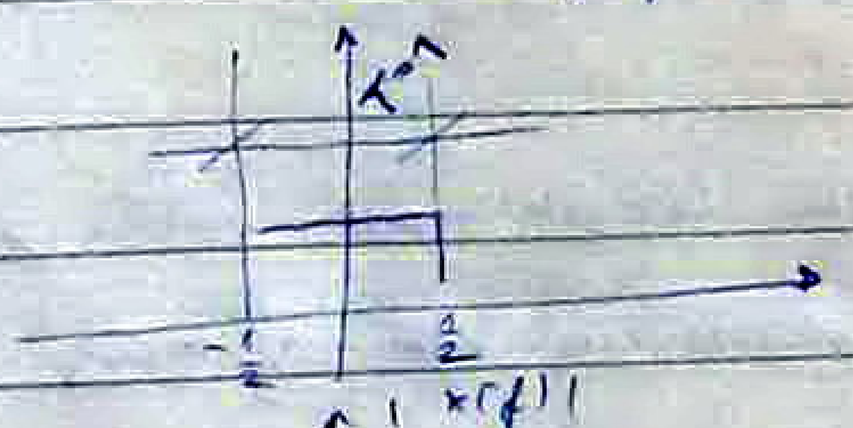
2.) absolútna ideálna synchronizácia neexistuje (vždy je prítomná neistota v synchronizácii - "jitter")

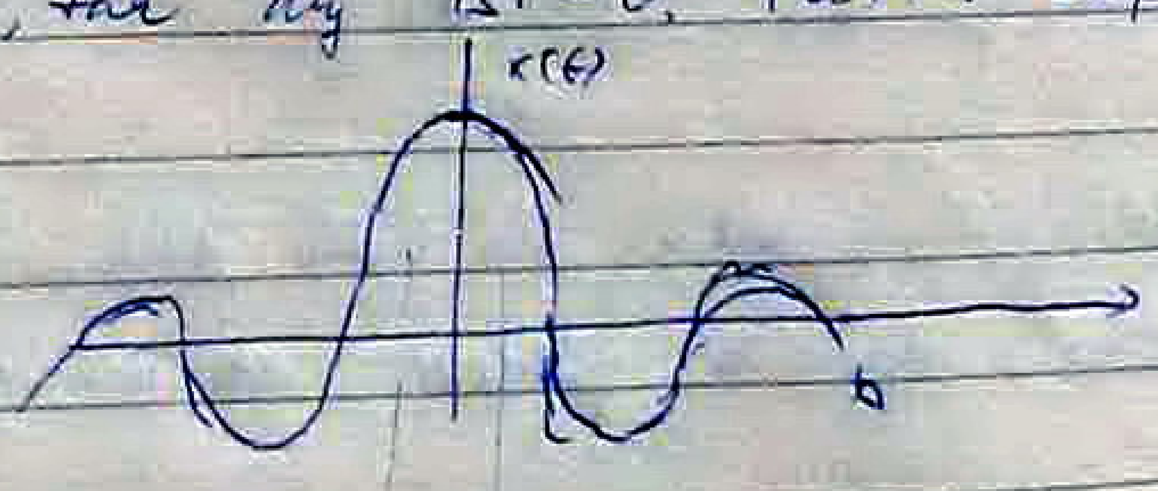
Ak chcem dosiahnuť stav $R_S = 2W_0$ pri $ISI \overset{!}{=} 0$ potom možno:

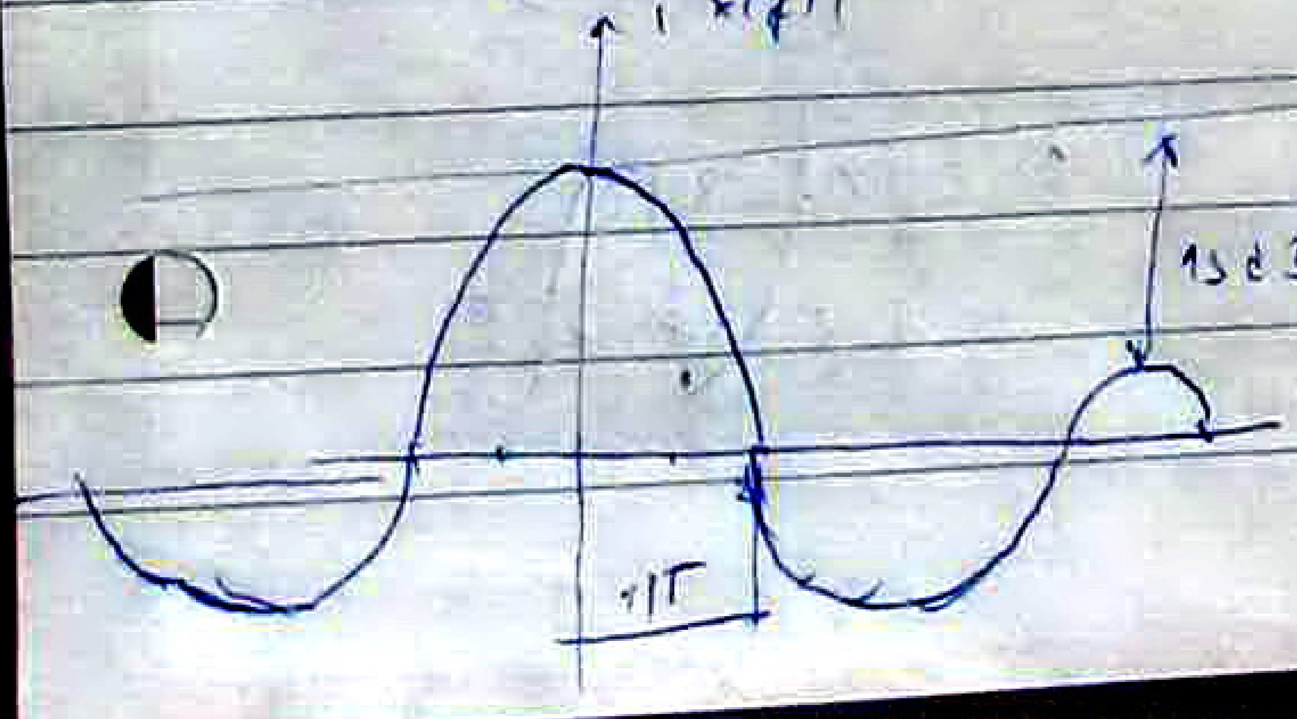
1.) zväčšenie reálnej šírky pásma $W > W_0$ a použijeme impulzy tvarované pomocou Raised Cosine Filtra (RCF)

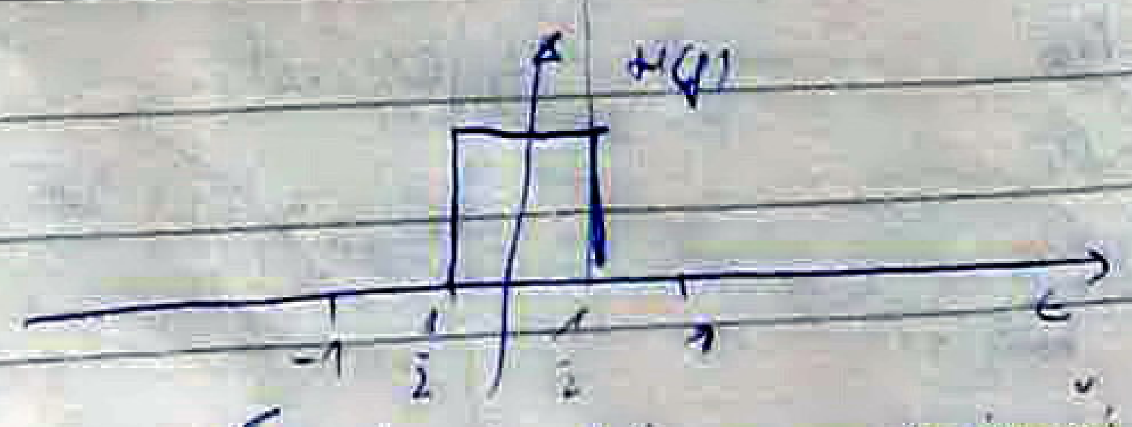
2.) $W = W_0$ a na vysielacej strane vnesieme korelovanú $ISI \neq 0$, kt. na strane prijímača vieme odstrániť, tak aby $ISI \overset{!}{=} 0$. Partial Response signaliz.

a)









↳ impulz bl. má sa magnitúde tvare

$\frac{1}{2T}$

$\frac{1}{2T}$

Nyquistove imp.

- $\mathrm{sinc}\left(\frac{t}{T}\right) *$ reálna časť funkcie:

1. Trieda Nyquistových imp. sa nazýva Raised Cosine Imp.

Roll - off faktor $\qquad \rho = \frac{W - W_0}{W_0} \rightarrow$

- pomerné zvýšenie real. šírky oproti ideálnej oproti W_0

$\rho = \langle 0, 1 \rangle \qquad \rho = \begin{cases} 0 & \text{čist. } \cancel{\text{signál}} \text{ sinc impulz v časovej obl.} \\ 1 & \end{cases}$

čistý $\sqcap$ impulz.

Reálna šírka pásma:

$W = \frac{1}{2}(1 + \rho) R_S \qquad$ pre ZP

$W = \cancel{1/2}\,(1 + \rho) \qquad$ pre AP

$\wedge$ prax: $\rho = \langle 0{,}2,\ 0{,}4 \rangle$

1) RCF

$$h_{RCF}(t) = \underbrace{2W_0\, \mathrm{sinc}(2W_0 t)}_{\text{si imp}}\ \underbrace{\frac{\cos[2\pi(W - W_0)]}{1 - [4(W - W_0)t]^2}}_{\text{cos. korekcia} \rightarrow \text{rýchle doznievanie}}$$

$\wedge$ nalepenie prechody

nulami v $\frac{k}{T}$ $\quad k = 1, 2 \ldots$

$$H_{RCF}(f) = \begin{cases} 1 & |f| < 2W_0 - W \\ \cos^2\left\{\frac{\pi}{4}\, \frac{|f| - W + 2W_0}{W - W_0}\right\} & 2W_0 - W \leq |f| < W \\ 0 & |f| > 0 \end{cases}$$

realizácia: (vys. strana) $\quad \leftarrow$ prijímacia strana

$\boxed{\sqrt{RCF}}$ — $\boxed{\text{kanál}}$ — $\boxed{\sqrt{RCF}}$

RRCF - root RCF

$H_{celk} = RCF(f)$

- Pr Sat. transpondér má šírku pásma $W = 36\,MHz$. Používa QPSK mod. Aká je max. možná prenosová rýchlosť ak na tvarovanie impulzov použijem RCF $\rho = 0{,}3$

$W_0 \rho = W - W_0$

$\cancel{W_0 \rho + W_0 = W}$

$\cancel{W_0 = \frac{W}{1 + \rho}}$

$W_0 = 2{,}769 \cdot 10^7$

$R_{S_{max}} = 5{,}538 \cdot 10^7$

$W = \frac{1}{2}(1 + \rho) R_S$

$= 5\ 5{,}38 \cdot 10^6$

1/2 tu nie je ρ, lebo je to preloz. pásmo

$W = (1+\rho) R_S$ lebo je QPSK mod tak $R_b = 2\cdot R_s$, lebo QPSK má 4 stavy, t.j. 2 bit.

$R_S = \frac{W}{1+\rho} = 27\cdot 10^6$ baud

$R_b = 2R_s = 55$ Mbps

• Aké je min teor. pásmo v ZP potrebné na prenos 10 Mbps ak signál má 16 ~~bit~~ úrovní

b, Aké je reál. pásmo v ZP potrebné na prenos 10 Mbps ak na tvarovanie použijem RCF s $\rho = 0,1$

$W = \frac{1}{2}(1+\rho)R_S$

$R_S = \frac{10\ Mbps}{4} = 2,5$ Mbaud

modulač. rýchl.

~~$W = \frac{1}{2}(1+$~~ $W_0 = \frac{R_S}{2} = 1,25$ MHz

b)

$W = \frac{1}{2}(1+\rho) R_S$

$W = \frac{1}{2}(1+0,1)\, 2,5$ Mbaud

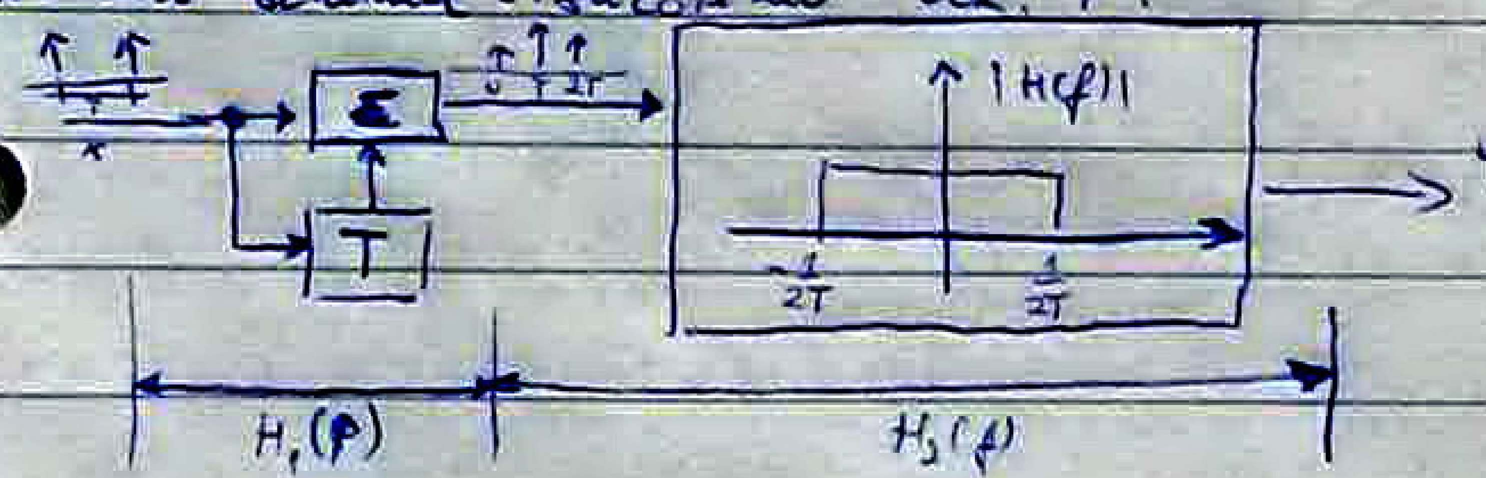
$W = 1,375$ MHz

PR Signalizácia:

$W = W_0$, 1963 – Adam Lender (duobinárna metóda)

tvarovanie pomocou cosinusového filtra (nerealizovateľný, ale dá sa aproximovať)

Blokové schéma číslicového lin. F.



$H_1(f) = 1 + e^{-j2\pi fT}$

$H_2(f) = \begin{cases} T & |f| \le \frac{1}{2T} \\ 0 & |f| > \frac{1}{2T} \end{cases}$

$H_c(f) = H_1(f)\cdot H_2(f)$

$|H_c(f)| = \begin{cases} 2T\cos \pi f T & |f| \le \frac{1}{2T} \\ 0 & |f| > \frac{1}{2T} \end{cases}$

• Pr Systém s BPNRZ $A = \pm 1$ a využíva PR signalizáciu. Na základe analýzy činnosti do obrodu ... prvotné dekódovanie.

	poč. podm.							
x_k	0	0	1	1	0	1	0	0
BPNRZ	-1	-1	1	1	-1	1	-1	-1
y_k		-2	0	2	0	0	0	-2

BPNRZ – 0 → -1, 1 → 1

$\tilde{y}_k$	-2	0	0	0	0	0	-2
$\hat{x}_k$	0	1	0	1	0	1	0

chyba

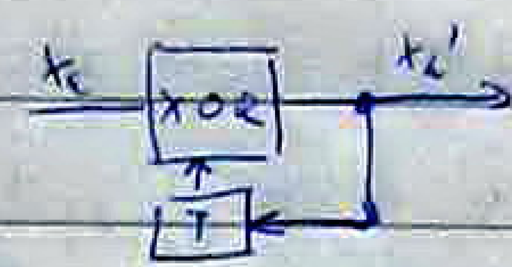
pre dekódovanie:

$y_k = -2 \rightarrow x_k = 0$ $\quad y_k = 0 \rightarrow x_k = \overline{x_{k-1}}$

$y_k = 2 \rightarrow x_k = 1$

predkódovanie

doplnenie pred prediktč. obvod



x_k	00	0	1	1	0	1	0	0
x_k':	0	0	1	0	0	1	1	1
BANEZ		-1	1	-1	1	+1	+1	+1
Y_k		-2	0	0	-2	0	2	2
$\hat{Y}_k$		-2	0	2	-2	0	2	2
x_k		0	1	$\boxed{0}$	0	1	0	0

↳ chyba sa ďalej nešíri.

dekód: $Y_k = -2 \rightarrow x_k = 0$

$Y_k = 2 \rightarrow x_k = 0$

$Y_k = 0 \rightarrow x_k = 1$