$\left(\left(f_1(x)\right)^{f_2(x)}\right)'$ $\quad x = e^{\ln x}$

$(x^n)' = n x^{n-1}$

$(x^2) = 2x^{2-1} = 2x$

$\left((4x-1)^{\cos x}\right)' = \left(e^{\cos x \cdot \ln(4x-1)}\right)'$

$\left((\sin x)^{\cos x}\right)' = \left(e^{\cos x \cdot \ln \sin x}\right)'$

$\sin\left(\frac{x+4}{\mathrm{tg}\, x + \mathrm{cotg}\, x} \cdot \ln\left(\mathrm{arctg}\left(7x + \cos x\right)\right)\right)$

$\sin\left(\cos\left(\mathrm{tg}\left(x^2 + \ln x^3\right)\right)\right)$

$\boxed{\text{Def.}}$ Derivácia funkcie $f(x)$ v bode x_0:

∇ 0 5k teória

$$(f(x))' = \frac{\partial f}{\partial x} = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

$\boxed{\text{Pr.}}$

$\alpha^2 - \beta^2 = (\alpha - \beta) \cdot (\alpha + \beta)$

$f(x) = x^2$

$$f'(x_0) = \lim_{x \to x_0} \frac{(x^2 - x_0^2)}{x - x_0} = \lim_{x \to x_0} \frac{\cancel{(x - x_0)}(x + x_0)}{\cancel{x - x_0}} = \lim_{x \to x_0} (x + x_0) =$$

$$= x_0 + x_0 = 2x_0$$

Derivácie elementárnych funkcií

12

1) $f(x) = c$ $\qquad (4)' = 0$

$f'(x) = 0$ $\qquad (7{,}5)' = 0$

2) $f(x) = x^n \qquad n \in N$

$(f(x))' = (x^n)' = n \cdot x^{n-1}$

$(x^5)' = 5x^4 \qquad (x^{-5}) = -5x^{-6}$

$(x^7)' = 7x^6 \qquad \left(\frac{1}{x^6}\right)' = (x^{-6}) = -6x^{-7}$

trigonom. f.

3) $(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(tg\, x)' = \frac{1}{\cos^2 x}$

$(cotg\, x)' = \frac{1}{-\sin^2 x}$

exponenciál. f.

$(a^x)' = a^x \cdot \ln a$

$(e^x)' = e^x$

4) cyklometr. f.

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$

$(arctg\, x)' = \frac{1}{1+x^2}$

$(arccotg\, x)' = -\frac{1}{1+x^2}$

$f(x) = x^r \qquad r \in (0;\infty)$

$(x^r)' = r\, x^{r-1}$

$(x^{3,5})' = 3{,}5 \cdot x^{2,5}$

5) logaritmické f.

$(\ln x)' = \frac{1}{x}$

$x > 0$

$(\log_a x)' = \frac{1}{x \ln a}$

1 MZ
25.9.2008

①

$$(\alpha_1 f_1(x) + \alpha_2 f_2(x) + \ldots + \alpha_n f_n(x))' =$$
$$= \alpha_1 f_1'(x) + \alpha_2 f_2'(x) + \ldots \pm \alpha_n f_n'(x)$$

Pr. $(3 \sin x + 5x^4 + 3 \ln x)' = 3 \cos x + 20x^3 + 3\frac{1}{x}$

$$[f_1(x) \cdot f_2(x)]' = f_1'(x) \cdot f_2(x) + f_1(x) \cdot f_2'(x)$$

Pr. $(3x^2 \cdot \ln x)' = 6x \cdot \ln x + 3x^2 \cdot \frac{1}{x}$

$$\left(\frac{f_1(x)}{f_2(x)}\right)' = \frac{f_1'(x) \cdot f_2(x) - f_1(x) \cdot f_2'(x)}{(f_2(x))^2}$$

Pr. $(\operatorname{tg} x)' = \left(\frac{\sin x}{\cos x}\right)' = \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

$$(f_1(x))^{f_2(x)} = e^{f_2(x) \cdot \ln f_1(x)}$$

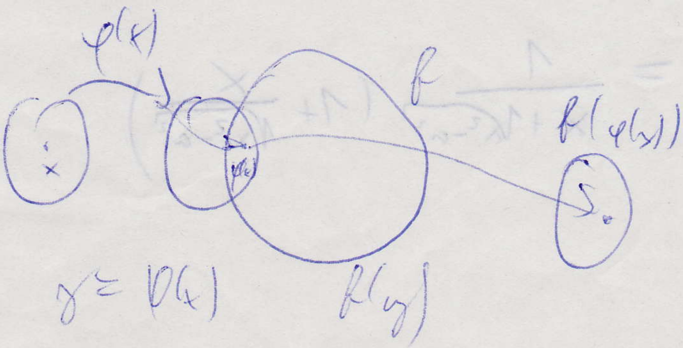
Pr. $(3x - \ln x)^{\operatorname{arctg} x} = e^{\operatorname{arctg} x \cdot \ln(3x - \ln x)}$

$$\left((\sin x)^{\operatorname{arctg} x}\right)' = \left(e^{\operatorname{arctg} x \cdot \ln(\sin x)}\right)'$$

Deriv. zloženej funkcie:

$F(x) = f(\varphi(x))$

$F(x) = \sin(2x)$



②

$d(F(x)) = f'(\varphi(x)) \cdot \varphi'(x)$

$y = f(u) \qquad u = \varphi(x)$

$y = f(\varphi(x)) \qquad \frac{dy}{du} = \frac{dy}{du} \cdot \frac{du}{dx}$

Pr.) $(\sin 2x)' = (\cos(2x)) \cdot (2x)' = (\cos 2x) \cdot 2$

$(\sin(\cos x))' = \cos u \cdot -\sin x = \cos(\cos x) - \sin x$

$(2\ln(x^2+3))' = 2(\ln(x^2+3))' = 2 \cdot \frac{1}{x^2+3} \cdot (x^2+3)' =$

$= 2 \frac{1}{x^2+3} \cdot 2x = 4 \frac{x}{x^2+3}$

$(2 \cdot x^2)' = 2(x^2)' = 2 \cdot 2x = 4x$

$\sin 2x' = 2 \cdot (\sin 2x) \cdot (\sin 2x)' =$

$= 2\sin 2x \cdot (\cos 2x) \cdot (2x)' =$

$= 2\sin 2x \cdot \cos 2x \cdot 2$

$f(x) = \ln(x + \sqrt{x^2 - a^2})$

$f'(x) = \frac{1}{x+\sqrt{x^2-a^2}} \cdot (x + \sqrt{x^2-a^2})' = \frac{1}{x+\sqrt{x^2-a^2}} \cdot (1 + (\sqrt{x^2-a^2})') =$

$= \frac{1}{x+\sqrt{x^2-a^2}} \left(1 + \frac{x}{\sqrt{x^2-a^2}}\right)$

2 M2
25.9.2008

$$(e^x)' = e^x$$

$$\left(e^{(4x^2+1)}\right)' = e^{(4x^2+1)} \cdot 8x$$

$$\left[(\sin(2x+1))^{tg(x^3+\ln x)}\right]' = \left(e^{tg(x^3+\ln x)\cdot\ln(\sin(2x+1))}\right)' =$$

$$= e^{tg(x^2+\ln x)\cdot\ln(\sin(2x+1))} \cdot \underbrace{\left[tg(x^3+\ln x)\cdot\ln(\sin 2x+1)\right]'}$$

$$\frac{1}{\cos(x^3+\ln x)}\cdot(x^3+\ln x)'\cdot\ln\sin(2x+1) +$$

$$tg(x^3+\ln x)\cdot\frac{1}{(\sin 2x+1)}\cdot\underbrace{(\sin 2x+1)'}_{[\cos(2x+1)\cdot 2]}$$

$$f(x) = \frac{1+x}{\sqrt{1-x}}$$

$$\left(\sqrt{1-x}\right)' = \left((1-x)^{\frac{1}{2}}\right)' =$$

$$= \frac{1}{2}(1-x)^{-\frac{1}{2}}\cdot(-1) = -\frac{1}{2}\,\frac{1}{\sqrt{1-x}}$$

$$f'(x) = \frac{1\cdot\sqrt{1-x} - (1+x)\cdot\left(-\frac{1}{2}\cdot\frac{1}{\sqrt{1-x}}\right)}{1-x}$$

$$\left(\sin^2\left(1+\frac{1}{\cos^2(4x+tg\,x)}\right)\right)' = \left[2\cdot\sin\left(1+\frac{1}{\cos^2(4x+tg\,x)}\right)\right]\cdot\left[\sin\left(1+\frac{1}{\cos^2(4x+tg\,x)}\right)\right]'$$

$$\left[\sin\left[1+\frac{1}{\cos^2(4x+tg\,x)}\right]\right]' = \cos\left(1+\frac{1}{\cos^2(4x+tg\,x)}\right)\cdot\left(1+\frac{1}{\cos^2(4x+tg\,x)}\right)'$$

P. v.
Nájsť priebeh funkcie

1 | M2
2. 10. 2008

$f(x) = x^3 - 5x^2 + 3x - 5$

$f'(x) = 3x^2 - 10x + 3$

$f''(x) = 6x - 10$

$f'(x) = 0$

$3x^2 - 10x + 3 = 0$

$D = b^2 - 4ac = 100 - 4 \cdot 3 \cdot 3 = 64$

$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{10 \pm \sqrt{64}}{6}$ $\begin{cases} 3 \\ \frac{1}{3} \end{cases}$

$6x - 10 = 0$

$2(3x - 5) = 0$

$x = \frac{5}{3}$

	$(-\infty; \frac{1}{3})$	$(\frac{1}{3}; \frac{5}{3})$	$(\frac{5}{3}; 3)$	$(3; \infty)$
f'	+ ↗	− ↘	+ ↗	+ ↗
f''	−	−	+	+

$f(\frac{5}{3}) = -\frac{250}{27}$ l. max

$f(\frac{1}{3}) = -23$ min

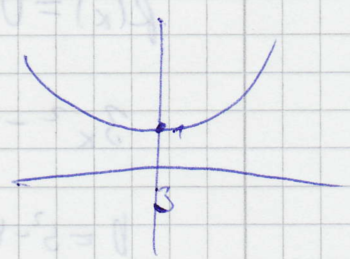
$f(x) = x^2 - 6x + 10$

$f'(x) = 2x - 6 \longrightarrow 2x - 6 = 0$

$x = 3$

$f''(x) = 2$

	$(-\infty; 3)$	$(3; \infty)$
f'	$-$	$+$



Neurčité integrály:

Ak $F(x)$, $f(x)$ sú def. na určitom intervale a platí $F'(x) = f(x) \Rightarrow F(x)$ je primitívna k funkcii $f(x)$.

Ak je $F(x)$ primit. k $f(x)$ na intervale J, tak aj $F(x) + C$ je primit. k $f(x)$

c – ľubovoľná konštanta

Ak existuje jedna primit. $\Rightarrow$ exist. aj nekonečne mnoho

Veta: ak je $F(x)$ primit. k $f(x)$ $\Rightarrow$ každá primit. $G(x)$ k $f(x)$

$G(x) = F(x) + c$

Veta: ak $F(x)$ je primit k f a G je

2 | M2

7. 10. 2008

Primit. funkciu k $f(x)$ nazývame neurčitý integrál funkcie $f(x)$ na intervale J a označujeme

$$\int f(x)\,dx$$

$$\left(\int f(x)\,dx\right)' = f(x)$$

Ak je F jedna z primit. ostatné sú $\int f(x)\,dx = F(x) + C$

$$\int 2x\,dx = x^2 + c$$

$F(x)$ $\quad x^2$

$F'(x) = 2x$

---

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$ $\quad \int \sin x\,dx = -\cos x + c$ $\quad \int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + c$
2. $\int \frac{1}{x}\,dx = \ln|x| + c$ $\quad \int \cos x\,dx = \sin x + c$ $\quad \int \frac{1}{1+x^2}\,dx = \operatorname{arctg} x + c$
3. $\int a^x\,dx = \frac{a^x}{\ln a} + c$ $\quad \int \frac{1}{\cos^2 x}\,dx = \operatorname{tg} x + c$
4. $\int e^x\,dx = e^x + c$ $\quad \int \frac{1}{\sin^2 x}\,dx = -\operatorname{cotg} x + c$

$F(x) = k_1 F_1(x) + k_2 F_2(x) + \dots k_n F_n(x)$

$F'(x) = k_1 F_1'(x) + k_2 F_2'(x) + \dots k_n F_n'(x) =$

$= k_1 f_1(x) + k_2 f_2(x) + \dots k_n f_n(x)$

Metóda Per Partes:

$$\int u(x) \cdot v'(x)\,dx = u(x)\,v(x) - \int u'(x) \cdot v(x)$$

$$\int \underbrace{x}_{u(x)} \cdot \underbrace{e^x}_{v'(x)}\,dx = x \cdot e^x - \int 1 \cdot e^x$$

$u'(x) = 1 \qquad v(x) = e^x$

$$\int \underbrace{x^2}_{u(x)} \cdot \underbrace{\sin x}_{v'(x)}\,dx = x^2 - \cos x + \int \overset{u(x)}{2x}\overset{v(x)}{\cos x}\,dx = x^2 - \cos x + 2\left(x \sin x - \int 1 \sin x\,dx\right) =$$

$$= x^2 \cdot \cos x + 2x \sin x - 2\int \sin x\,dx =$$

$u'(x) = 2x \quad -\cos x$

$$= x^2 - \cos x + 2x \sin x + 2\cos x + C$$

$$\int \sin^2 x \, dx = \int \underset{u(x)}{\sin x} \cdot \underset{v'(x)}{\sin x} \cdot dx = \sin x \cdot -\cos x - \int -\cos^2 x \, dx =$$

$u'(x) = \cos x \qquad v(x) = -\cos x$

$$= -\sin x \cos x + \int \cos^2 x \, dx =$$

$$= -\sin x \cos x + \int (1 - \sin^2 x) \, dx =$$

$$= -\sin x \cos x + \int 1 \, dx - \int \sin^2 x \, dx$$

$$2\int \sin^2 x \, dx = -\sin x \cos x + \int 1 \, dx =$$

$$2\int \sin^2 x \, dx = -\sin x \cos x + x$$

$$\int \sin^2 x \, dx = \frac{1}{2}(-\sin x \cos x + x)$$

$$\int \ln^2 x \, dx = \int \underset{v'(x)}{1} \cdot \underset{u(x)}{\ln^2 x} \, dx = x \ln^2 x - \int x \, 2\ln x \, \frac{1}{x} =$$

$$= x \ln^2 x - 2\int \ln x \, dx =$$

$v(x) = x$

$$= x \cdot \ln^2 x - 2x(\ln x - 1) + C$$

$u'(x) = (2 \ln x) \frac{1}{x}$

$$\int \ln x \, dx = \int \underset{v'(x)}{1} \underset{u(x)}{\ln x} \, dx = x \ln x - \int x \frac{1}{x} \, dx =$$

$$= x \ln x - \int 1 \, dx =$$

$v(x) = x$

$$= x \cdot \ln x - x = x(\ln x - 1)$$

$u'(x) = \frac{1}{x}$

---

1 | M2
6. 10. 2008

$f(x) = 3x^2 - 6x - 9 \Rightarrow 3.(x^2 - 6x - 9) = 0$

$f(x) = 6x - 6 = 6.(x-1)$

$x \neq 1$

$f''(x) = 6$

	$(-\infty; 1)$	$(1; \infty)$
$f'(x)$	$-$	$+$
$f''(x)$	$+$	

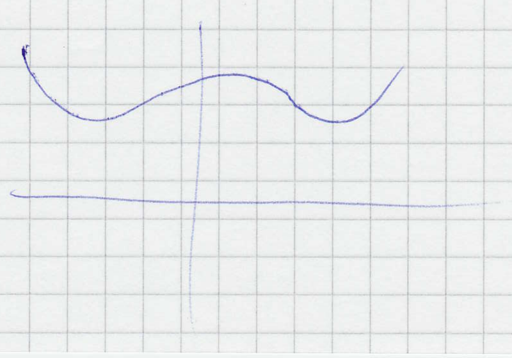
$f(x) = x^4 - 2x^2 + 3$

$f'(x) = 4x^3 - 4x \Rightarrow 4x.(x^2 - 1)$ $\quad x^2 - 1 = 0,\ x = \pm 1$ $\quad 4x = 0,\ x = 0$

$f''(x) = 12x^2 - 4 \Rightarrow 4.(3x^2 - 1)$ $\quad 3x^2 - 1 = 0,\ x = \pm \frac{1}{\sqrt{3}}$

	$(-\infty; -1)$	$(-1; 0)$	$(0; 1)$	$(1; \infty)$
f'	$-$	$+$	$-$	$+$

	$(-\infty; -\frac{1}{\sqrt{3}})$	$(-\frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}})$	$(\frac{1}{\sqrt{3}}; \infty)$
f''	$+$	$-$	$+$



1 | M2 | 9.10.2008

$$\int \frac{1}{\cos^2 x \cdot \sin^2 x} = \int \frac{\cos^2 x + \sin^2 x}{\cos^2 x \cdot \sin^2 x} \cdot dx = \int \frac{\cancel{\cos^2 x}}{\cancel{\cos^2 x} \sin^2 x} dx + \int \frac{\cancel{\sin^2 x}}{\cos^2 x \cancel{\sin^2 x}} =$$

$1 = \cos^2 x + \sin^2 x$

$$= \int \frac{1}{\sin^2 x} dx + \int \frac{1}{\cos^2 x} dx = -\operatorname{cotg} x + \operatorname{tg} x + c$$

$\cos 2x = \cos^2 x - \sin^2 x$

$\sin 2x = 2 \sin x \cdot \cos x$

$$\int \sqrt{x}\, dx = \int x^{\frac{1}{2}} dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3} \cdot \sqrt{x^3} + c$$

$$\int x^{\alpha} = \frac{x^{\alpha+1}}{\alpha+1} + c$$

$$\int 10^x = \frac{10^x}{\ln a} + c$$

$$\int \left(\frac{1-z}{z}\right)^2 dz = \int \frac{(1-z)^2}{z^2} dz = \int \frac{1 - 2z + z^2}{z^2} =$$

$$= \int \frac{1}{z^2} dz - \int \frac{2\cancel{z}}{z^{\cancel{2}}} dz + \int \frac{z^2}{z^2} dz =$$

$$= \int z^{-2} dz - 2 \int \frac{1}{z} dz + \int 1\, dz = \frac{z^{-1}}{-1} - 2 \ln|z| + z + c$$

$$\int \left(\frac{1 + \sqrt{x}}{\sqrt[3]{x}}\right)^3 dx$$

$$\frac{\sqrt{x}}{\sqrt[3]{x}} = x^{\frac{1}{2}} \cdot x^{-\frac{1}{3}} = x^{\frac{3-2}{6}} = x^{\frac{1}{6}}$$

$$\int \frac{\cos 2x}{\cos^2 x \cdot \sin^2 x} dx = \int \frac{\cos^2 x - \sin^2 x}{\cos^2 \sin^2 x} dx$$

$$\int \operatorname{tg}^2 x\, dx = \int \frac{\sin^2 x}{\cos^2 x} dx = \int \frac{1 - \cos^2 x}{\cos^2 x} dx = \int \frac{1}{\cos^2 x} dx - \int \frac{\cos^2 x}{\cos^2 x} dx =$$

$$= \operatorname{tg}^2 x - x + c$$

$$\int \frac{1}{\cos^2 x + \sin^2 x}\,dx = \int \frac{1}{\cos^2 x - \sin^2 x + \sin^2 x}\,dx = \int \frac{1}{\cos^2 x}\,dx = tg\,x + c$$

$$\int u(x) \cdot v'(x) = u(x) \cdot v(x) - \int u'(x) \cdot v(x)\,dx$$

$$\int \underset{u(x)}{f(x)}\, \underset{v'(x)}{g'(x)}$$

Substitučný spôsob integrovania

ide o integrál typu $\int f(\varphi(x)) \cdot \varphi'(x)\,dx$

$$\int \frac{1}{\cos x}\,dx = \int \frac{\cos^2 x + \sin^2 x}{\cos x}\,dx = \int \frac{\cos^2 x}{\cos x} + \int \frac{\sin^2 x}{\cos x}\,dx = \int \cos x$$

Majme $\int f(\varphi(x)) \cdot \varphi'(x)\,dx$

Nech $F(u)$ je primit. k $f(U)$: $F'(u) = f(u)$

$$\int f(u)\,du = F(u)$$

ak $u = \varphi(x)$

$$F'(\varphi(x)) = f(\varphi(x)) \cdot \varphi'(x)$$

$$(F'\varphi(x)) \cdot \varphi'(x) = f\,\varphi(x) \cdot \varphi'(x)$$

$$(F(\varphi(x)))' = F'(\varphi(x)) \cdot \varphi'(x)$$

2 | M2
9.10-2008

$$\boxed{\int f(\varphi(x))\cdot\varphi'(x)\,dx = F(\varphi(x)) + c}$$

$$\int \frac{(\ln x)^4}{x}\,dx = \int \frac{1}{x}(\ln x)^4\,dx = (\ln x)^5 + c$$

$\ln x = u$

$f(u) = u^4$

$F(u) = \frac{u^5}{5}$

$$\int 2x\sqrt{x^2+1}\,dx = F(\varphi(x)) + c = \frac{2}{3}\sqrt{x^2+1}$$

$f(\varphi(x)) = \sqrt{x^2+1}$

$\varphi(x) = x^2+1$

$u = \varphi(x)$

$f(u) = \sqrt{u}$

$F'(u) = f(u)$

$F(u) = \int f(u)\,du$

$F(u) = \int \sqrt{u}\,du = \int u^{\frac{1}{2}}\,du =$

$= \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2\sqrt{u^3}}{3}$

$$\int \frac{1}{(2x-3)^5} = \frac{1}{2}\int 2\cdot\frac{1}{(2x-3)^5} =$$

$f(\varphi(x)) = \frac{1}{(2x-3)^5}$

$\varphi(x) = 2x-3$

$f(u) = (u)^5$

$F(u) = \int u^5 = \frac{u^6}{6}$

$F(\varphi(x)) = \frac{1}{6}(4x-3)^6.$

1 M2
16-10-2008

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int \cot g\,dx = \int \frac{\cos x}{\sin x}\,dx = \ln|\sin x| + c$$

$$\int \frac{x}{x^2+3}\,dx = \frac{1}{2}\int \frac{\overset{f'(x)}{2x}}{\underset{f(x)}{x^2+3}}\,dx = \frac{1}{2}\ln|x^2+3|$$

Vzorec

$$\int \frac{1}{1-x^2}\,dx = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| + c$$

racional $\frac{f(x)}{g(x)}$ A

Veta: každá racionálna funkcia sa dá napísať ako polynóm + vyldzoracionálna
- získa sa to delením vichaého dohyým polyn.

Elementárne zlomky

- sú racionálne funkcie typu $\frac{A}{(x-\alpha)^n}$ alebo $\frac{ax+b}{(x^2+px+q)^n}$

$$\frac{f(x)}{g(x)} = \frac{a_n x^n + a_{n-1}x^{n-1} + \dots a_0}{b_k x^k + b_{k-1}x^{k-1} + \dots b_0}$$

$$\frac{6x-2}{(x-1)^2(x+7)^3(x^2-3x+4)^2(3x^2+x+1)} = \frac{A_1}{(x-1)} + \frac{A_2}{(x-1)^2} + \frac{B_1}{(x+7)} + \frac{B_2}{(x+7)^2} + \frac{B_3}{(x+7)^3} +$$

$$+ \frac{C_1x+D_1}{(4x^2-3x+4)} + \frac{C_2x+D_2}{(4x^2-3x+4)^2} +$$

$$+ \frac{Fx+k}{3x^2+x+1}$$

1 M2 20.10.2007

Tomáš Pátek

$$\frac{5x^3-15x^2+15x-3}{x^3-8x^2+17x-10}$$

$$5x^3-15x^2+15x-3 : x^3-8x^2+17x-10 = 5 + \frac{25x^2-70x+47}{x^3-8x^2+17x-10}$$

$$\underline{-(5x^3-40x^2+85x-50)}$$

$$0+25x^2-70x+47$$

x^2-8x

	1	-8	17	-10
		8	0	
8	8	0	17	

$$\frac{x^3+x-1}{x(x^2+1)} = \frac{x^3+x-1}{x^3+x} = 1-\frac{1}{x^3+x} = 1-\frac{1}{x(x^2+1)} =$$

$$= 1-\left[\frac{A}{x}+\frac{Bx+C}{(x^2+1)}\right] = 1-\frac{A}{x}-\frac{Bx+C}{x^2+1}$$

$$\frac{2x^2+7}{(x+1)^2(x+3)(x^2+2x+5)^3(x^2+x+4)} = \frac{A}{(x+1)}+\frac{B}{(x+1)^2}+\frac{C}{x+3}+\frac{D_1x+E_1}{(x^2+2x+5)}+$$

$$+\frac{D_2x+E_2}{(x^2+2x+5)^2}+\frac{D_3x+E_3}{(x^2+2x+5)^3}+\frac{Fx+G}{(x^2+x+4)}$$

$$\frac{x^6-12x^5+55x^4-137x^3+194x+40}{(x-1)(x-2)^3(x^2-2x+2)^2} = \frac{A}{x-1}+\frac{B}{x-2}+\frac{C}{(x-2)^2}+\frac{D}{(x-2)^3}+$$

$$+\frac{Ex+F}{(x^2-2x+2)}+\frac{Gx+H}{(x^2-2x+2)^2}$$

$$\frac{x^3+x-1}{x(x^2+1)} = 1 - \frac{A}{x} - \frac{Bx+C}{x^2+1} \qquad \Big/ \cdot x(x^2+1)$$

$$1 - \frac{1}{x} - \frac{-x}{x^2+1}$$

$$x^3+x-1 = 1x(x^2+1) - A(x^2+1) - (Bx+C)x$$

$$x^3+x-1 = x^3+x-Ax^2-A-Bx^2-Cx$$

$$Bx^2+1x^3+x-1 = 1x^3+(-A-B)x^2+(1-C)x-A$$

$-A-B=0 \qquad B=-A$

$1-C=1 \qquad B=-1$

$C=0$

$-A=-1 \qquad A=1$

1 | M2
23.12.2008

$$\int \frac{Ax+B}{(x^2+px+q)^8}\,dx \qquad \int \frac{A}{(x-3)^{10}}\,dx$$

$$\int tg\,x\,dx = \int \frac{\sin x}{\cos x}\,dx = -\int \frac{-\sin x}{\cos x}\,dx = \qquad \begin{aligned}\cos x &= t\\ -\sin x\,dx &= 1dt\end{aligned}$$

$$= \ln|\cos x| + c$$

$$\int \sin^5 x\,dx = \int \sin x \cdot \underbrace{\overbrace{\sin^4 x}^{(\sin^2 x)^2}}_{(1-\cos^2 x)^2}\,dx = -\int (1-t^2)^2\,dt$$

$$\cos x = t$$

$$\int \frac{(3x+2)}{(x-1)^2(x^2+3x-4)}\,dx$$

$$\frac{3x+2}{(x-1)^2(x^2+3x-4)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{(x^2+3x-4)} \quad \Big/ \cdot (x-1)^2(x^2+3x-4)$$

$$3x+2 = A(x-1)(x^2+3x-4) + B(x^2+3x-4) + (Cx+D)(x-1)^2$$

. . . . . .

$$\int \frac{A}{(x-B)^k}\,dx = A\int \frac{1}{(x-B)^k}\,dx = A\int \frac{1}{t^k}\,dt =$$

$$x - B = t \qquad = A\int t^{-k}\,dt = A\,\frac{t^{-k+1}}{-k+1} + c =$$

$$(1-0)\,dx = 1dt$$

$$dx = dt \qquad = \frac{A(x-B)^{-k+1}}{-(k+1)} = \frac{A}{k+1}\cdot\frac{1}{(x-B)^{1-k}}$$

$$\int \frac{1}{x^2+a^2}\,dx = \int \frac{1}{a^2\left(\frac{x^2}{a^2}+1\right)}\,dx = \frac{1}{a^2}\int \frac{1}{\left(\frac{x}{a}\right)^2+1}\,dx =$$

$$= \frac{1}{a^2}\int \frac{1}{t^2+1}\,dt$$

$\frac{x}{a} = t$ $\qquad$ $\frac{1}{a}\,dx = dt$

$\frac{1}{a}\cdot x = t$

1 M2 27.10.2008

$$\int \frac{4x+7}{x^2-6x+5}$$

$D = 36-20 = 16$

$$\int \frac{1}{x^2+a^2}\,dx = \int \frac{1}{a^2\left(\frac{x^2}{a^2}+1\right)}\,dx = \frac{1}{a^2}\int \frac{1}{\left(\frac{x}{a}\right)^2+1}\,dx =$$

$\frac{x}{a} = t$

$x = at$

$dx = a\,dt$

$$= \frac{1}{a^2}\int \frac{a\,dt}{1+t^2} = \frac{1}{a}\int \frac{1}{1+t^2}\,dt =$$

$$= \operatorname{arctg} t = \operatorname{arctg}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{x^2+3x+4}\,dx = \int \frac{1}{\left(x+\frac{3}{2}\right)^2+\frac{7}{4}}\,dx$$

$D = 9-16 = -7$

$$\int \frac{4x+7}{x^2-6x+10} = \int \frac{4x}{x^2-6x+10} + \int \frac{7}{x^2-6x+10} =$$

$$= 2\int \frac{2x-6}{x^2-6x+10}\,dx + 12\int \frac{6}{x^2-6x+10}\,dx + \int \frac{7}{x^2-6x+10} =$$

$$= 2\int \frac{2x-6}{x^2-6x+10}\,dx + \int \frac{12+7}{x^2-6x+10}\,dx =$$

$$\boxed{\int \frac{4x}{x^2-6x+10}\,dx = 4\int \frac{x}{x^2-6x+10}\,dx = 2\int \frac{2x-6}{x^2-6x+10} + 12\int \frac{6}{x^2-6x+10}}$$

$$= 2\int \frac{2x-6}{x^2-6x+10}\,dx + 19\int \frac{1}{x^2-6x+10}\,dx$$

$$2\ln|x^2-6x+10| + 19\operatorname{arctg}(x-3) + c$$

$$\int \frac{1}{3-2x^2}\,dx$$

$$\frac{1}{3-2x^2} = \frac{1}{3\cdot(1-\frac{2}{3}x^2)} = \frac{1}{3}\int \frac{1}{1-\left(\sqrt{\frac{2}{3}}x\right)^2}\,dx = \frac{1}{3}\int \frac{1}{1-t^2}\cdot\sqrt{\frac{3}{2}}\,dt =$$

$$\sqrt{\frac{2}{3}}x = t$$

$$\sqrt{\frac{3}{2}}\,dx = dt$$

$$dx = \sqrt{\frac{3}{2}}\,dt$$

$$= \frac{1}{3}\,\frac{\sqrt{3}}{\sqrt{2}}\int\frac{1}{1-t^2}\,dt$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \frac{a}{a}\int\frac{1}{\sqrt{1-t^2}}\,dt$$

$$\sqrt{a^2-x^2} = \sqrt{a^2\left(1-\frac{x^2}{a^2}\right)} = \sqrt{a^2}\cdot\sqrt{1-\left(\frac{x}{a}\right)^2} = a\sqrt{1-\left(\frac{x}{a}\right)^2}$$

$$\frac{x}{a} = t \qquad x = at$$
$$dx = a\,dt$$
