FREKVENČNÉ CHARAKTERISTIKY LDKI SYSTÉMOV

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$H(z)$ – prenosová funkcia

$H(z) \rightarrow H(e^{j\Omega}) = H(\Omega)$

Poznáme 4 charakteristiky:

- $A(\Omega)$ - amplitúdová frekvenčná charakteristika
- $\varphi(\Omega)$ - amplitúdová fázová frekvenčná charakteristika
- $M(\Omega)$ - magnitúdová frekvenčná charakteristika
- $\phi(\Omega)$ - magnitúdová fázová frekvenčná charakteristika

Pre tieto 4 charakteristiky platia nasledovné 3 vzťahy:

I. $|A(\Omega)| = M(\Omega)$

II. $H(\Omega) = A(\Omega) \cdot e^{j\varphi(\Omega)}$

III. $H(\Omega) = M(\Omega) \cdot e^{j\phi(\Omega)}$

Príklad 1

H(z) = 1 – z^{-3}

$$= 1 - e^{-j3\Omega} = e^{\frac{-3j\Omega}{2}} \bullet \left(e^{\frac{+3j\Omega}{2}} - e^{\frac{-3j\Omega}{2}} \right) = 2je^{\frac{-3j\Omega}{2}} \bullet \left(\frac{e^{\frac{+3j\Omega}{2}} - e^{\frac{-3j\Omega}{2}}}{2j} \right)$$

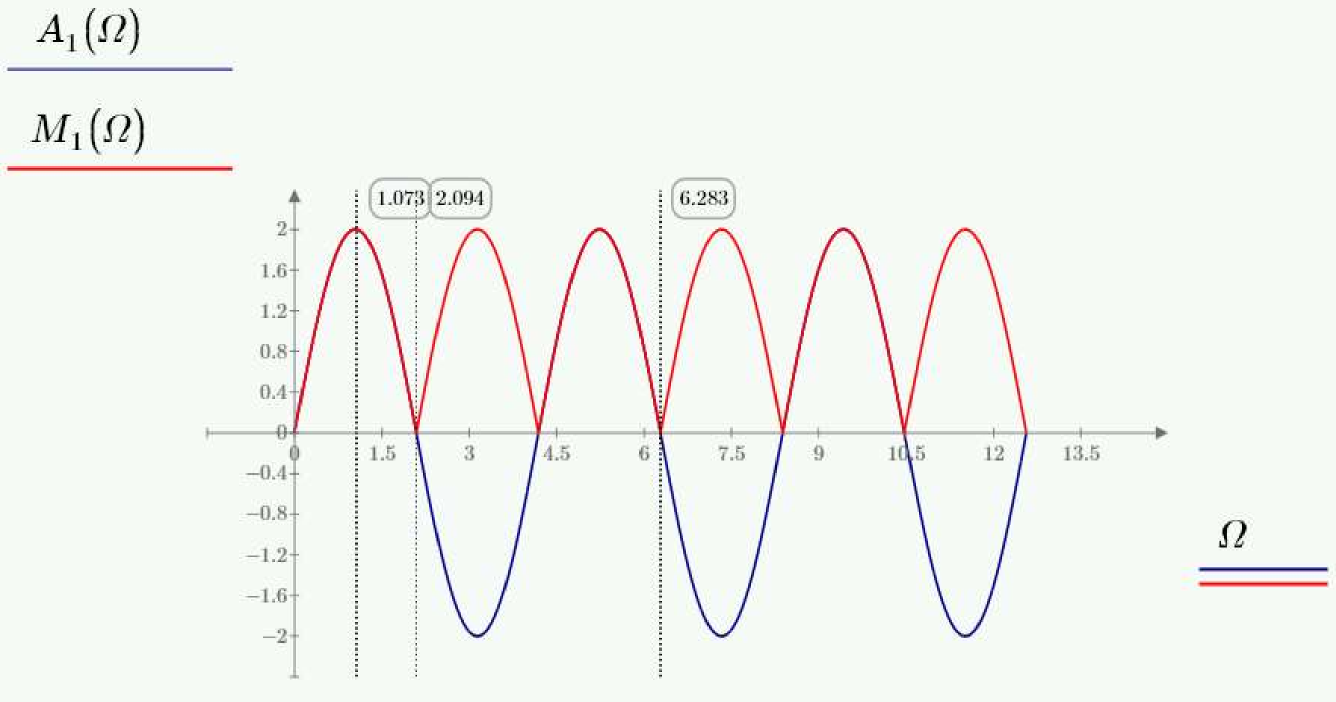
$$= 2je^{\frac{-3j\Omega}{2}} \bullet \sin\left(\frac{3\Omega}{2}\right) = 2j\sin\left(\frac{3\Omega}{2}\right) \bullet e^{\frac{-3j\Omega}{2}} = 2\sin\left(\frac{3\Omega}{2}\right) \bullet e^{\frac{-3j\Omega}{2}} \bullet e^{\frac{j\pi}{2}}$$

$$= \underbrace{2\sin\left(\frac{3\Omega}{2}\right)}_{A(\Omega)} \bullet \underbrace{e^{j\left(\frac{-3\Omega}{2}+\frac{\pi}{2}\right)}}_{e^{j\varphi(\Omega)}}$$

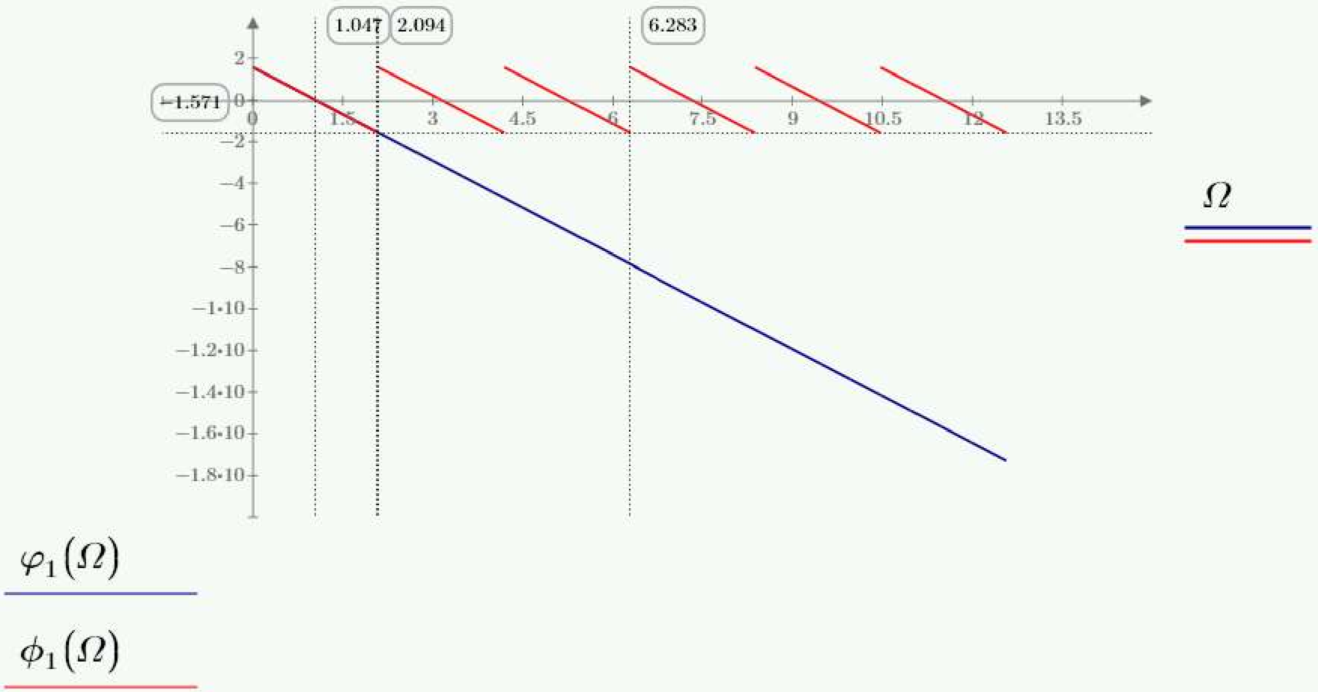
$$\sin x = \frac{e^{jx} - e^{-jx}}{2j}$$

$$j = e^{\frac{j\pi}{2}}$$

Príklad 1



$A_1(\Omega)$
$M_1(\Omega)$
1.073
2.094
6.283
Ω



1.047
2.094
6.283
-1.571
Ω
$\varphi_1(\Omega)$
$\phi_1(\Omega)$

Príklad 2

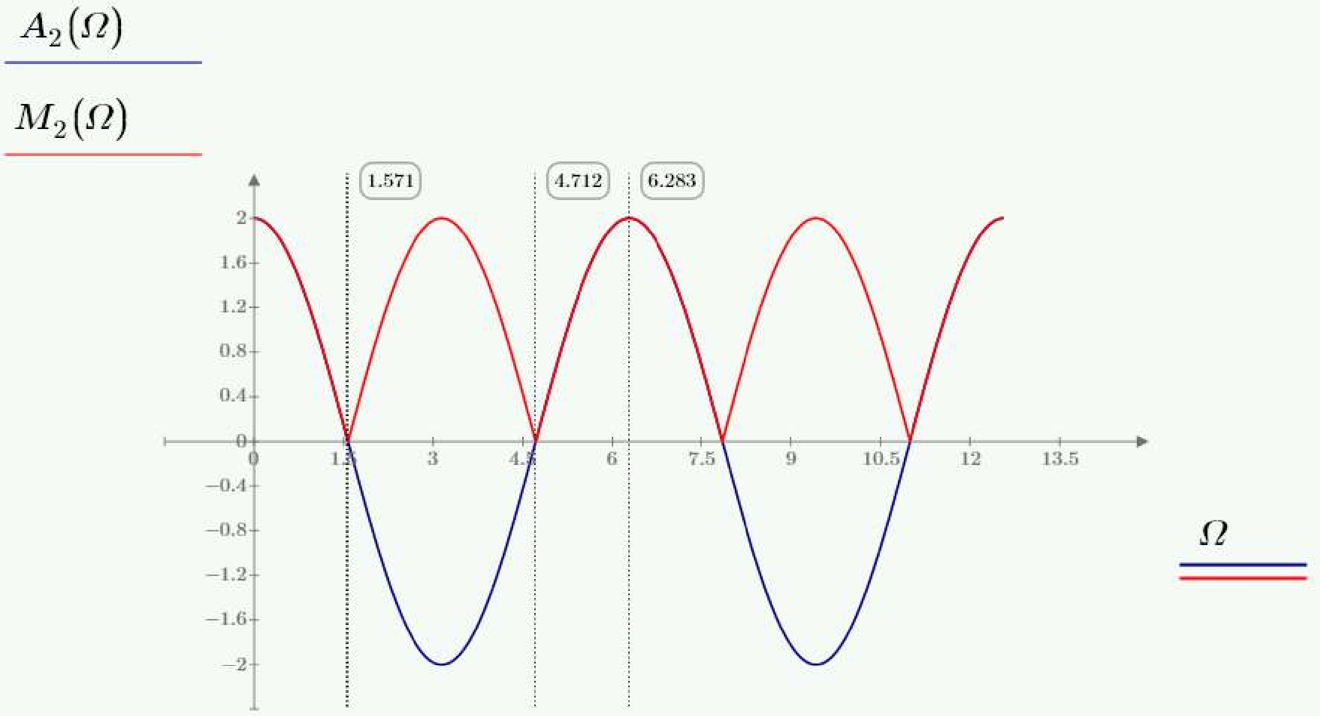
H(z) = 1 + z^{-2}

$$= 1 + e^{-j2\Omega} = e^{\frac{-2j\Omega}{2}} \bullet \left(e^{\frac{+2j\Omega}{2}} + e^{\frac{-2j\Omega}{2}} \right) = 2e^{\frac{-2j\Omega}{2}} \bullet \left(\frac{e^{\frac{+2j\Omega}{2}} + e^{\frac{-2j\Omega}{2}}}{2} \right)$$

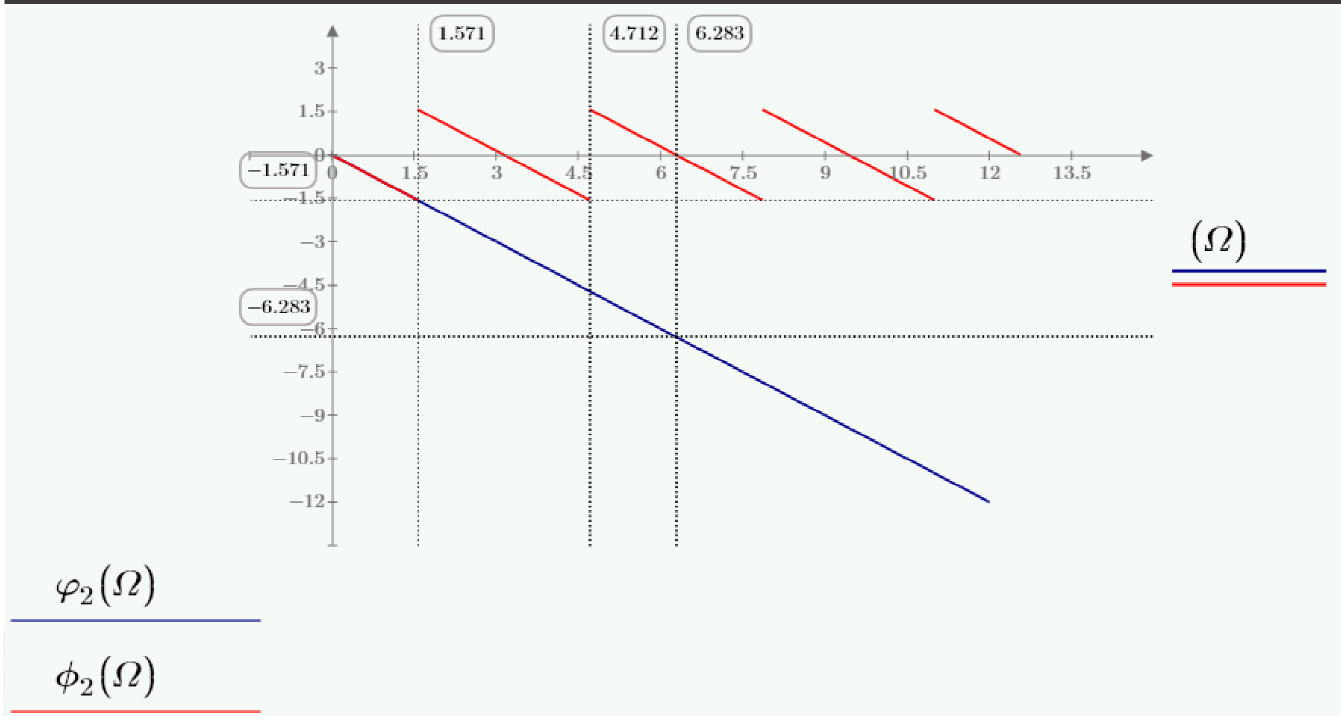
$$= 2e^{\frac{-2j\Omega}{2}} \bullet \cos\left(\frac{2\Omega}{2}\right) = \underbrace{2\cos(\Omega)}_{A(\Omega)} \bullet \underbrace{e^{j(-\Omega)}}_{e^{j\varphi(\Omega)}}$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$

Príklad 2



$A_2(\Omega)$
$M_2(\Omega)$
1.571
4.712
6.283
2
1.6
1.2
0.8
0.4
0
−0.4
−0.8
−1.2
−1.6
−2
0
1.5
3
4.5
6
7.5
9
10.5
12
13.5
Ω



1.571
4.712
6.283
3
1.5
0
−1.571
−1.5
−3
−4.5
−6.283
−6
−7.5
−9
−10.5
−12
0
1.5
3
4.5
6
7.5
9
10.5
12
13.5
(Ω)
$\varphi_2(\Omega)$
$\phi_2(\Omega)$

Príklad 3

H(z) = 2 – 2e^jπ/4 • z^-1

$$= 2 - 2e^{\frac{j\pi}{4}} \bullet e^{-j\Omega} = 2 - 2e^{j\left(\frac{\pi}{4}-\Omega\right)} = e^{\frac{j\left(\frac{\pi}{4}-\Omega\right)}{2}} \bullet \left(2e^{\frac{-j\left(\frac{\pi}{4}-\Omega\right)}{2}} - 2e^{\frac{j\left(\frac{\pi}{4}-\Omega\right)}{2}} \right)$$

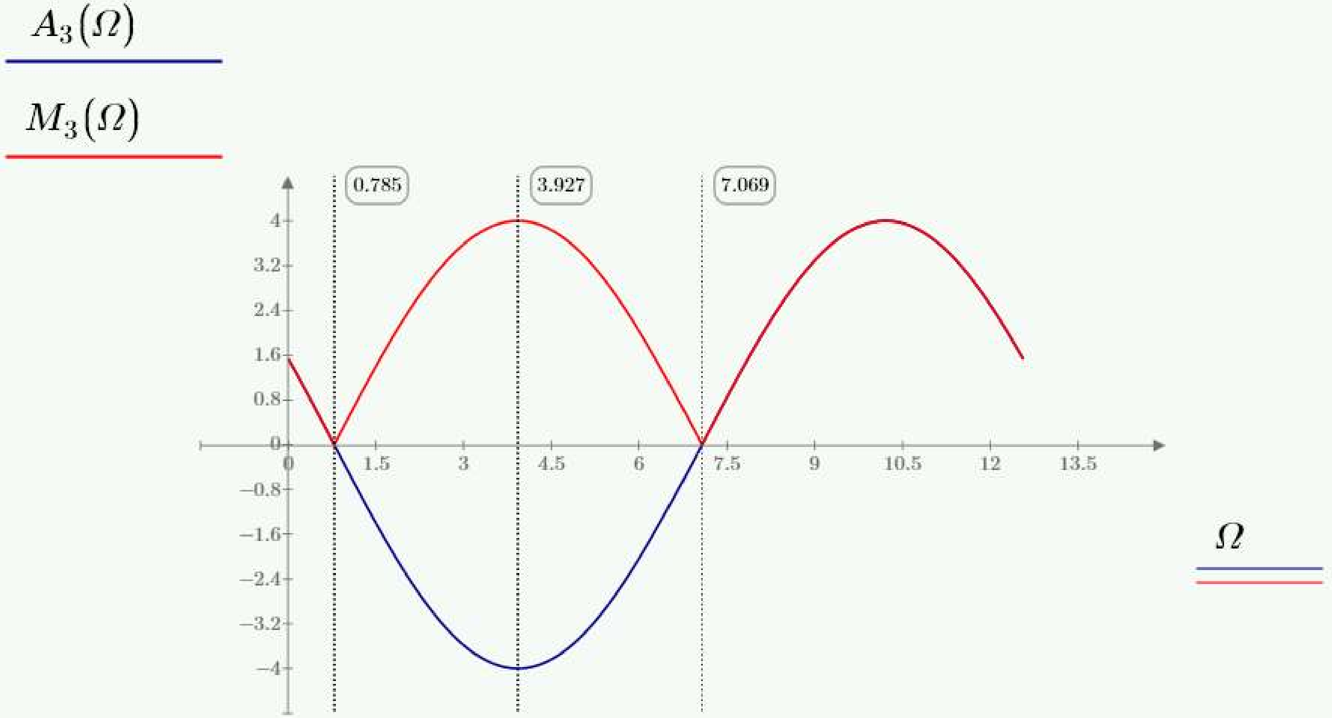
$$= -4je^{\frac{j\left(\frac{\pi}{4}-\Omega\right)}{2}} \bullet \left(\frac{e^{\frac{j\left(\frac{\pi}{4}-\Omega\right)}{2}} - e^{\frac{-j\left(\frac{\pi}{4}-\Omega\right)}{2}}}{2j} \right) = -4je^{\frac{j\left(\frac{\pi}{4}-\Omega\right)}{2}} \bullet \sin\left(\frac{\left(\frac{\pi}{4}-\Omega\right)}{2} \right) = -4j\sin\left(\frac{\left(\frac{\pi}{4}-\Omega\right)}{2} \right) \bullet e^{\frac{j\left(\frac{\pi}{4}-\Omega\right)}{2}}$$

$$= 4\sin\left(\frac{\left(\frac{\pi}{4}-\Omega\right)}{2} \right) \bullet e^{\frac{j\left(\frac{\pi}{4}-\Omega\right)}{2}} \bullet e^{\frac{-j\pi}{2}} = \underbrace{4\sin\left(\frac{\left(\frac{\pi}{4}-\Omega\right)}{2} \right)}_{A(\Omega)} \bullet \underbrace{e^{j\left(-\frac{\Omega}{2}-\frac{3\pi}{8}\right)}}_{e^{j\varphi(\Omega)}}$$

$$\sin x = \frac{e^{jx} - e^{-jx}}{2j}$$

$$-j = e^{\frac{-j\pi}{2}}$$

Príklad 3



$A_3(\Omega)$
$M_3(\Omega)$
0.785
3.927
7.069
Ω

0.785
3.927
7.069
Ω
$\varphi_3(\Omega)$
$\phi_3(\Omega)$

Príklad 4

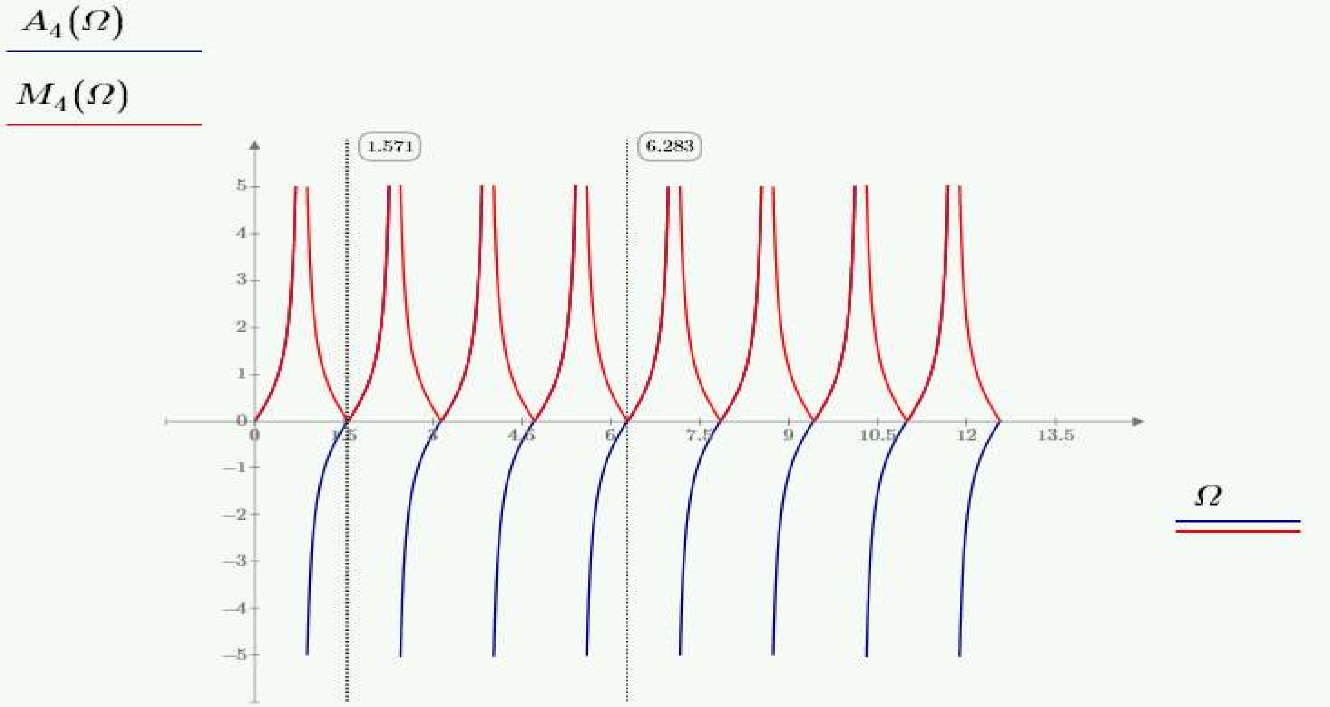
H(z) = (1 - z^{-4})/(1+z^{-4})

$$= \frac{1-e^{-j4\Omega}}{1+e^{-j4\Omega}} = \frac{e^{-2\Omega}\bullet(e^{j2\Omega}-e^{-j2\Omega})}{e^{-2\Omega}\bullet(e^{j2\Omega}+e^{-j2\Omega})}$$

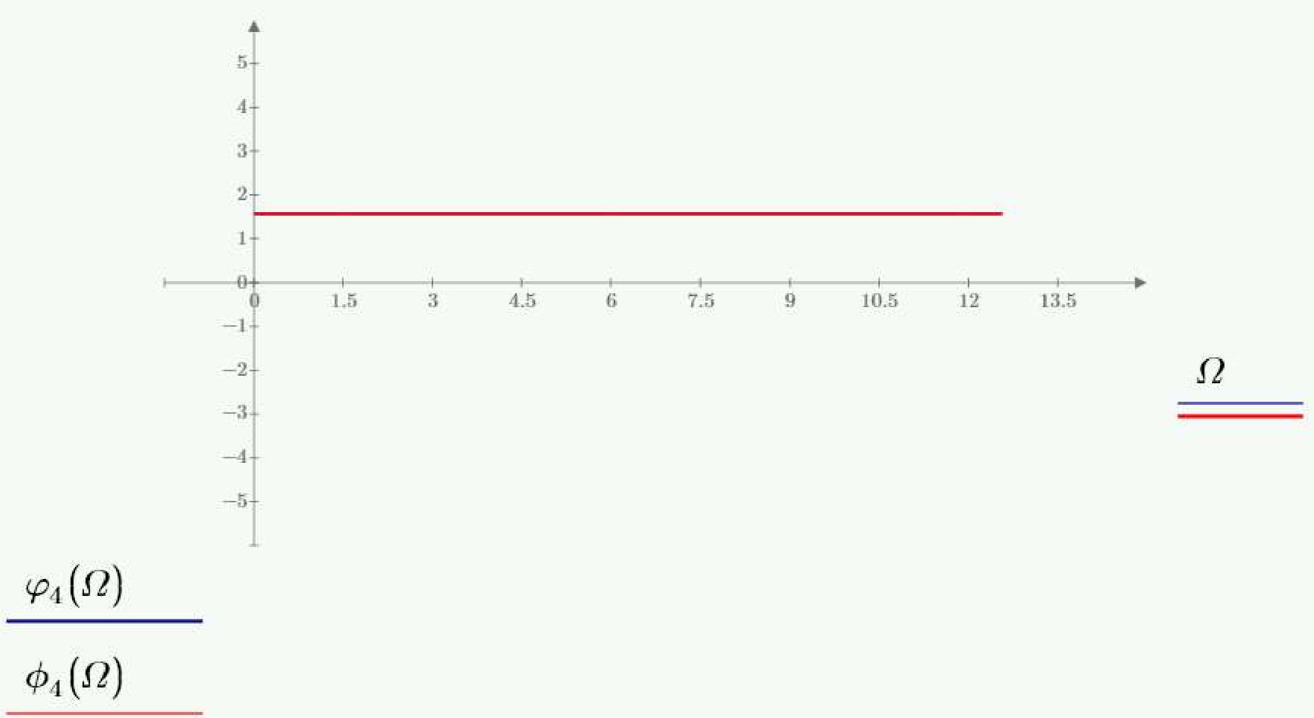
$$= \frac{2j\sin(2\Omega)}{2\cos(2\Omega)} = j\tan(2\Omega) = \underbrace{\tan(2\Omega)}_{A(\Omega)}\bullet \underbrace{e^{\frac{j\pi}{2}}}_{e^{j\varphi(\Omega)}}$$

$$\sin x = \frac{e^{jx}-e^{-jx}}{2j} \qquad \cos x = \frac{e^{jx}+e^{-jx}}{2} \qquad j = e^{\frac{j\pi}{2}}$$

Príklad 4



$A_4(\Omega)$
$M_4(\Omega)$
1.571
6.283
Ω



Ω
$\varphi_4(\Omega)$
$\phi_4(\Omega)$