Fourierov rad

$$x(\lambda) = \tfrac{1}{2}a_0 + a_1 \cos \lambda + a_2 \cos 2\lambda + a_3 \cos 3\lambda + \cdots + b_1 \sin \lambda + b_2 \sin 2\lambda + b_3 \sin 3\lambda + \cdots \quad \text{(A.1)}$$

exponenciálny tvar:

$$x(t) = \frac{a_0}{2} + \frac{1}{2}\sum_{n=1}^{\infty}[(a_n - b_n)e^{j2\pi nf_0 t} + (a_n + b_n)e^{-j2\pi nf_0 t}] \quad \text{(A.11)}$$

komplexné koeficienty:

$$c_n = \begin{cases} \frac{1}{2}(a_n - jb_n) & n > 0 \\ \frac{a_0}{2} & n = 0 \\ \frac{1}{2}(a_n + jb_n) & n < 0 \end{cases} \quad \text{(A.12)}$$

$$c_n = |c_n|\, e^{j\theta_n} \quad \text{(A.17)}$$

$$c_{-n} = |c_n|\, e^{-j\theta_n} \quad \text{(A.18)}$$

potom:

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi nf_0 t} \quad \text{(A.13)}$$

$$c_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)e^{-j2\pi nf_0 t}\, dt \quad \text{(A.14)}$$

nakoľko:

$$\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} e^{j(n-m)2\pi f_0 t}\, dt = \delta_{nm} = \begin{cases} 1 & n = m \\ 0 & n \neq m \end{cases} \quad \text{(A.15)}$$

$$\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)e^{-j2\pi mf_0 t}\, dt = \sum_{n=-\infty}^{\infty} c_n \delta_{nm} = c_m \quad \text{(A.16)}$$

Fourierova transformácia

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi ft}\, df$$

$x(t)$	$X(f)$
1. $\delta(t)$	1
2. 1	$\delta(f)$
3. $\cos 2\pi f_0 t$	$\frac{1}{2}[\delta(f - f_0) + \delta(f + f_0)]$
4. $\sin 2\pi f_0 t$	$\frac{1}{2j}[\delta(f + f_0) - \delta(f - f_0)]$
5. $\delta(t - t_0)$	$\exp(-j2\pi ft_0)$
6. $\exp(j2\pi f_0 t)$	$\delta(f - f_0)$
7. $\exp(-a\lvert t\rvert),\ a > 0$	$\frac{2a}{a^2 + (2\pi f)^2}$
8. $\exp\left[-\pi\left(\frac{t}{T}\right)^2\right]$	$T\exp[-\pi(fT)^2]$
9. $u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
10. $\exp(-at)\,u(t),\ a > 0$	$\frac{1}{a + j2\pi f}$
11. $t\exp(-at)\,u(t),\ a > 0$	$\frac{1}{(a + j2\pi f)^2}$
12. $\operatorname{rect}\left(\frac{t}{T}\right)$	$T\operatorname{sinc} fT$
13. $\cos 2\pi f_0 t\left[\operatorname{rect}\left(\frac{t}{T}\right)\right]$	$T\operatorname{sinc}(f - f_0)T$
14. $W\operatorname{sinc} Wt$	$\operatorname{rect}\left(\frac{f}{W}\right)$
15. $\begin{cases} 1 - \frac{\lvert t\rvert}{T} & \lvert t\rvert \le T \\ 0 & \lvert t\rvert > T \end{cases}$	$T\operatorname{sinc}^2 fT$
16. $\sum_{m=-\infty}^{\infty}\delta(t - mT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{\infty}\delta\left(f - \frac{n}{T_0}\right)$
$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{f}{a}\right)$
$x(t - t_0)$	$X(f)\exp(-j2\pi ft_0)$
$x(t)\exp(j2\pi f_0 t)$	$X(f - f_0)$
$\frac{d^n x}{dt^n}$	$(j2\pi f)^n X(f)$
$(-jt)^n x(t)$	$\frac{d^n X}{df^n}$
$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{j2\pi f}X(f) + \frac{1}{2}X(0)\delta(f)$
$x_1(t) * x_2(t)$	$X_1(f)X_2(f)$
$x_1(t)x_2(t)$	$X_1(f) * X_2(f)$

Často používané vzťahy

$\operatorname{rect}(f/2W) = 1$ pre $-W < f < W$, 0 pre $\lvert f\rvert > W$,
$\operatorname{sinc} x = (\sin \pi x)/\pi x$.

$Si(x) = Sin x/x$

$$\cos x \cos y = \tfrac{1}{2}\cos(x + y) + \tfrac{1}{2}\cos(x - y) \quad \text{(D.1)}$$

$$\sin x \sin y = -\tfrac{1}{2}\cos(x + y) + \tfrac{1}{2}\cos(x - y) \quad \text{(D.2)}$$

$$\sin x \cos y = \tfrac{1}{2}\sin(x + y) + \tfrac{1}{2}\sin(x - y) \quad \text{(D.3)}$$

$$\cos x \sin y = \tfrac{1}{2}\sin(x + y) - \tfrac{1}{2}\sin(x - y) \quad \text{(D.4)}$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y \quad \text{(D.5)}$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y \quad \text{(D.6)}$$

$$\cos^2 x = \tfrac{1}{2}(1 + \cos 2x) \quad \text{(D.7)}$$

$$\sin^2 x = \tfrac{1}{2}(1 - \cos 2x) \quad \text{(D.8)}$$

$$\sin x \cos x = \tfrac{1}{2}\sin 2x \quad \text{(D.9)}$$

$$\sin x + \sin y = 2\sin\tfrac{1}{2}(x + y)\cos\tfrac{1}{2}(x - y) \quad \text{(D.10)}$$

$$\sin x - \sin y = 2\cos\tfrac{1}{2}(x + y)\sin\tfrac{1}{2}(x - y) \quad \text{(D.11)}$$

$$\cos x + \cos y = 2\cos\tfrac{1}{2}(x + y)\cos\tfrac{1}{2}(x - y) \quad \text{(D.12)}$$

$$\cos x - \cos y = -2\sin\tfrac{1}{2}(x + y)\sin\tfrac{1}{2}(x - y) \quad \text{(D.13)}$$

$$\sin x = \frac{e^{jx} - e^{-jx}}{2j} \quad \text{(D.14)}$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2} \quad \text{(D.15)}$$

$$\left.\begin{aligned} \int_{-\pi}^{\pi}\sin m\lambda\, d\lambda &= 0 \\ \int_{-\pi}^{\pi}\cos m\lambda\, d\lambda &= 0 \\ \int_{-\pi}^{\pi}\sin m\lambda \cos n\lambda\, d\lambda &= 0 \end{aligned}\right\} \quad m \wedge n \in \mathbb{Z} \quad \text{(A.2)}$$

$$\left.\begin{aligned} \int_{-\pi}^{\pi}\sin m\lambda \sin n\lambda\, d\lambda &= 0 \\ \int_{-\pi}^{\pi}\cos m\lambda \cos n\lambda\, d\lambda &= 0 \end{aligned}\right\} \quad m \neq n \quad \text{(A.3)}$$

$$\left.\begin{aligned} \int_{-\pi}^{\pi}(\sin m\lambda)^2\, d\lambda &= \pi \\ \int_{-\pi}^{\pi}(\cos m\lambda)^2\, d\lambda &= \pi \end{aligned}\right\} \quad m = n \quad \text{(A.4)}$$