SLOVAK UNIVERSITY OF TECHNOLOGY IN BRATISLAVA
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OFDM basics

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Orthogonal Frequency Division Multiplexing

- OFDM is present in
  - LTE
  - Mobile WiMax IEEE 802.16e
  - xDSL
  - Wireless LAN IEEE 802.11a,g,n

- OFDM basics
- Multipath channel & OFDM
- Redundancies in OFDM

What is OFDM?

- Orthogonal FDM – it’s multiplexing
  - Classic FDM + major improvements
- It’s more:
  - Multi Carrier
  - Digital modulation (PSK, QAM)
  - Digital processing
- Demultiplexing

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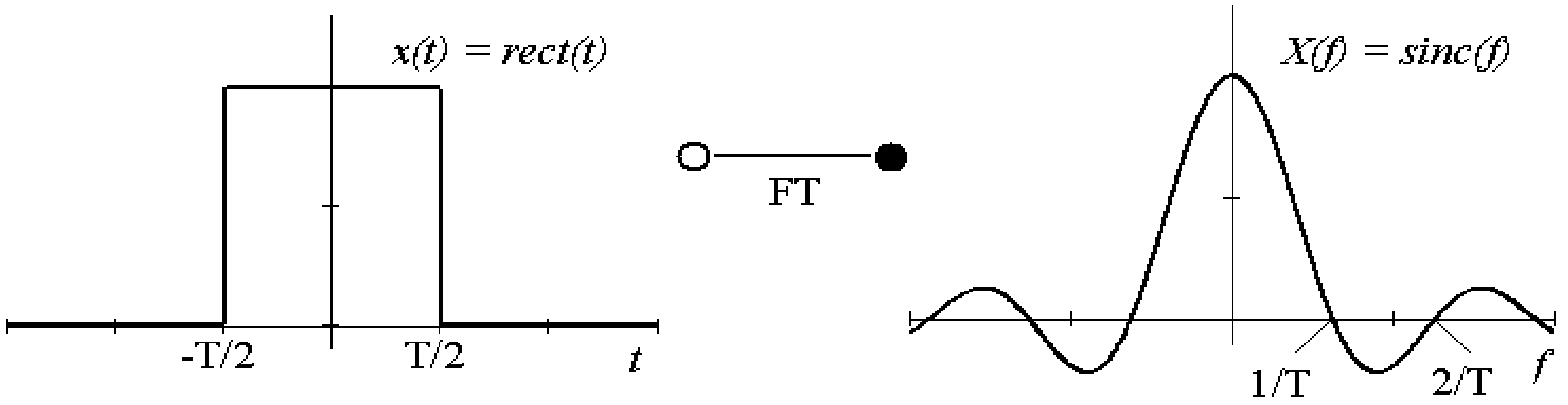
Takmer Definícia

- OFDM je digitálna prenosová technika využívajúca demultiplexovanie jedného rýchleho dátového toku na niekoľko pomalších, paralelne prenášaných (multiplexovaných) subtokov, oddelených moduláciou podľa princípu FDM, používajúca ortogonálne rozostupy medzi nosnými frekvenciami jednotlivých subtokov, definujúc tým celú množinu ďalších principiálnych parametrov systému

Základné princípy

- Demultiplexovanie
- Ortogonalita FDM
- Digitálne spracovanie signálov (DSP)

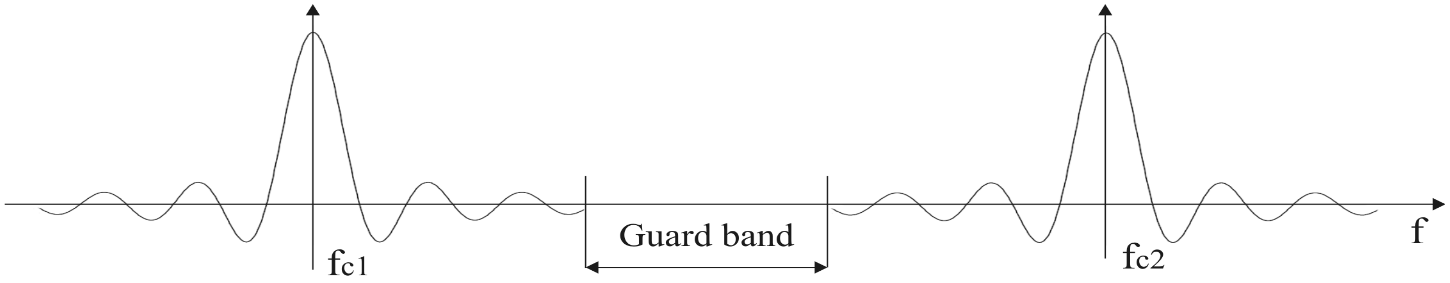
Pásmo obdĺžnikového impulzu



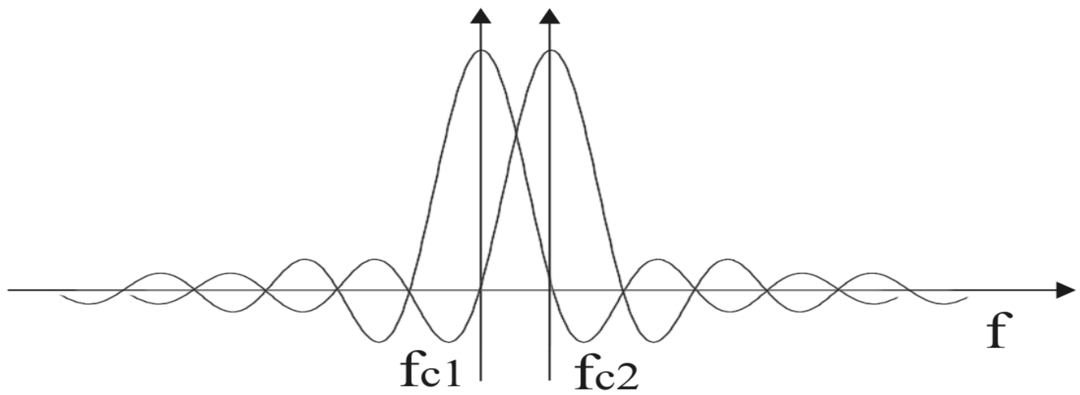
- W = 1/T

Orthogonal FDM

- FDM wastes frequency band

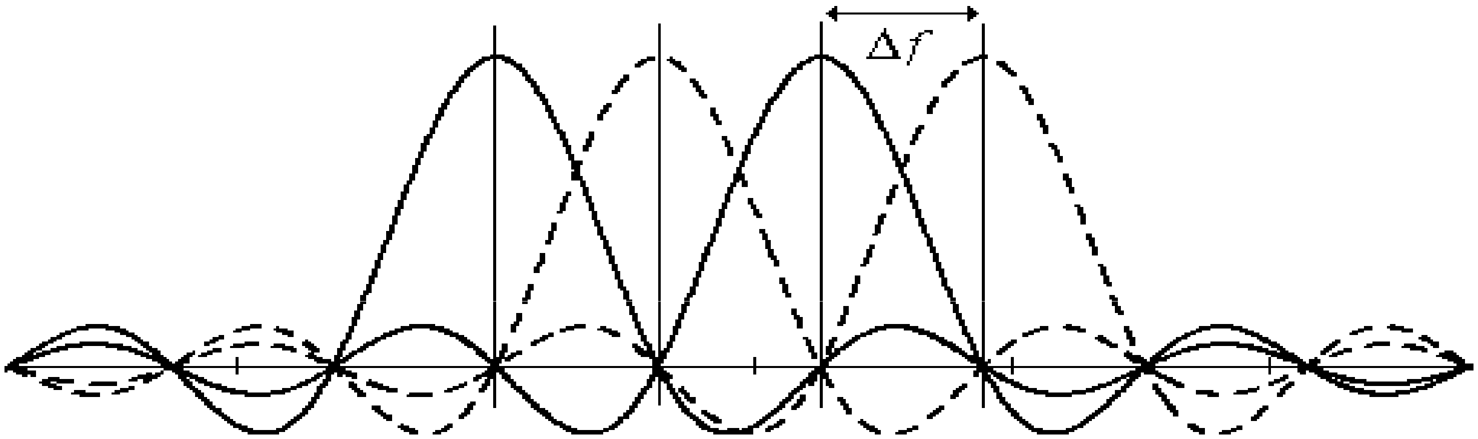


- OFDM: orthogonal spacing of carriers – high spectral effectivity



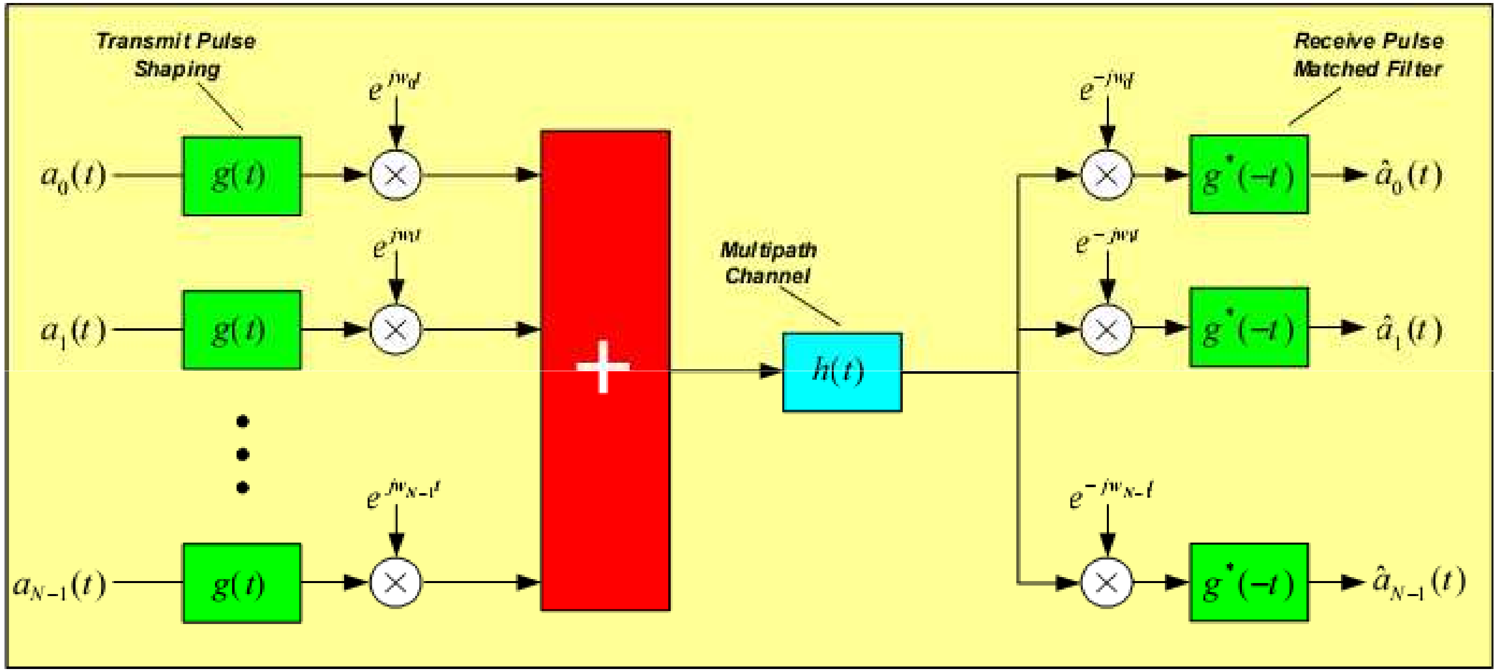
- Only discrete frequencies can be utilized , samples at $\Delta f = 1/T$

Orthogonal FDM



• Spektrá sa prekrývajú. Nulová interferencia nastáva len v diskrétnych bodoch nosných frekvencií subtokov.

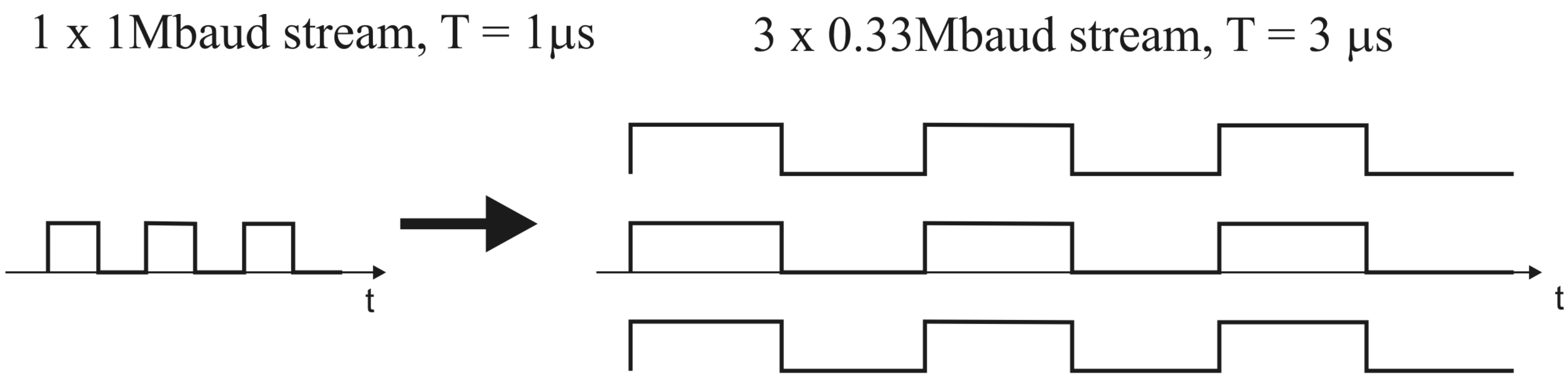
Basic OFDM system architecture



- Carriers are equally spaced by Δf = 1/T
- Amplitude and phase carries information

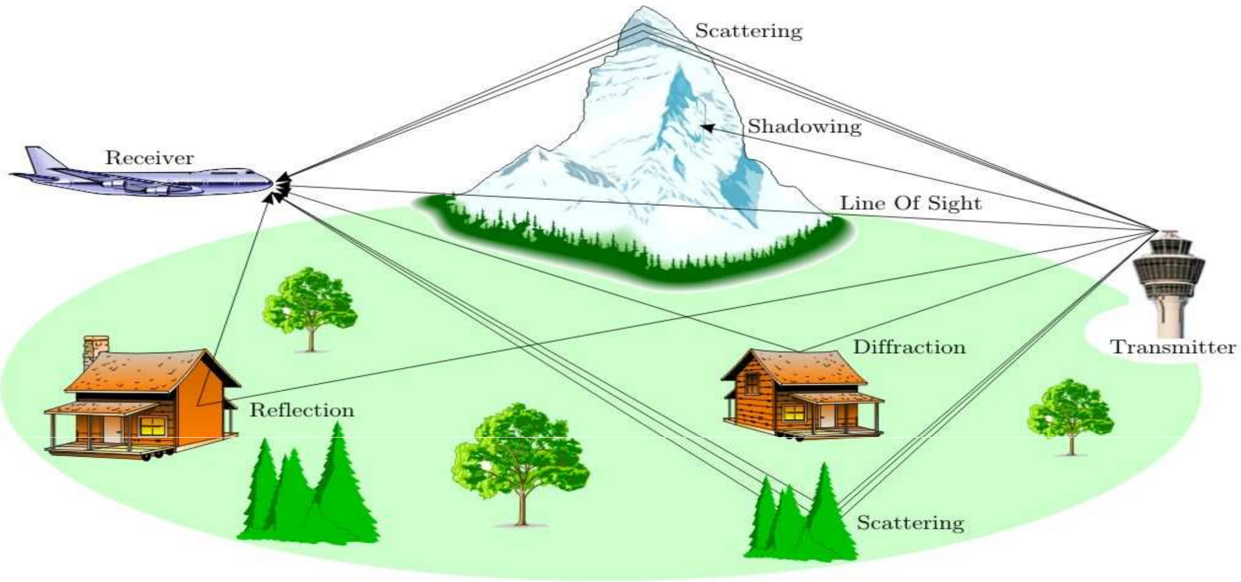
MultiCarrier

- Demultiplexing one stream to many streams
  - Each stream modulated separately



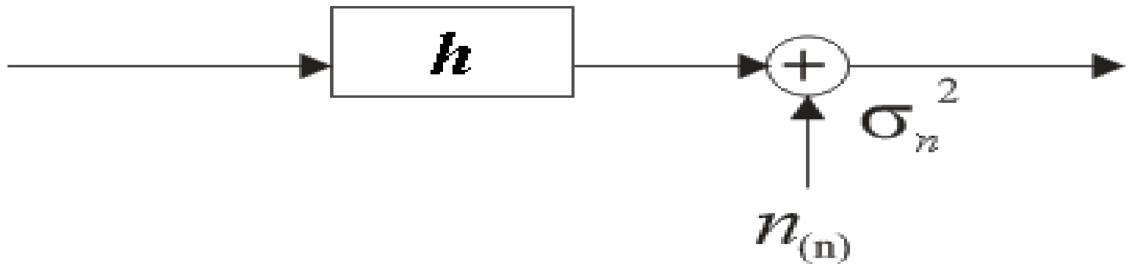
- Resistance to Inter Symbol Interference

Multipath Channel



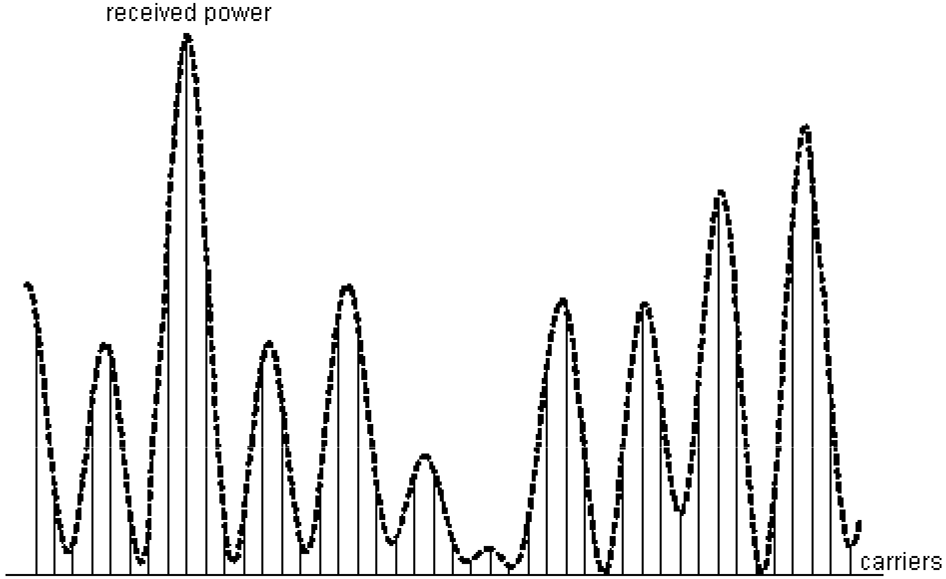
- RX observes sum of delayed copies of transmitted symbol
- Channel impulse response $h(t)$
- Transmission modeled by convolution $r(t) = h(t) * s(t)$

Modelovanie kanála



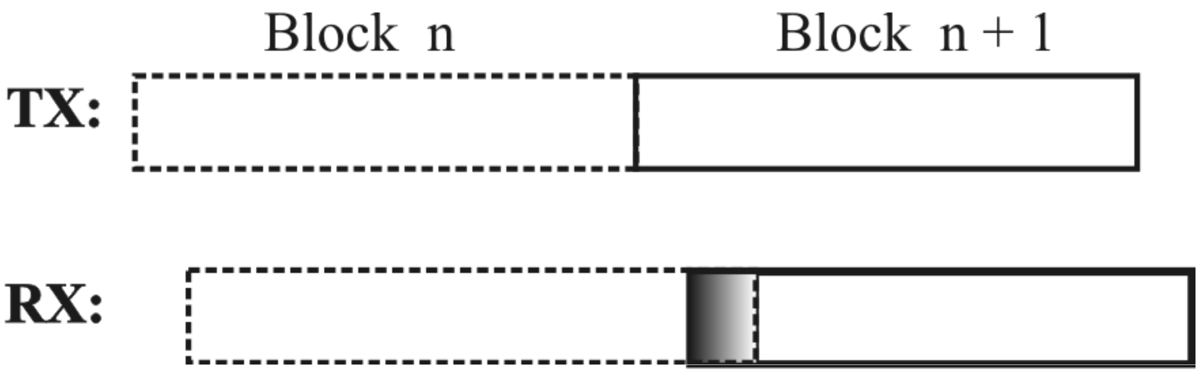
- Model kanála v dikrétnom čase
- $r(n) = h(n) * s(n)$

Frekvenčne selektívny kanál



- OFDM signál je širokopásmový (MHz)
- *H(f)* nie je konštantná
- Je treba zabezpečiť ekvalizáciu.

Inter**S**ymbol & **I**nter**B**lock **I**nterference (ISI & IBI)



- Channel convolution prolongs transmitted blocks
- Blocks are transmitted in serial way
- In RX
  - blocks overlap – IBI
  - Samples inside one block interfere - ISI

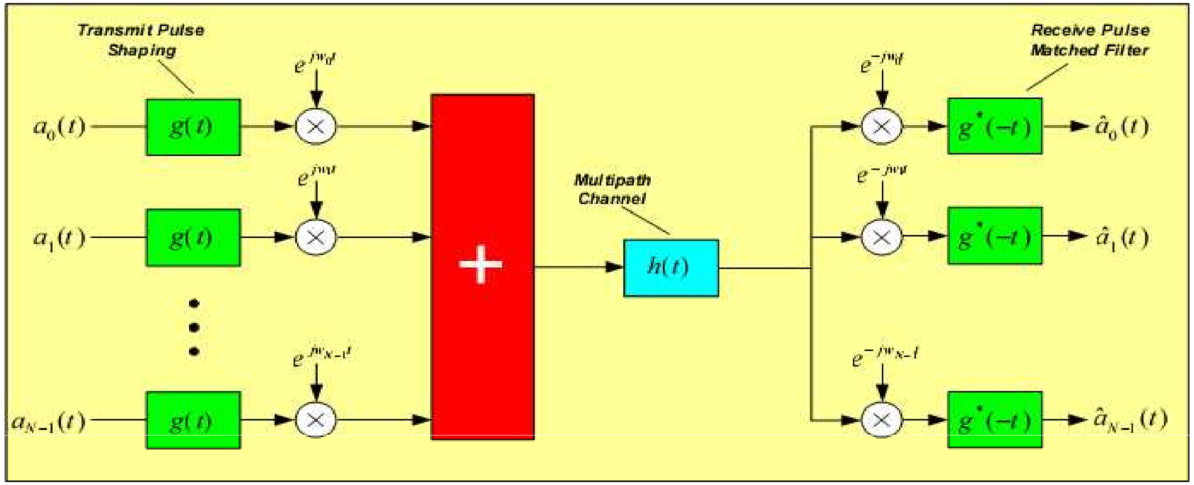
OFDM ISI/IBI tolerance example

- Tolerance to ISI because of MultiCarrier
  - One data stream splits to 1024 streams [Wmx]
  - Delay spread for large open space environment 1800 ns [Deb]

- One fast stream R_m = 10Mbaud, $T = 1/R_m$
  - T = 100 ns, 18 succesive symbols overlap, ISI 1800% of T

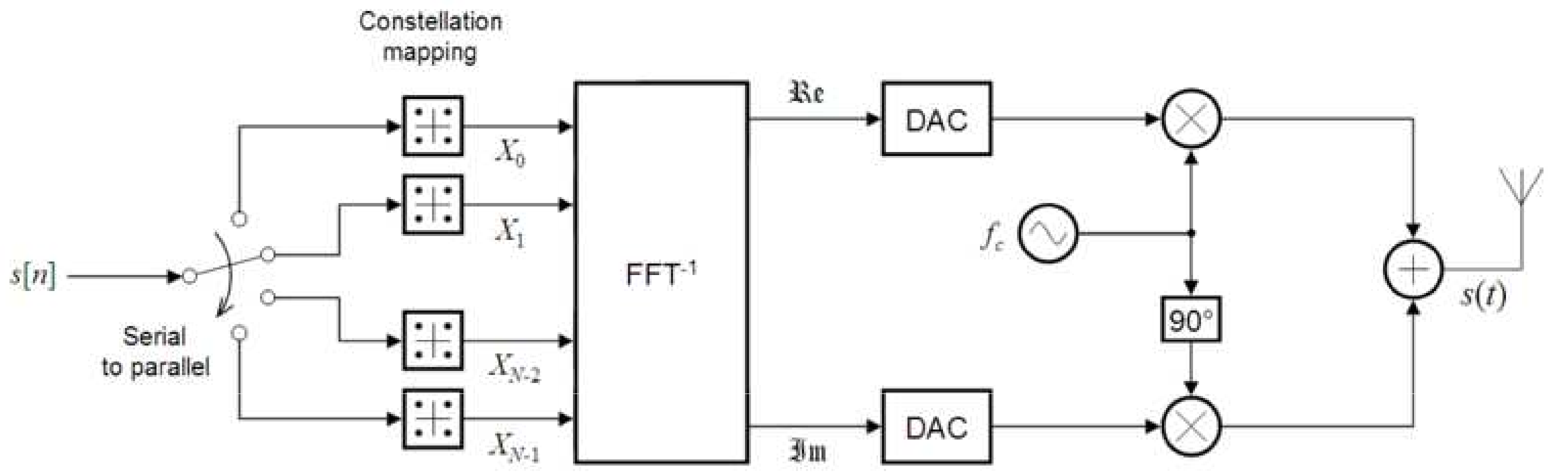
- 1000 slow streams, 10kbaud each
  - T = 100 μs, 2 succesive symbols overlap, ISI 1.8% of T

Zhrnutie – banka TX a RX



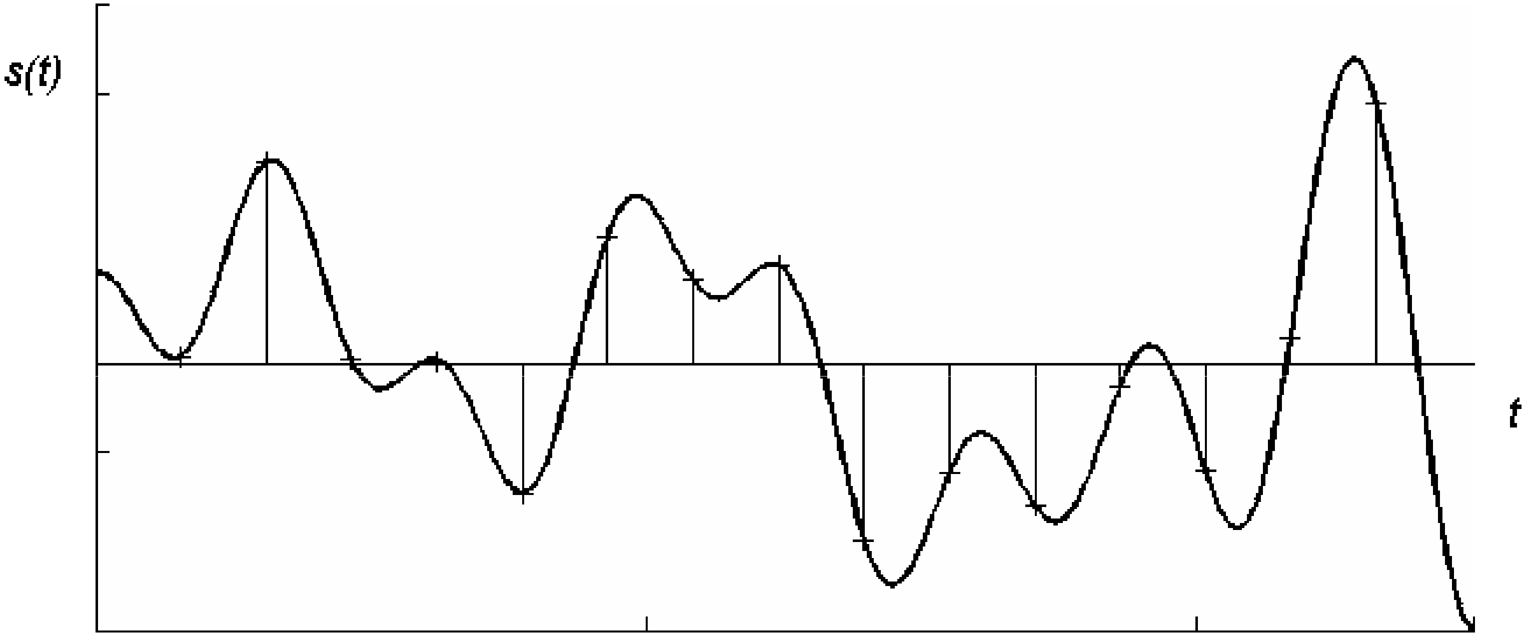
- Demultiplexovanie (MC)
  - odolnosť voči ISI
- Ortogonalita
  - spektrálna efektivita
- Implementačne nepohodlné

More Practical simple OFDM transmitted

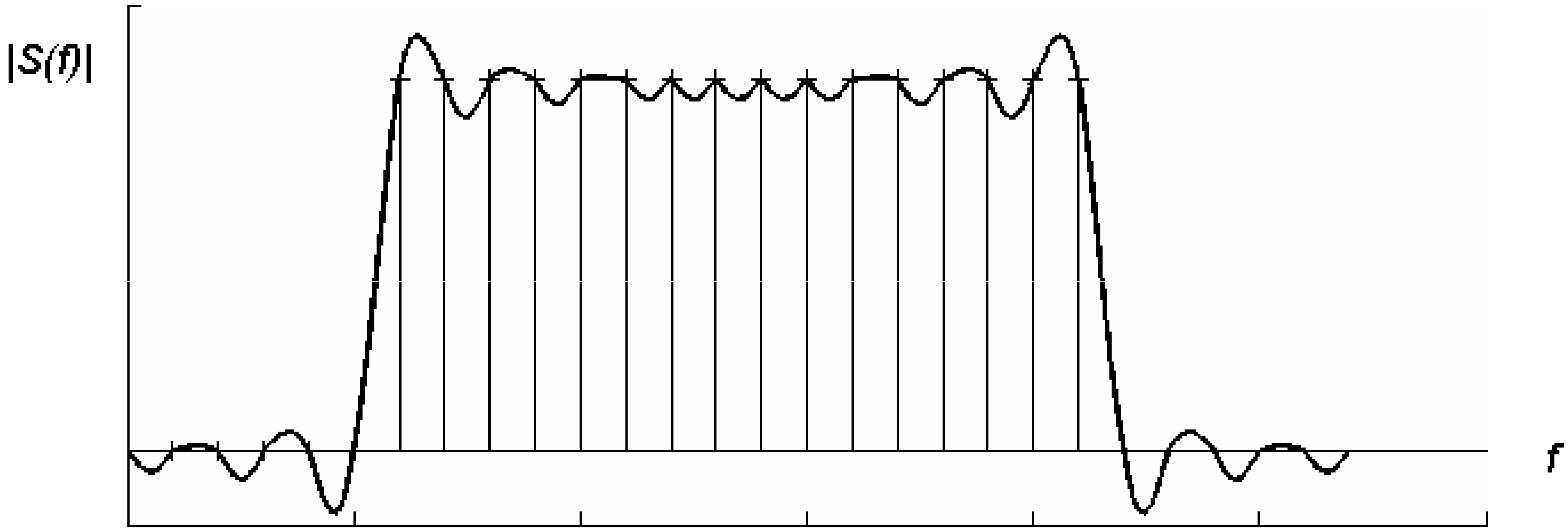


- Oscillator Bank replaced by IDFT
- OFDM symbol is a block of samples
- SW Processing in discrete time and freq. domain

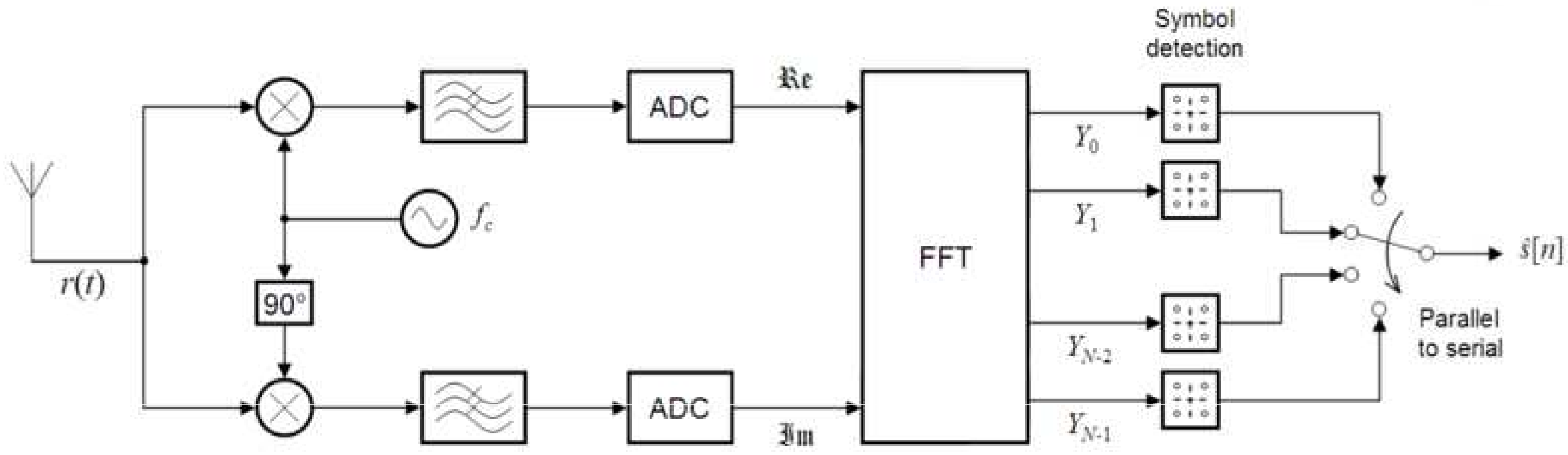
OFDM prototyp v časovej oblasti



OFDM prototyp vo frekvenčnej oblasti



More Practical simple OFDM receiver



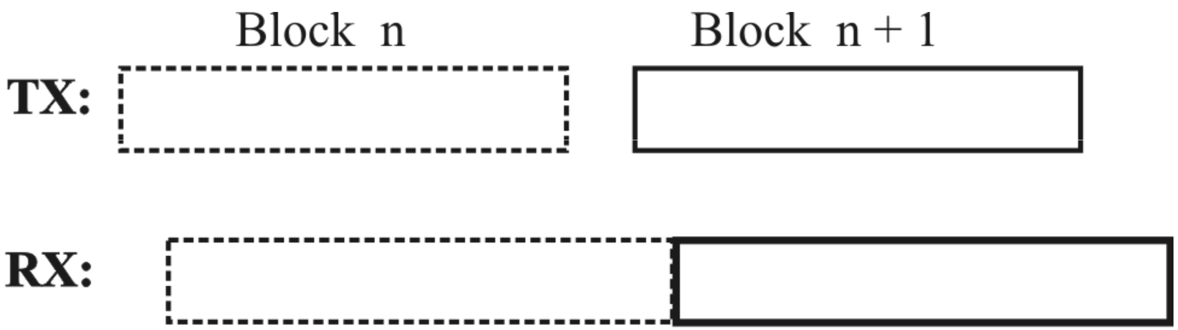
- Oscillator Bank replaced by DFT
- Symbol detection in frequency domain

OFDM ISI/IBI tolerance

- IEEE 802.16e OFDM parameters:
  - Usefull symbol time 91.4 μs
  - Number of samples per symbol 1024 (11.2 MHz f_s)
  - Blocks overlap by 89 samples (approx. 9 %)
- Additional tolerance necessary ► prefix

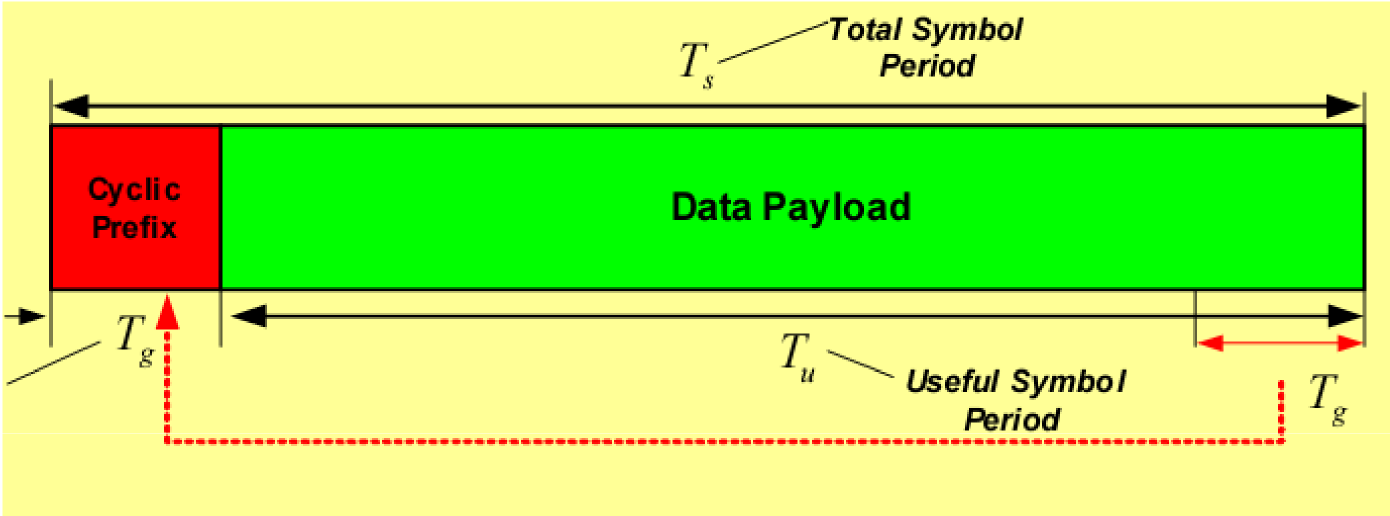
Improving ISI/IBI tolerance

- Use of guard interval
  - Zero prefix



- In real OFDM systems
  - Cyclic prefix

Cyclic prefix in OFDM



- Mobile WiMax (IEEE 802.16e):
  - T_g = 1/4, 1/8, 1/16, 1/32 of T_u

Redundancies in OFDM

- Cyclic prefix
  - Designed for easy FDE
- Pilots
  - used for channel estimation
- Upper layer headers
  - IP & TCP headers

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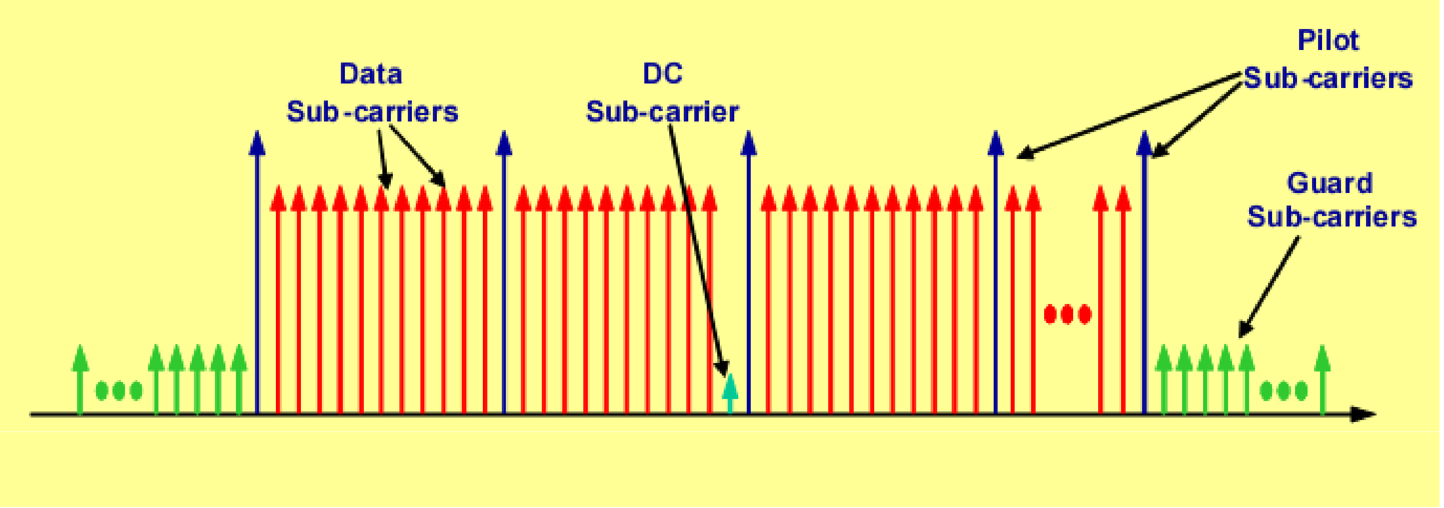
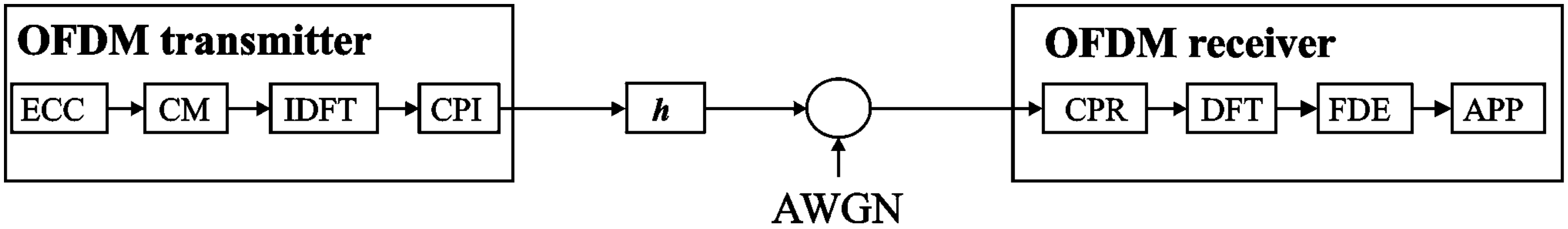
Pilots



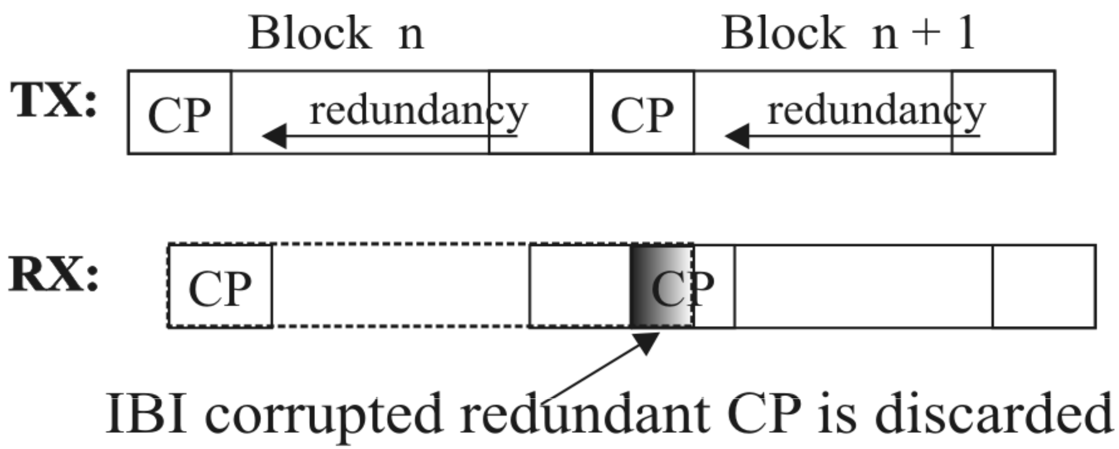
- IEEE 802.16e groups sub-carriers to clusters
  – Cluster = 48 data carriers + 8 pilots

Practical OFDM system



- ECC – Error Control Code
- CM – Constellation Mapping
- CPI – Cyclic Prefix Insertion
- CPR – Cyclic Prefix Removal
- FDE – Frequency Domain Equalization
- APP – Posterior Probability Decoder

The "Cyclic prefix" situation



- Blocks with CP are corrupted by IBI
- If delay spread < CP size => all OK
- CP designed for worst case
- **Why did we do it ?**

The “Why?” of the Cyclic Prefix

- Multipath channel
  - Transmission modeled by convolution $\boldsymbol{r}(t) = \boldsymbol{h}(t) * \boldsymbol{t}(t)$
  - In discrete time $\boldsymbol{r}(n) = \boldsymbol{h}(n) * \boldsymbol{t}(n)$
  - Simple per-component multiplication in frequency domain (DFT)
- Channel convolution – ***linear*** convolution:

$$r_{(n)} = s_{(n)} * h_{(n)} = \sum_{m=-\infty}^{\infty} h_{(m)} \cdot s_{(n-m)}$$

The “Why?” of the Cyclic Prefix II

- DFT theory:
  - Requires periodic discrete-time signals
  - Transfers ***circular*** convolution in time-domain to simple multiplication in frequency-domain

$$r_{(n)} = t_{(n)} \otimes h_{(n)} = \sum_{m=0}^{N-1} h_{(m)} \cdot t_{[(n-m)\,\mathrm{mod}N]}$$

$$\boldsymbol{R}(k) = \boldsymbol{H}(k) \times \boldsymbol{T}(k)$$

- Cyclic Prefix insertion in TX and removal in RX ensures that the linear channel convolution appears as circular convolution.

Matrix model of linear convolution

$\mathbf{H_c}$ =

1	0	0	0	0	0	0	0	0	0
0.9	**1**	0	0	0	0	0	0	0	0
0.4	**0.9**	**1**	0	0	0	0	0	0	0
0	**0.4**	**0.9**	**1**	0	0	0	0	0	0
0	0	**0.4**	**0.9**	**1**	0	0	0	0	0
0	0	0	**0.4**	**0.9**	**1**	0	0	0	0
0	0	0	0	**0.4**	**0.9**	**1**	0	0	0
0	0	0	0	0	**0.4**	**0.9**	**1**	0	0
0	0	0	0	0	0	**0.4**	**0.9**	**1**	0
0	0	0	0	0	0	0	**0.4**	**0.9**	**1**
0	0	0	0	0	0	0	0	**0.4**	**0.9**
0	0	0	0	0	0	0	0	0	**0.4**

size($\mathbf{H_c}$) = 12 rows, 10 columns

- $\boldsymbol{h}(n)$ = {1, 0.9, 0.4}
- **t** – input vector (transmitted block)
- **r** – output vector (received block)
- $\mathbf{H_c}$ – convolution matrix

- Linear convolution can be modeled by matrix multiplication:

  $\mathbf{r} = \mathbf{H_c} \times \mathbf{t}$ (1)

Size of $\mathbf{H_c}$ dictated by **t** and $\boldsymbol{h}(n)$

Linear & cyclic convolution matrices

$\mathbf{H_c}$ =

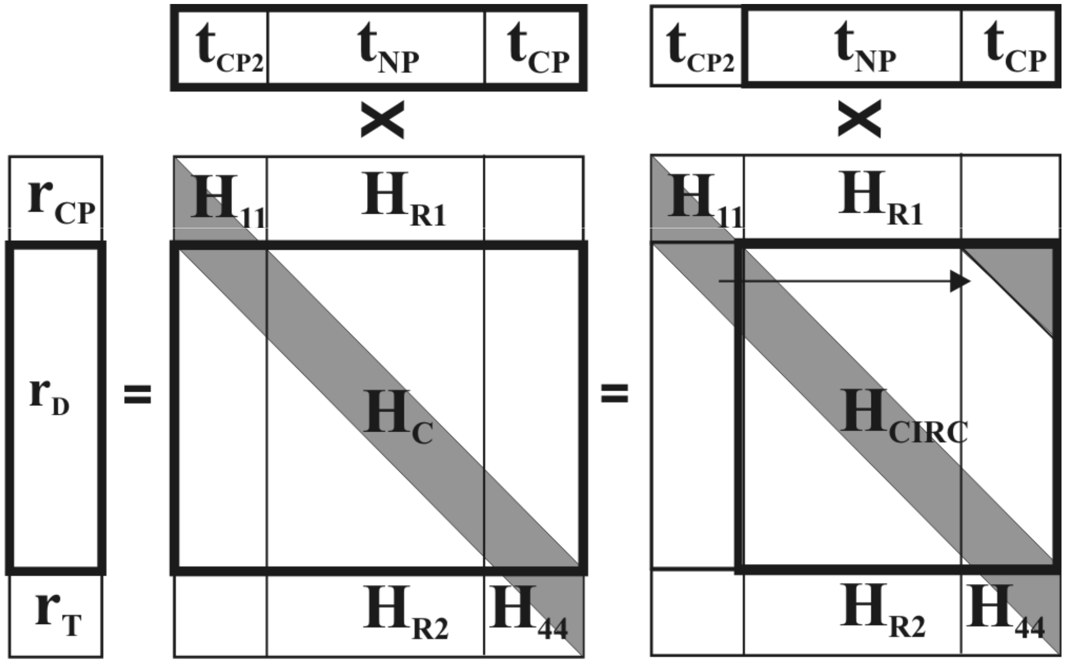
1	0	0	0	0	0	0	0	0	0
0.9	**1**	0	0	0	0	0	0	0	0
0.4	**0.9**	**1**	0	0	0	0	0	0	0
0	**0.4**	**0.9**	**1**	0	0	0	0	0	0
0	0	**0.4**	**0.9**	**1**	0	0	0	0	0
0	0	0	**0.4**	**0.9**	**1**	0	0	0	0
0	0	0	0	**0.4**	**0.9**	**1**	0	0	0
0	0	0	0	0	**0.4**	**0.9**	**1**	0	0
0	0	0	0	0	0	**0.4**	**0.9**	**1**	0
0	0	0	0	0	0	0	**0.4**	**0.9**	**1**
0	0	0	0	0	0	0	0	**0.4**	**0.9**
0	0	0	0	0	0	0	0	0	**0.4**

$\mathbf{H_{circ}}$ =

1	0	0	0	0	0	0	0	**0.4**	**0.9**
0.9	**1**	0	0	0	0	0	0	0	**0.4**
0.4	**0.9**	**1**	0	0	0	0	0	0	0
0	**0.4**	**0.9**	**1**	0	0	0	0	0	0
0	0	**0.4**	**0.9**	**1**	0	0	0	0	0
0	0	0	**0.4**	**0.9**	**1**	0	0	0	0
0	0	0	0	**0.4**	**0.9**	**1**	0	0	0
0	0	0	0	0	**0.4**	**0.9**	**1**	0	0
0	0	0	0	0	0	**0.4**	**0.9**	**1**	0
0	0	0	0	0	0	0	**0.4**	**0.9**	**1**

Linear to cyclic transform

- $\mathbf{t}_D$ - original data withouth the cyclic prefix
- $\mathbf{r}_D$ - subblock of received samples, selected for further processing



$$\mathbf{r}_D = \mathbf{H_C} \times \mathbf{t} = \mathbf{H_{CIRC}} \times \mathbf{t}_D \qquad (2)$$

Matrix model of channel convolution

- OFDM transmission can be modelled by matrix multiplication:

$$\mathbf{r}_D = \mathbf{H}_\mathbf{C} \times \mathbf{t} = \mathbf{H}_\mathbf{CIRC} \times \mathbf{t}_D \qquad (2)$$

- A circulant matrix can be diagonalized [Toepl]

$$\mathbf{D}_\mathbf{h} = \mathbf{F} \times \mathbf{H}_\mathbf{circ} \times \mathbf{F}^{-1} \qquad (3)$$

- **F** - Fourier transform matrix
- $\mathbf{F}^{-1}$ - inverse Fourier transform matrix
- $\mathbf{D}_h$ - diagonal matrix:

$$\text{diag}(\mathbf{D}_h) = \boldsymbol{H}(k)$$

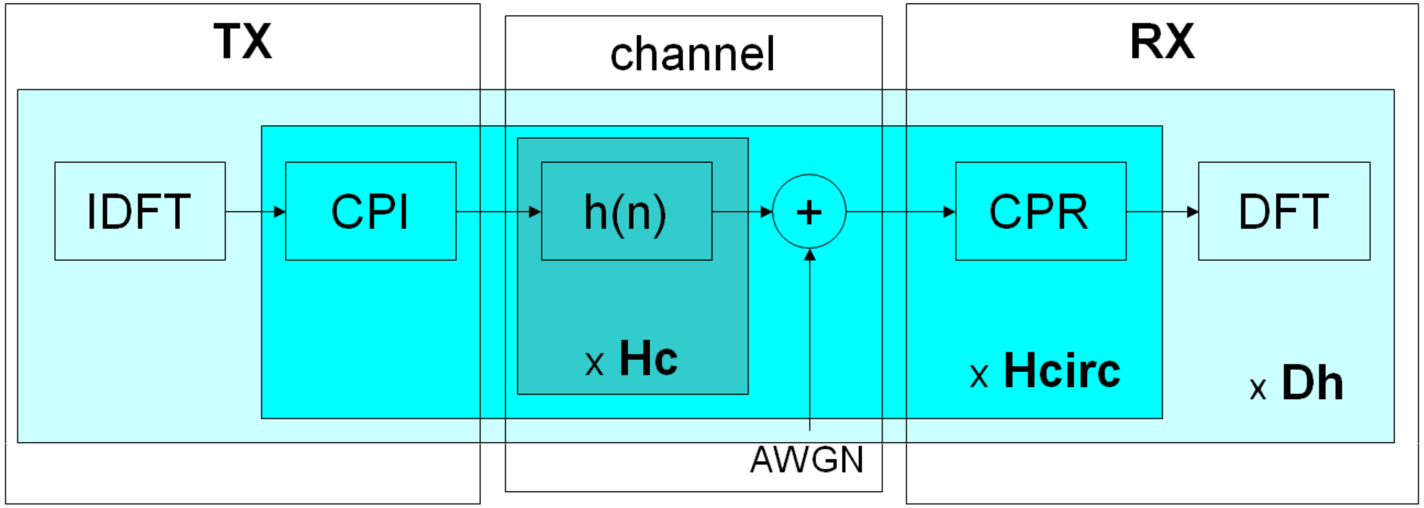
- $\boldsymbol{H}(k)$ is N-point DFT of the channel impulse response $\boldsymbol{h}(n)$

(Discrete) Fourier Transform Matrix

$$W = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega & \omega^2 & \omega^3 & \cdots & \omega^{N-1} \\ 1 & \omega^2 & \omega^4 & \omega^6 & \cdots & \omega^{2(N-1)} \\ 1 & \omega^3 & \omega^6 & \omega^9 & \cdots & \omega^{3(N-1)} \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 1 & \omega^{N-1} & \omega^{2(N-1)} & \omega^{3(N-1)} & \cdots & \omega^{(N-1)(N-1)} \end{bmatrix}$$

- N-point DFT
- ω is a primitive N^{th} root of unity exp(-2πj / N)

OFDM transmission matrix models



- Time domain models:

$$\mathbf{r}_D = \mathbf{H}_{\mathrm{C}} \times \mathbf{t} = \mathbf{H}_{\mathrm{CIRC}} \times \mathbf{t}_D \qquad (2)$$

- Frequency domain model:

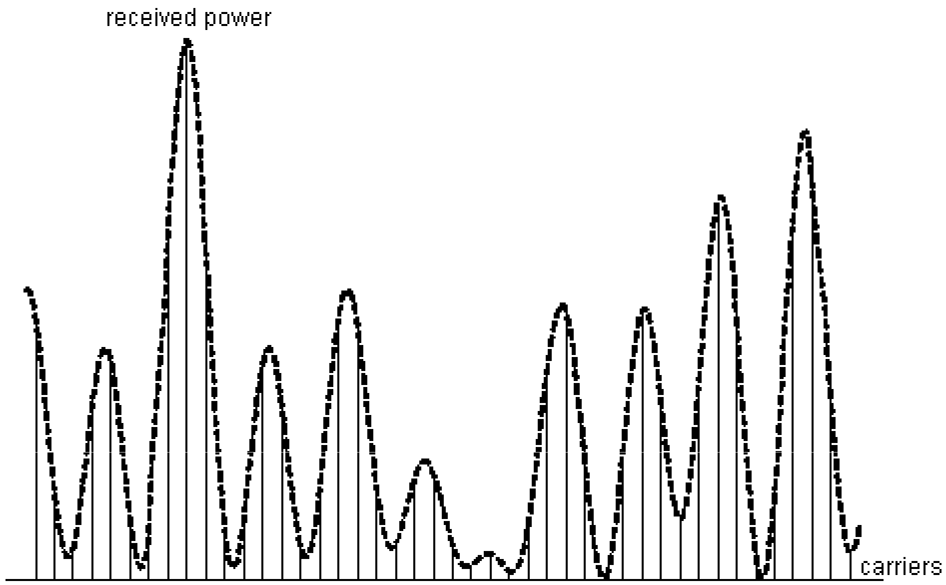
$$\mathbf{R}_D = \mathbf{D}_{\mathrm{h}} \times \mathbf{T}_D = \boldsymbol{H}_{(k)} \times \boldsymbol{T}_{(k)} = \boldsymbol{R}_{(k)} \qquad (4)$$

Simple **F**requency **D**omain **E**qualization (FDE)

- In time domain models
  - each RX sample is a linear combination of several TX samples
- In frequency domain model
  - Each RX sample depends on one TX sample
  - If CSI is know in RX, equalization is multiplication:

$$\hat{\boldsymbol{T}}_{(k)} = (\boldsymbol{H}_{(k)})^{-1} \times \boldsymbol{R}_{(k)} \qquad (5)$$

Frekvenčne selektívny kanál



- OFDM signál je širokopásmový (MHz)
- *H(k)* nie je konštantná
- Je treba zabezpečiť ekvalizáciu.

Výhody a nevýhody OFDM

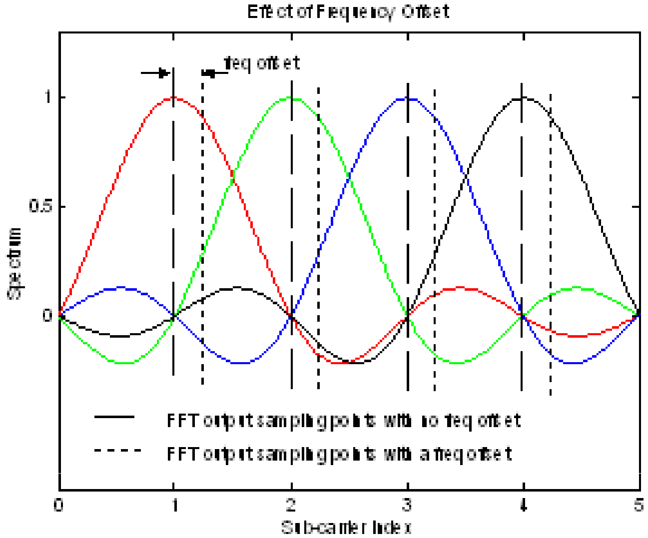
+

- Vysoká spektrálna efektivita (+ veľká prenosová kapacita)
- Prirodzená odolnoť voči ISI
- Jednoduchá frekvenčná ekvalizácia
- Perspektíva z hľadiska SDR

-

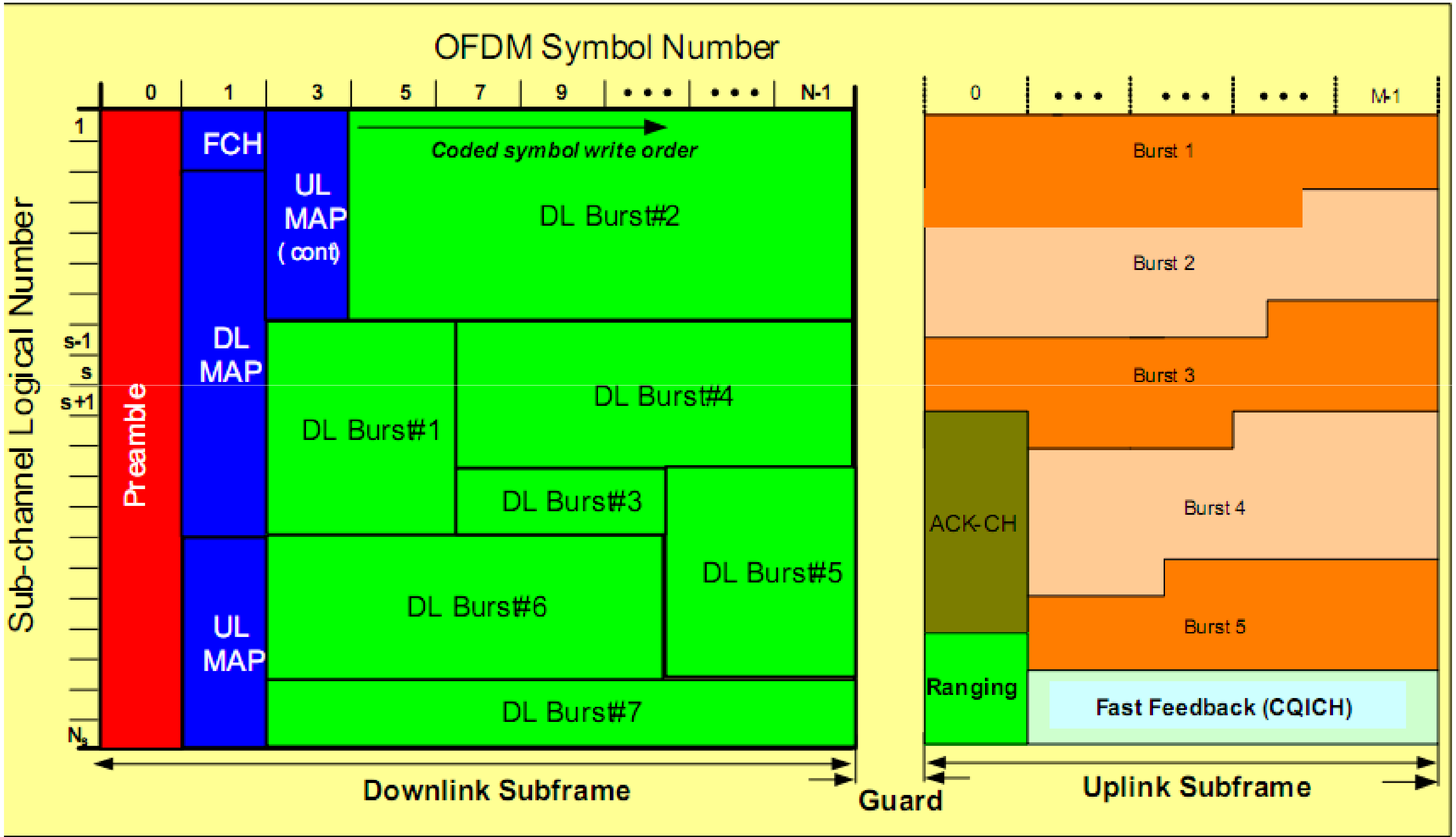
- Vysoké nároky na analógové obvody
  - Výstupný zosilňovač vysielača
    - (Veľký dynamický rozsah výstupného signálu)
  - Fluktuácia frekvencie a fázy oscilátorov
- Citlivosť na frekvenčný a časový offset

Časová a frekvenčná synchronizácia



- Rx nevzorkuje v správnych okamihoch – posunutie o δt
- Rx prijíma frekvenčne posunutý prototyp – vplyvom Doplerovho efektu
  - Inter subcarrier interference
- Odstraňovanie pomocou vkladania redundancie
  - Piloty
  - Synchronizačný OFDM symbol na začiatku rámca

Štruktúra rámca TDD v IEEE 802.16e



OFDM basics

Thank you for your attention…

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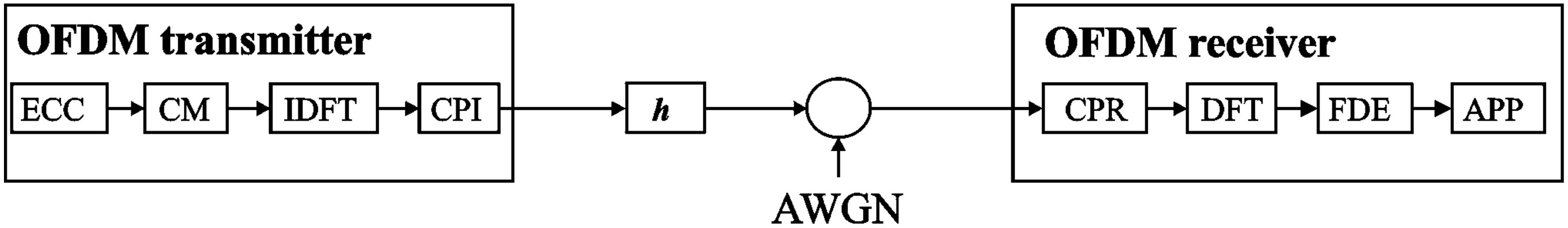
Exploiting of CP redundancy

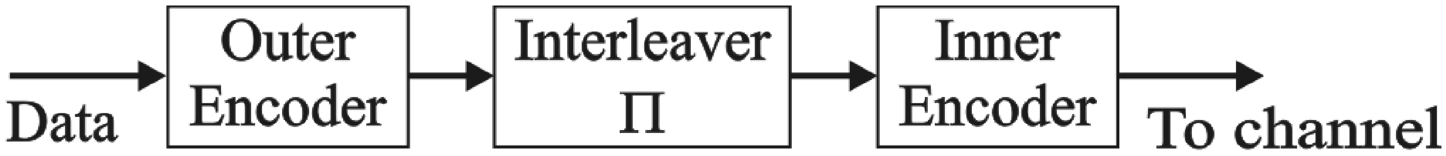


- There is intentional redundancy in OFDM – Cyclic Prefix
- Used for
  - IBI protection
  - Simple FDE
- It can also be used to improve the error performance of the posterior probabilistic decoder of the ECC
- Practical OFDM is in fact COFDM

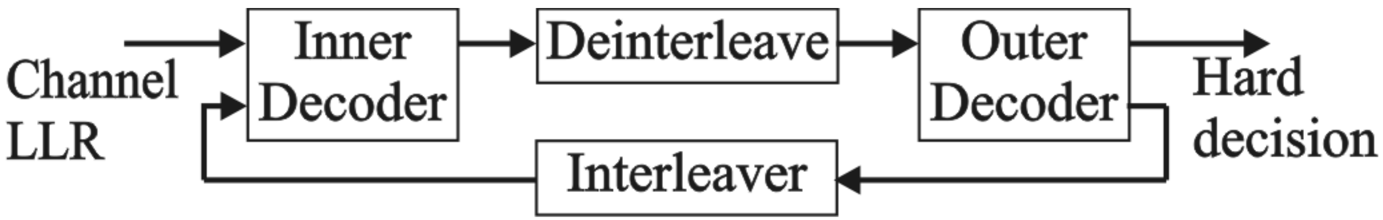
The CP is ECC

- The insertion of cyclic prefix is a partial repetition code
- IEEE 802.16e: T_g = 1/4, 1/8, 1/16, 1/32 of T_u
- Hagenauer: serial concatenation of codes:

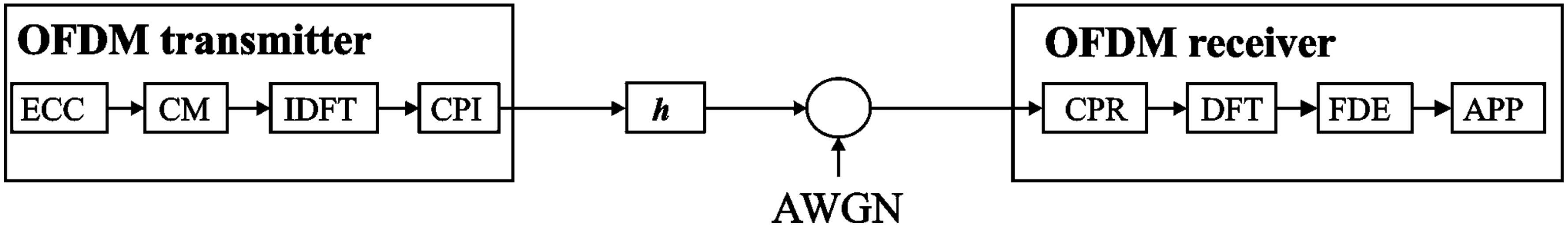
Encoder:



Iterative SISO decoder :



Exploiting of CP redundancy

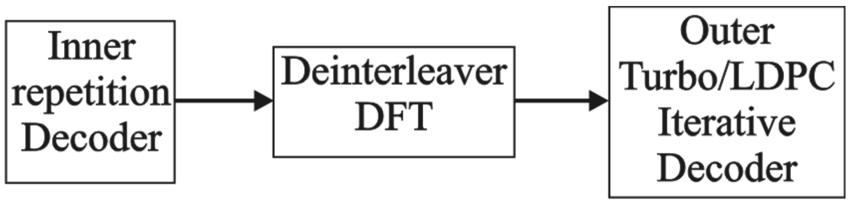


OFDM transmitter is a serially encoded system:

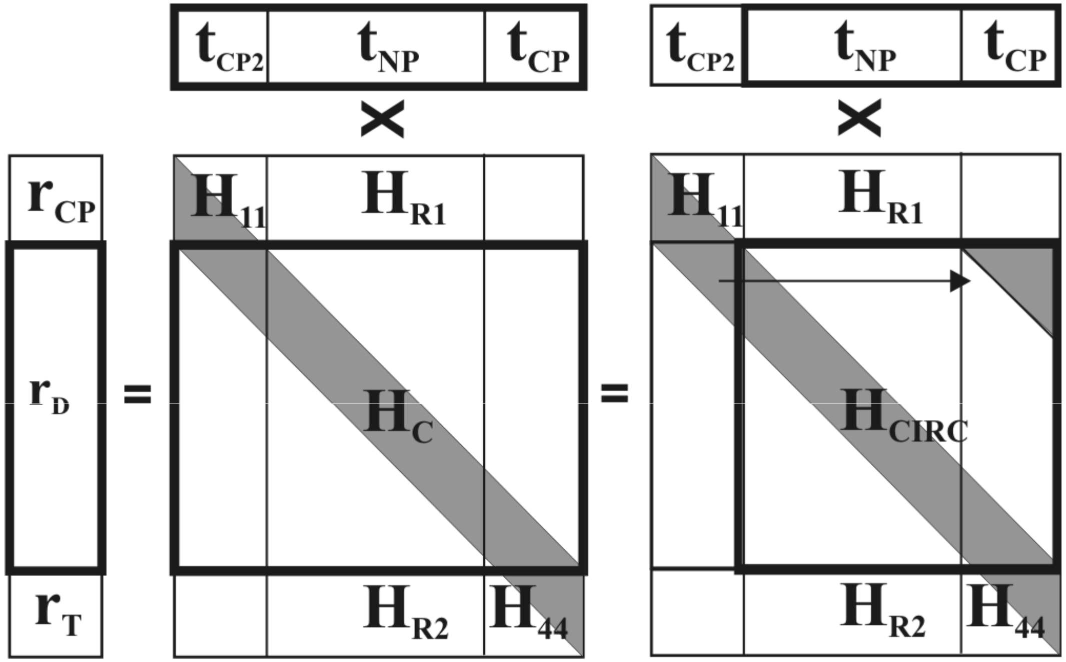
- ECC - outer encoder, Powerfull Code – Turbo or LDPC
- CPI - inner encoder, weak partial repetition code
- IDFT - Interleaver

The goal is to modify the receiver to extract the redundancy

Use non-iterative form:



Matrix models revisited



- $\mathbf{r}_{\mathrm{CP}}$ & $\mathbf{r}_{\mathrm{T}}$ are discarded in RX because of IBI

$$\mathbf{r}_O = \mathbf{r}_{CP(n)} + \mathbf{r}_{T(n-1)} = \mathbf{H}_{\mathbf{11}(n)} \times \mathbf{t}_{CP2(n)} + \mathbf{H}_{\mathbf{44}(n-1)} \times \mathbf{t}_{CP(n-1)}$$

Recovering the redundant samples

$$\mathbf{r}_O = \mathbf{r}_{CP(n)} + \mathbf{r}_{T(n-1)} = \mathbf{H}_{\mathbf{11}(n)} \times \mathbf{t}_{CP2(n)} + \mathbf{H}_{\mathbf{44}(n-1)} \times \mathbf{t}_{CP(n-1)}$$

- It is possible to apply subtractive correction:

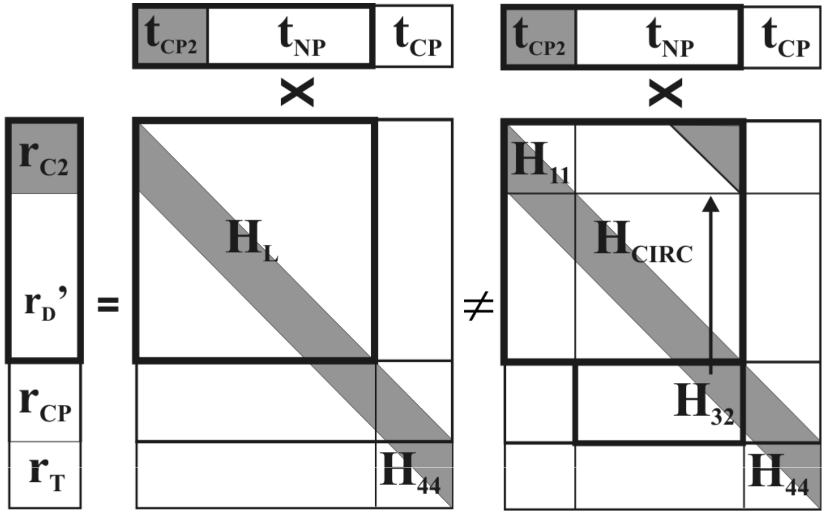
$$\mathbf{r}_{cor1(n-1)} = \mathbf{H}_{\mathbf{44}(n-1)} \times \mathbf{t}_{CP(n-1)}$$

- The transmitted samples can be recovered:

$$\mathbf{t}_{CP2(n)} = (\mathbf{H}_{\mathbf{11}}^{-1}) \times (\mathbf{r}_O - \mathbf{r}_{cor1(n-1)})$$

- This will do no good:
  - For APP decoding, the LLR of samples in frequency domain are necessary!

Prefix extraction



$$\mathbf{r}_D' = \mathbf{H_L} \times \mathbf{t}_D'$$

If a "*good estimate*" of transmitted vector $\mathbf{t}_D$ is known, a correction to

$$\mathbf{r}_{cor2(n)} = \mathbf{H}_{\mathbf{32}(n)} \times \mathbf{t}_{NP(n)}$$

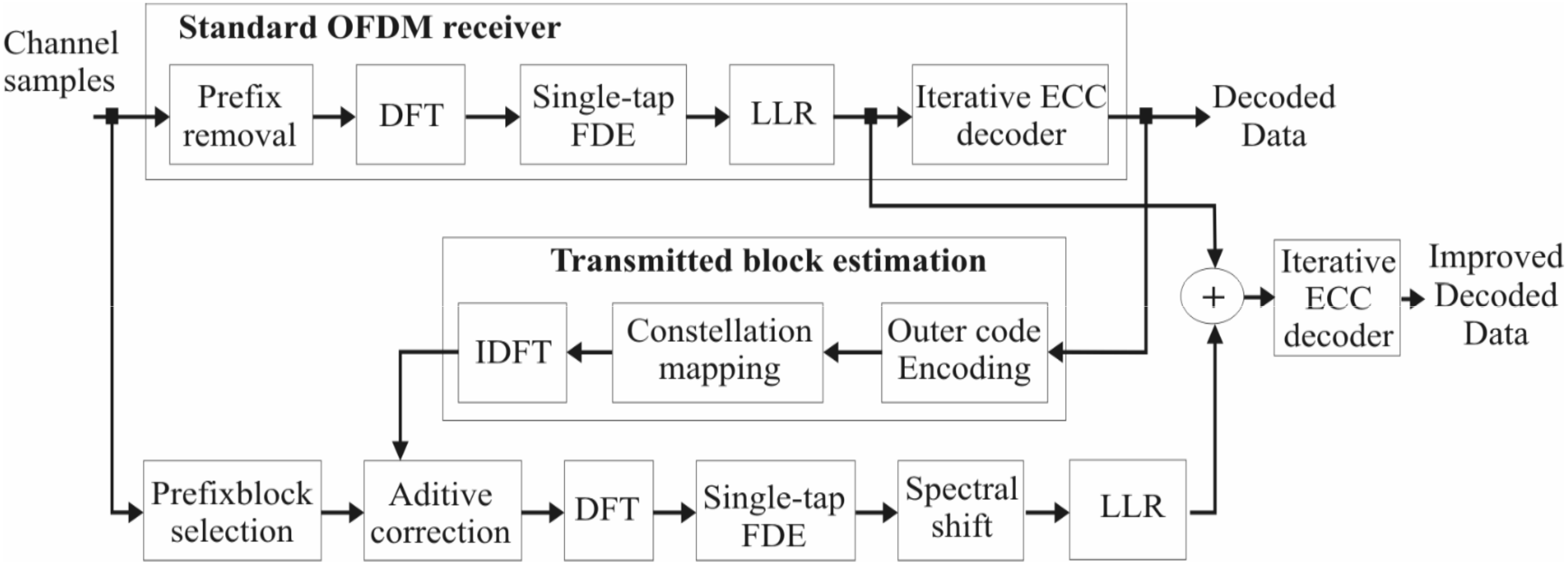
could be applied to the received subvector $\mathbf{r}_D$', such that:

$$\mathbf{r}_D' = \mathbf{H_{CIRC}} \times \mathbf{t}_D'$$

Transmitted block estimate

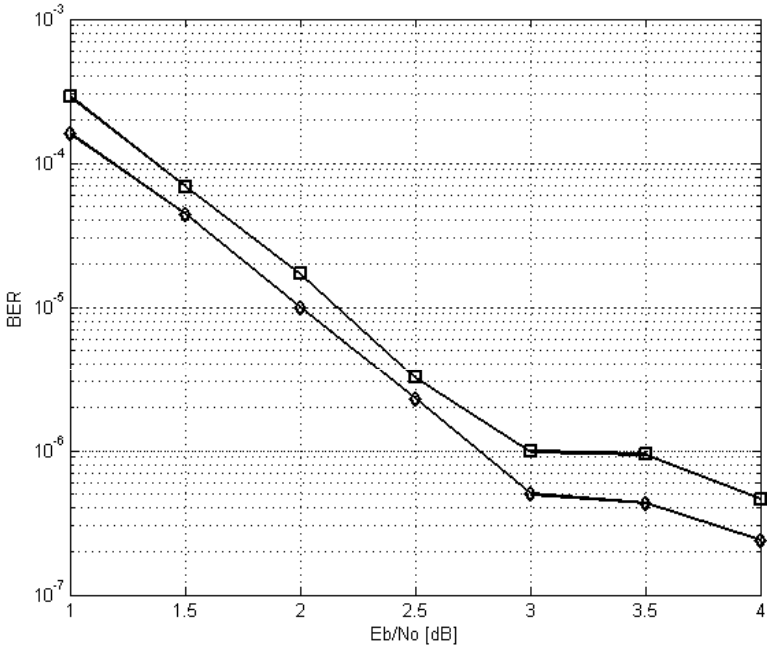
- The *“good estimate”* of transmitted vector $\mathbf{t}_D$, can be obtained by utilizing the already decoded data.
- RX must emulate the TX operation to get a reliable estimate of the transmitted block.
- Using this estimate, both corrections can be computed.
- The second received subblock is processed in the same way as the first subblock.

Suboptimal receiver with CP redundancy extraction



- The modification adds a new branch of processing to existing OFDM RX
- The modified receiver is fully compatible with existing standards

Performance of the suboptimal modified receiver



- Setup: 1024 carriers, T_g = 1/8 T_u, UMTS defined CTC R = 1/3
- Only a small improvement of approx 0.1 dB

Turbo Equalization

- C. Douillard *et al.*, “*Iterative correction of intersymbol interference:Turbo equalization*” Eur. Trans. Telecommun., vol. 6, pp. 507–511,Sept.–Oct. 1995.

- Iterative:

  Estimation -> Equalization -> Decoding -> Estimation…

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- [Fark] P. Farkaš, „*OFDM is an error control code*“, Journal of ELECTRICAL ENGINEERING vol. 54, NO 11-12, 2003

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- [Deb] M. Debbah, ”*Short introduction to OFDM*”, 2002,

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Circular Convolution

- N-point DFT
- ω is a primitive N^{th} root of unity $\exp(-2\pi j / N)$