Riešenia príkladov z predmetu ELEKTROTECHNIKA 2, riadny termín, 11.1.2008

ET2/RT/11.1.08

1. $i(t) = C\dfrac{du(t)}{dt} = ?$ $\quad C = 1\mu F = 10^{-6} F\ ;\ a = 10^{-3} s$, $U_m = 10^2 V$ $\Big\}\ \dfrac{CU_m}{a} = 0{,}1\,A$

a.)

$$u(t) = U_m\left[\left(1+\frac{t}{a}\right)1(t+a) - \frac{2t}{a}1(t) + \left(\frac{t}{a}-1\right)1(t-a)\right] =$$

$$= U_m\left[1(t+a) + \frac{t}{a}1(t+a) - \frac{2}{a}t\cdot 1(t) + \frac{1}{a}t\cdot 1(t-a) - 1(t-a)\right]$$

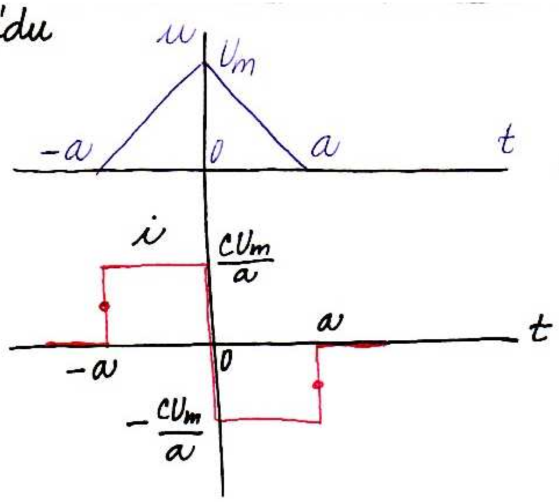
$$i(t) = C\cdot u'(t) = CU_m\Big[\cancel{\delta(t+a)} + \frac{1}{a}1(t+a) + \underbrace{\cancel{\frac{t}{a}\delta(t+a)}}_{-\delta(t+a)} - \frac{2}{a}1(t) - \frac{2}{a}\underbrace{t\delta(t)}_{0} +$$

$$+ \frac{1}{a}1(t-a) + \underbrace{\cancel{\frac{1}{a}\cdot t\cdot\delta(t-a)}}_{\delta(t-a)} - \cancel{\delta(t-a)}\Big] =$$

$f(x)\delta(x+a) = f(-a)\delta(x+a)$

$$= \frac{CU_m}{a}\left[1(t+a) - 2\cdot 1(t) + 1(t-a)\right] = 0{,}1\left[1(t+a) - 2\cdot 1(t) + 1(t-a)\right] = \begin{cases} \frac{CU_m}{a}\ ; & t\in(-a,0) \\ -\frac{CU_m}{a}\ ; & t\in(0,a) \\ 0\ ; & \text{inde} \end{cases}$$

časový priebeh prúdu v bode a.)



$a = 1\,ms$

$\dfrac{CU_m}{a} = 0{,}1\ A$

$U_m = 10^2\ V$

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b.) $$J(\omega) = \frac{C\cdot U_m}{a}\left[\left(\pi\delta(\omega)+\frac{1}{j\omega}\right)e^{-j\omega a} - \cancel{2\left(\pi\delta(\omega)+\frac{1}{j\omega}\right)} + \left(\pi\delta(\omega)+\frac{1}{j\omega}\right)e^{j\omega a}\right] =$$

veta o posunutí čas. funkcie
$f(t\pm a)\leftrightarrow F(\omega)e^{\mp j\omega a}$

$\cancel{= \pi\delta(\omega)}$ $\qquad \cancel{= \pi\delta(\omega)}$

$$= \frac{CU_m}{a}\left[\frac{e^{-j\omega a}+e^{j\omega a}}{j\omega} - \frac{2}{j\omega}\right] = \frac{CU_m}{a}\left[\frac{2}{j\omega}\cdot\underbrace{\frac{e^{j\omega a}+e^{-j\omega a}}{2}}_{\cos(\omega a)} - \frac{2}{j\omega}\right] =$$

$$= \frac{2CU_m}{j\omega a}\left[\cos(\omega a) - 1\right] = j\,\frac{2CU_m}{\omega a}\left[1-\cos(\omega a)\right] = j\,\frac{0{,}2}{\omega}\left[1-\cos(\omega\cdot 10^{-3})\right]$$

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2. $$I_0 = \frac{1}{T}\int_0^T i(t)\,dt = \int_0^a (-I_m)\,dt + \int_{3a}^{4a} I_m\,dt = -I_m\cdot a + I_m\cdot a = 0$$

$$\dot{I}_n = \frac{2}{T}\int_0^T i(t)\,e^{-jn\omega_0 t}\,dt = -j\,\frac{4}{T}\int_0^{T/2} i(t)\cdot\sin(n\omega_0 t)\,dt =$$

sínusové

časový priebeh $i(t)$ je nepárnou funkciou $\Rightarrow$ Four. rad obsahuje len imag. členy!

$$= -j\,\frac{4}{T}\int_0^{T/2}(-I_m)\cdot\sin(n\omega_0 t)\,dt = j\,\frac{\cancel{4}I_m}{\cancel{4}a}\int_0^a \sin(n\omega_0 t)\,dt =$$

$$= j\,\frac{I_m}{a}\left[-\frac{\cos(n\omega_0 t)}{n\omega_0}\right]_0^a = j\,\frac{I_m}{\cancel{a}}\left[\frac{1-\cos\left(n\frac{\cancel{2}\pi}{\cancel{4}_{2}\cancel{a}}\cancel{a}\right)}{n\frac{\cancel{2}\pi}{\cancel{4}_{2}\cancel{a}}}\right] =$$

$$= j\,\frac{2I_m}{n\pi}\left[1-\cos\left(n\frac{\pi}{2}\right)\right]$$

tento výsledok by sme dostali z úlohy 1, ak by sme namiesto ω dosadili $n\omega_0 = n\frac{2\pi}{T} = n\frac{2\pi}{4a} = n\frac{\pi}{2a}$ a súčasne označili súčin $2CU_m \equiv I_m$

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4. $$U(\xi) = U_2\cosh(\gamma\xi) + I_2 Z_0\sinh(\gamma\xi) = U_2\,\frac{e^{\gamma\xi}+e^{-\gamma\xi}}{2} + I_2 Z_0\,\frac{e^{\gamma\xi}-e^{-\gamma\xi}}{2} =$$

$$= \frac{1}{2}(U_2 + I_2 Z_0)e^{\gamma\xi} + \frac{1}{2}(U_2 - I_2 Z_0)e^{-\gamma\xi} = U_p(\xi) + U_s(\xi)$$

$$\rho(\xi) = \frac{U_s(\xi)}{U_p(\xi)} = \frac{\frac{1}{2}(U_2 - I_2 Z_0)e^{-\gamma\xi}}{\frac{1}{2}(U_2 + I_2 Z_0)e^{\gamma\xi}} = \frac{Z_2 - Z_0}{Z_2 + Z_0}\,e^{-2\gamma\xi} = \rho_2\cdot e^{-2\gamma\xi}$$

komplexný koeficient odrazu

$$\xi = l - x \Rightarrow \rho(x) = \frac{U_s(x)}{U_p(x)} = \frac{(U_2 - I_2 Z_0)e^{-\gamma[l-x]}}{(U_2 + I_2 Z_0)e^{\gamma[l-x]}} = \frac{Z_2 - Z_0}{Z_2 + Z_0}\,e^{-2\gamma[l-x]}$$

a.) $$\rho(x=l) = \frac{Z_2 - Z_0}{Z_2 + Z_0} = \frac{100-300}{100+300} = -\frac{200}{400} = -0{,}5 \equiv \rho_2$$ (ideálne homogénne vedenie!)

b.) $$\rho(x=0) = \frac{Z_2 - Z_0}{Z_2 + Z_0}\,e^{-2\gamma\cdot l} = -0{,}5\,e^{-j\,4\pi\cdot 3{,}75} = 0{,}5\,e^{j(-15\pi\pm\pi)} =$$

$$= 0{,}5\,\underbrace{e^{-j14\pi}}_{\text{alebo: } e^{-j16\pi}} = 0{,}5$$

$$\gamma = j\omega\sqrt{L_0 C_0} = j\alpha = j\,\frac{2\pi}{\lambda} = j2\pi$$

ideálne vedenie; $\lambda = 1\,m$

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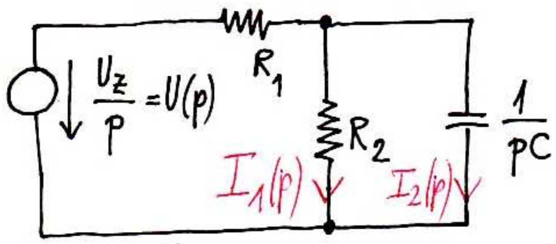
3.



$$I_1(p) = \frac{\frac{U_z}{p}\cdot\frac{1}{pC}}{R_1R_2+\frac{R_1}{pC}+\frac{R_2}{pC}} = \frac{U_z}{p}\cdot\frac{\cancel{\frac{1}{pC}}}{\frac{pCR_1R_2+R_1+R_2}{\cancel{pC}}} = \frac{U_z}{R_1R_2C}\cdot\frac{1}{p\left(p+\frac{R_1+R_2}{R_1R_2C}\right)} =$$

sérioparalelný obvod

$$= \frac{U_z}{C\cancel{R_1R_2}}\cdot\left\{\frac{1}{\frac{R_1+R_2}{\cancel{R_1R_2C}}p} - \frac{1}{\frac{R_1+R_2}{\cancel{R_1R_2C}}\left(p+\frac{R_1+R_2}{R_1R_2C}\right)}\right\} = \frac{U_z}{R_1+R_2}\left[\frac{1}{p} - \frac{1}{p+\frac{R_1+R_2}{R_1R_2C}}\right] \Leftrightarrow$$

$$\frac{1}{p(p+a)} = \frac{1}{ap} - \frac{1}{a(p+a)}$$

$$\Leftrightarrow \frac{U_z}{R_1+R_2}\left(1-e^{-\frac{[R_1+R_2]}{R_1R_2C}t}\right)1(t) = \left(1-e^{-10^3t}\right)1(t) = i_1^+(t)$$

$$\tau = \frac{R_1R_2C}{R_1+R_2} = 1\,ms$$

$$I_2(p) = \frac{\frac{U_z}{p}\cdot R_2}{R_1R_2+\frac{R_1}{pC}+\frac{R_2}{pC}} = \frac{U_zR_2}{\cancel{p}}\cdot\frac{\cancel{p}C}{pCR_1R_2+R_1+R_2} = \frac{U_z\cdot\cancel{R_2}\cdot\cancel{C}}{\cancel{C}\cdot R_1\cdot\cancel{R_2}}\cdot\frac{1}{p+\frac{R_1+R_2}{R_1R_2C}} \Leftrightarrow$$

$$\Leftrightarrow \frac{U_z}{R_1}e^{-\frac{(R_1+R_2)}{R_1R_2C}t}\cdot 1(t) = 1{,}5e^{-10^3t}1(t) = i_2^+(t)$$