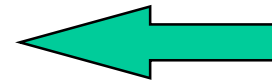
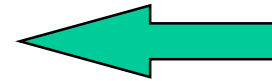


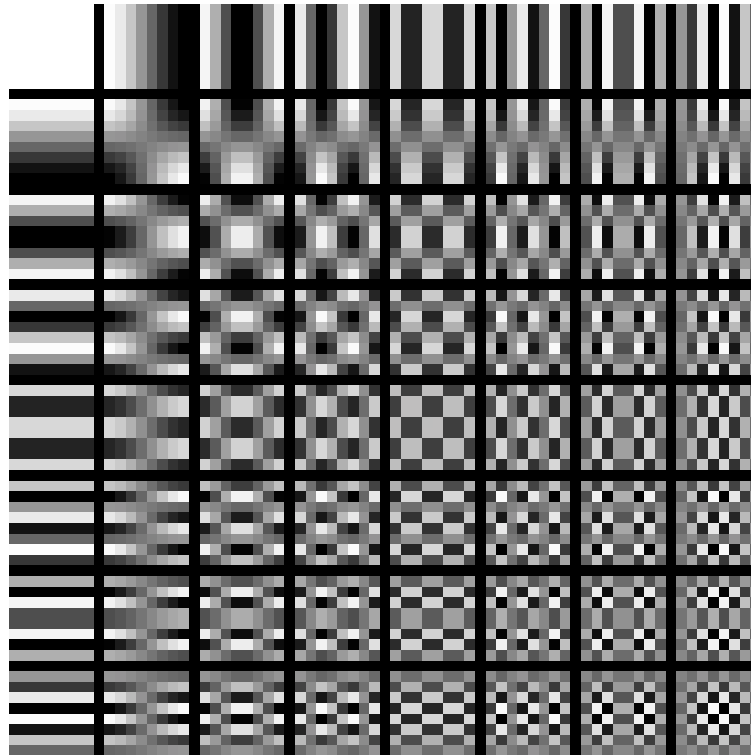
Aproximácia DCT v JPEG

JPEG:

- digitálny obraz
- bloky 8x8
- **DCT po blokoch**
- kompresia + kódovanie
- prenos
- dekódovanie + dekompresia
- **inverzná DCT po blokoch**



Aproximácia DCT v JPEG



Bázové funkcie DCT 8x8

Aproximácia DCT v JPEG

generátor jedného riadku DCT:

konst.

$$\alpha(i) = 2 \cos(\Omega) \alpha(i-1) - \alpha(i-2)$$

pričom: $\Omega = 2\pi f / f_{vz}$

$$\alpha(0) = A$$

$$\alpha(1) = A \cos(\Omega)$$

potom: $\alpha(i) = \{A, A \cos(\Omega), A \cos(2\Omega), A \cos(3\Omega), \dots\}$

Aproximácia DCT v JPEG

nahradenie násobenia: $\cos(\Omega) = 2^{-s}$

pričom: $s = 1, 2, 3, \dots$

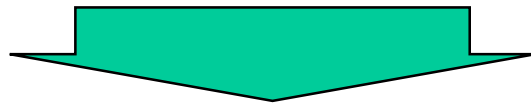
Príklad 1: $s = 1$ $\cos(\Omega) = 0.5$ $\Omega = \pi / 4$

$$N = f / f_{vz} = \frac{2\pi}{\arccos\left(\frac{1}{2}\right)} = 6 \quad \text{vzoriek na jednu periódu}$$

Aproximácia DCT v JPEG

Príklad 2:

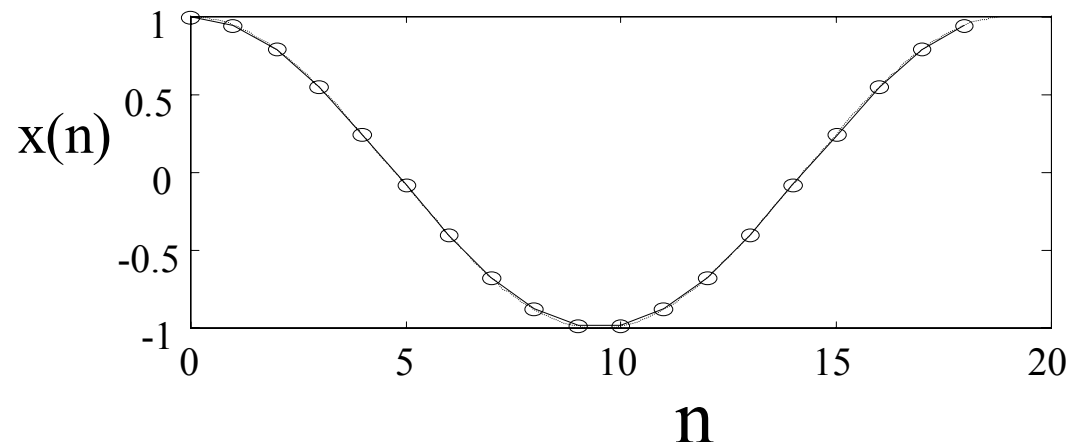
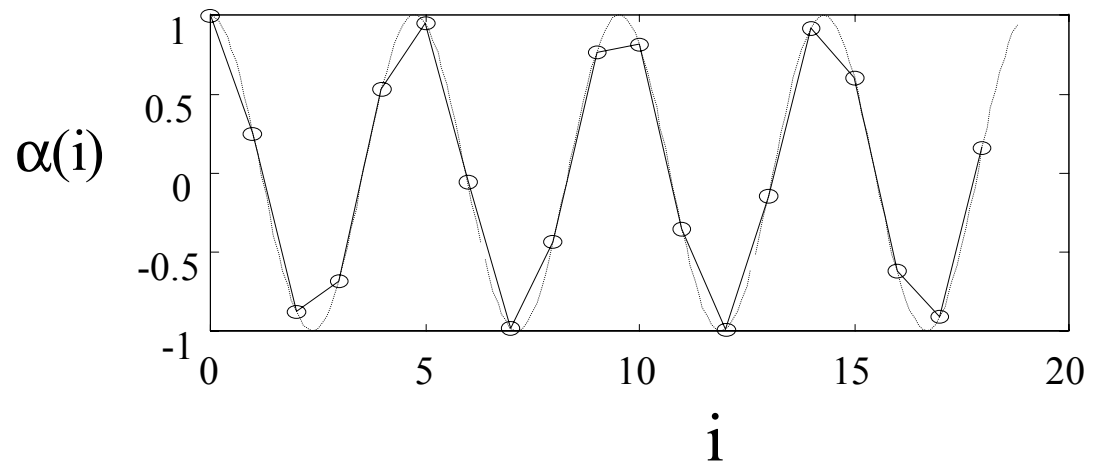
$$s = 2 \quad \rightarrow \quad \cos(\Omega) = 0.25 \quad \Omega \cong 1.318116$$



$$\alpha(i) = \frac{1}{2} \alpha(i-1) - \alpha(i-2)$$

$$N = f / f_{vz} = \frac{2\pi}{\arccos\left(\frac{1}{4}\right)} \cong 4.766792 \quad \text{vzoriek na jednu periódu}$$

Aproximácia DCT v JPEG



Aproximácia DCT v JPEG

Výpočet dvojrozmernej DCT:

- \mathbf{X} obrazový blok, \mathbf{Y} blok koeficientov
- $\tilde{\mathbf{X}}$ rekonštruovaný blok, $\tilde{\mathbf{Y}}$ blok koeficientov po kvantizácii
- \mathbf{K} kvantizačná konštanta, \mathbf{T} jadro DCT

$$\mathbf{Y} = \mathbf{T}\mathbf{X}\mathbf{T}^t \quad \tilde{\mathbf{X}} = \frac{1}{K^2} \mathbf{T}^t \tilde{\mathbf{Y}}\mathbf{T}$$

$$\mathbf{Y} = \frac{1}{K^2} \mathbf{T}\mathbf{X}\mathbf{T}^t \quad \tilde{\mathbf{X}} = \mathbf{T}^t \tilde{\mathbf{Y}}\mathbf{T}$$

$$\mathbf{Y} = \frac{1}{K} \mathbf{T}\mathbf{X}\mathbf{T}^t \quad \tilde{\mathbf{X}} = \frac{1}{K} \mathbf{T}^t \tilde{\mathbf{Y}}\mathbf{T}$$

Aproximácia DCT v JPEG

$$t(i, j) = \begin{cases} (1/n)^{1/2} & i = 0, 0 \leq j \leq n-1 \\ (2/n)^{1/2} \cos \frac{\pi(2i+1)j}{2n} & 1 \leq i \leq n-1, 0 \leq j \leq n-1 \end{cases}$$

$$\mathbf{T} = \begin{pmatrix} h & h & h & h & h & h & h & h \\ a & b & c & d & -d & -c & -b & -a \\ e & f & -f & -e & -e & -f & f & e \\ b & -d & -a & -c & c & a & d & -b \\ g & -g & -g & g & g & -g & -g & g \\ c & -a & d & b & -b & -d & a & -c \\ f & -e & e & -f & -f & e & -e & f \\ d & -c & b & -a & a & -b & c & -d \end{pmatrix}$$

$$a = 0.980785$$

$$b = 0.831469$$

$$c = 0.555570$$

$$d = 0.195090$$

$$e = 0.923879$$

$$f = 0.382683$$

$$g = 0.707107$$

$$h = 0.707107$$

Aproximácia DCT v JPEG

$$\text{DCT: } y(k, l) = \frac{1}{4} \sum_{i=0}^7 \sum_{j=0}^7 t(k, i) x(i, j) t(l, j)$$

$$\text{IDCT: } \tilde{x}(i, j) = \frac{1}{4} \sum_{k=0}^7 \sum_{l=0}^7 t(k, i) \tilde{y}(k, l) t(l, j)$$

Aproximácia DCT v JPEG

Dvojkroková aproximácia:

$$\alpha(0) = 1$$

$$\alpha(1) = 0.25$$

$$\alpha(2) = -0.875$$

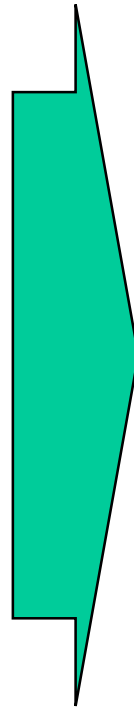
$$\alpha(3) = -0.6875$$

$$\alpha(4) = 0.53125$$

$$\alpha(5) = 0.953125$$

$$\alpha(6) = -0.054688$$

$$\alpha(7) = -0.980469$$



$$f = 0.382683$$

1 vzorkou:

$$f \cong \alpha(1) = 0.25$$

2 vzorkami:

$$f \cong -\alpha(2) - \alpha(4) = 0.34375$$

3 vzorkami:

$$f \cong -\alpha(1) - \alpha(3) + \alpha(6) = 0.382812$$

Aproximácia DCT v JPEG

$$\alpha(0) = 1 \quad \longrightarrow \quad (a, b, c, d, e, f, g, h)$$

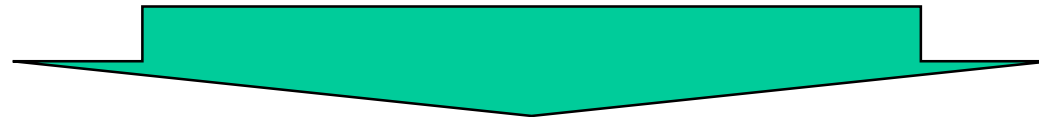
$$\alpha(0) = x \quad \longrightarrow \quad (xa, xb, xc, xd, xe, xf, xg, xh)$$

$$\begin{pmatrix} xaa & xab & xac & xad & xae & xaf & xag & xah \\ xba & xbb & xbc & xbd & xbe & xbf & xbg & xbh \\ xca & xcb & xcc & xcd & xce & xcf & xcg & xch \\ xda & xdb & xdc & xdd & xde & xdf & xdg & xdh \\ xea & xeb & xec & xed & xee & xef & xeg & xeh \\ xfa & xfb & xfc & xfd & xfe & xff & xfg & xfh \\ xga & xgb & xgc & xgd & xge & xgf & xgg & xgh \\ xha & xhb & xhc & xhd & xhe & xhf & xhg & xhh \end{pmatrix}$$

Aproximácia DCT v JPEG

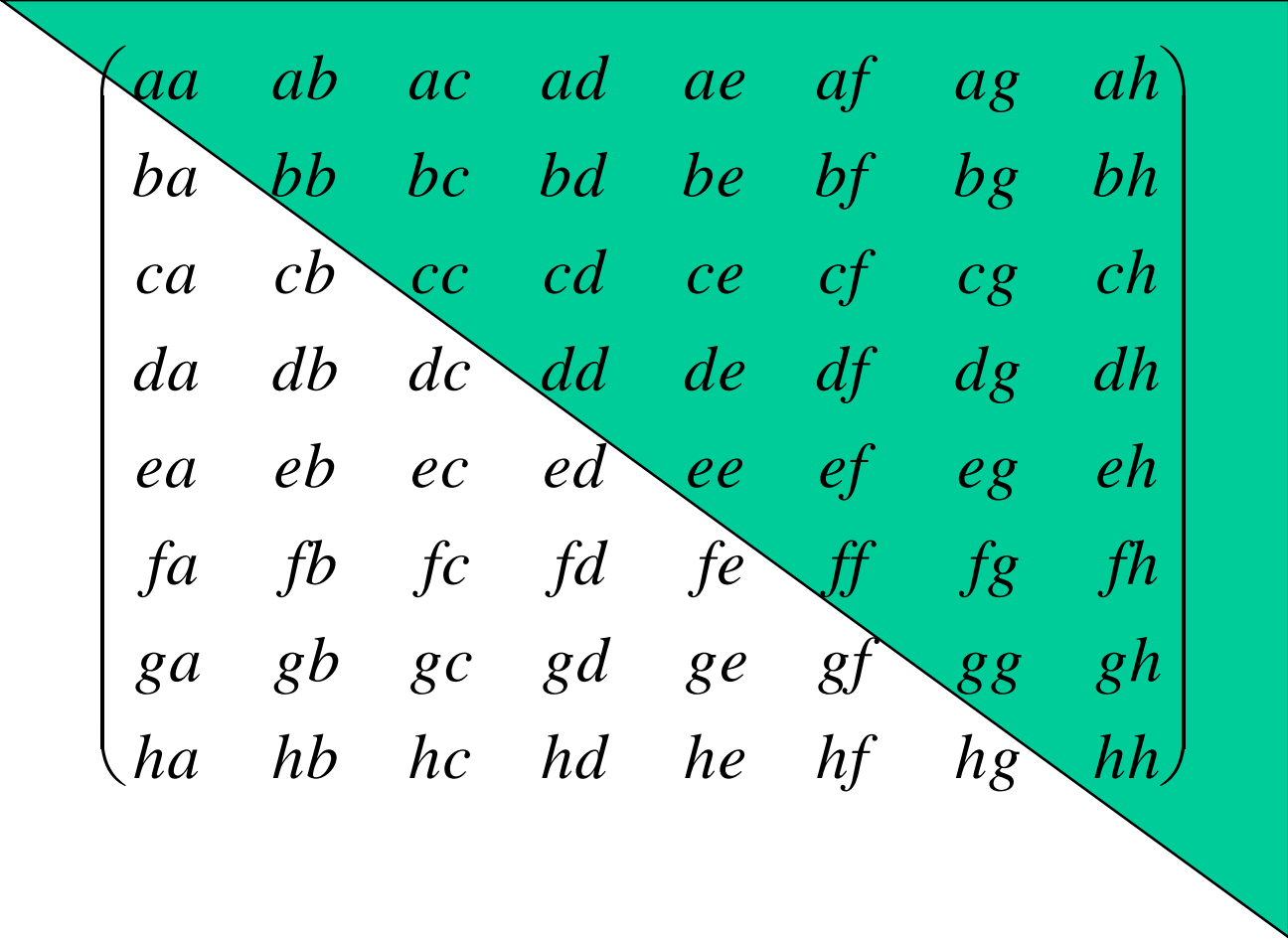
Nevýhody dvojkrokovej aproximácie:

- akumulácia chýb dvojnásobnou aproximáciou
- samostatný výpočet zrkadlových párov x_{pq} a x_{qp}



Jednokroková aproximácia

Aproximácia DCT v JPEG



The diagram illustrates an 8x8 grid of labels, likely representing a quantization table or a similar data structure. The labels are arranged in a grid from 'aa' to 'hh'. A green triangular shape is overlaid on the grid, starting from the top-left corner and extending towards the bottom-right corner. The labels are as follows:

<i>aa</i>	<i>ab</i>	<i>ac</i>	<i>ad</i>	<i>ae</i>	<i>af</i>	<i>ag</i>	<i>ah</i>
<i>ba</i>	<i>bb</i>	<i>bc</i>	<i>bd</i>	<i>be</i>	<i>bf</i>	<i>bg</i>	<i>bh</i>
<i>ca</i>	<i>cb</i>	<i>cc</i>	<i>cd</i>	<i>ce</i>	<i>cf</i>	<i>cg</i>	<i>ch</i>
<i>da</i>	<i>db</i>	<i>dc</i>	<i>dd</i>	<i>de</i>	<i>df</i>	<i>dg</i>	<i>dh</i>
<i>ea</i>	<i>eb</i>	<i>ec</i>	<i>ed</i>	<i>ee</i>	<i>ef</i>	<i>eg</i>	<i>eh</i>
<i>fa</i>	<i>fb</i>	<i>fc</i>	<i>fd</i>	<i>fe</i>	<i>ff</i>	<i>fg</i>	<i>fh</i>
<i>ga</i>	<i>gb</i>	<i>gc</i>	<i>gd</i>	<i>ge</i>	<i>gf</i>	<i>gg</i>	<i>gh</i>
<i>ha</i>	<i>hb</i>	<i>hc</i>	<i>hd</i>	<i>he</i>	<i>hf</i>	<i>hg</i>	<i>hh</i>

Aproximácia DCT v JPEG

Príklady

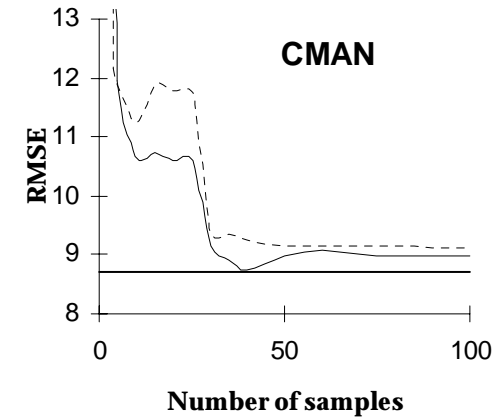
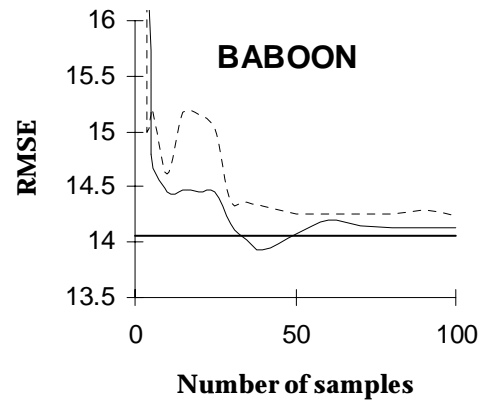
- \vdots
BABOON
- CMAN
- LENA
- ZEBRA

Spôsob hodnotenia:

$$RMSE = \frac{1}{256} \sqrt{\sum_{i=0}^{255} \sum_{j=0}^{255} (x_{i,j} - \tilde{x}_{i,j})^2}$$

Aproximácia DCT v JPEG

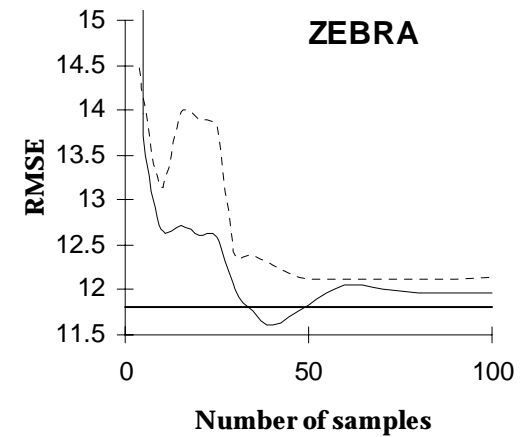
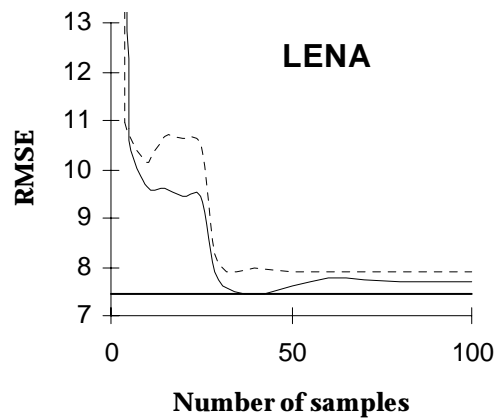
Aproximácia
1 koeficientom
generátora



dvojkroková

jednokroková

DCT



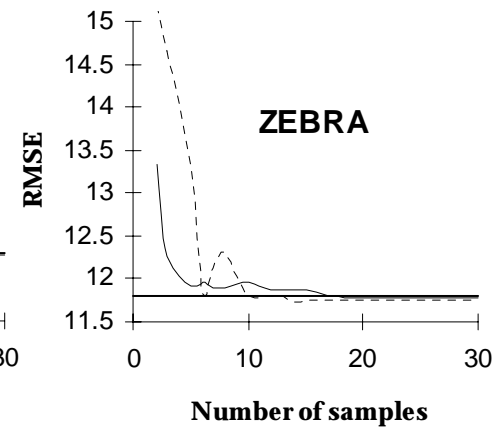
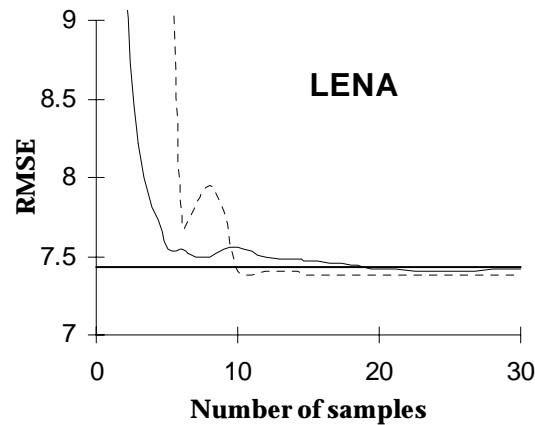
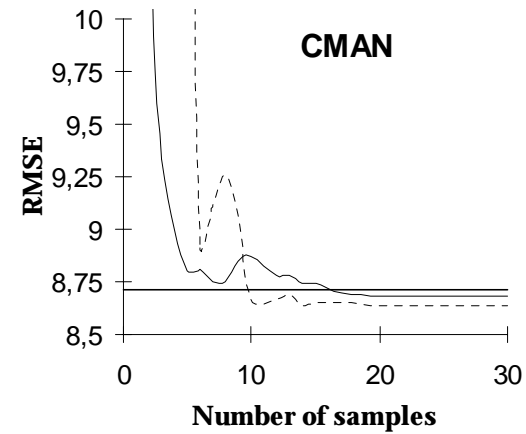
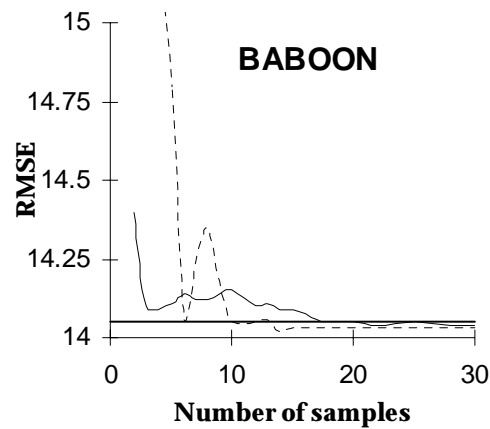
Aproximácia DCT v JPEG

Aproximácia
2 koeficientami
generátora

dvojkroková

jednokroková

DCT



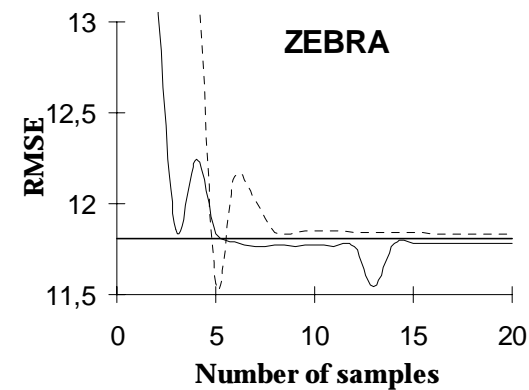
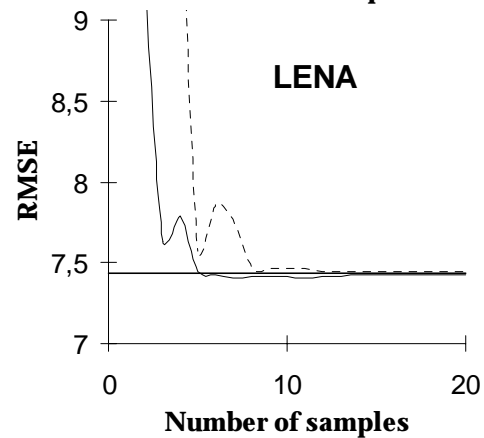
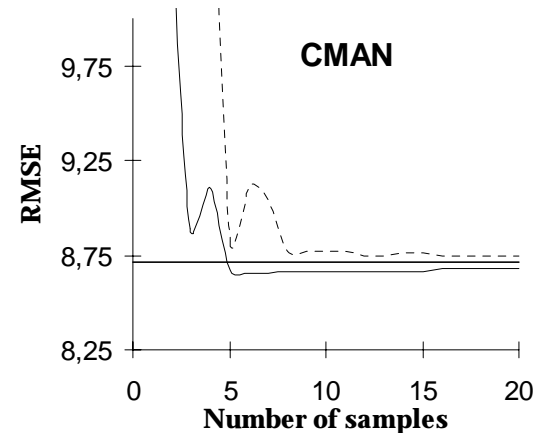
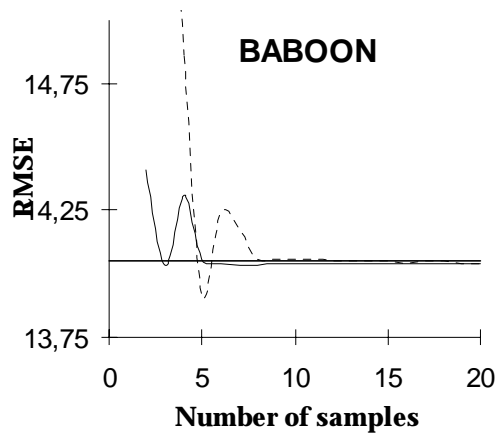
Aproximácia DCT v JPEG

Aproximácia
3 koeficientami
generátora

dvojkroková

jednokroková

DCT



Aproximácia DCT v JPEG

Prečo nie sú priebehy klesajúce?

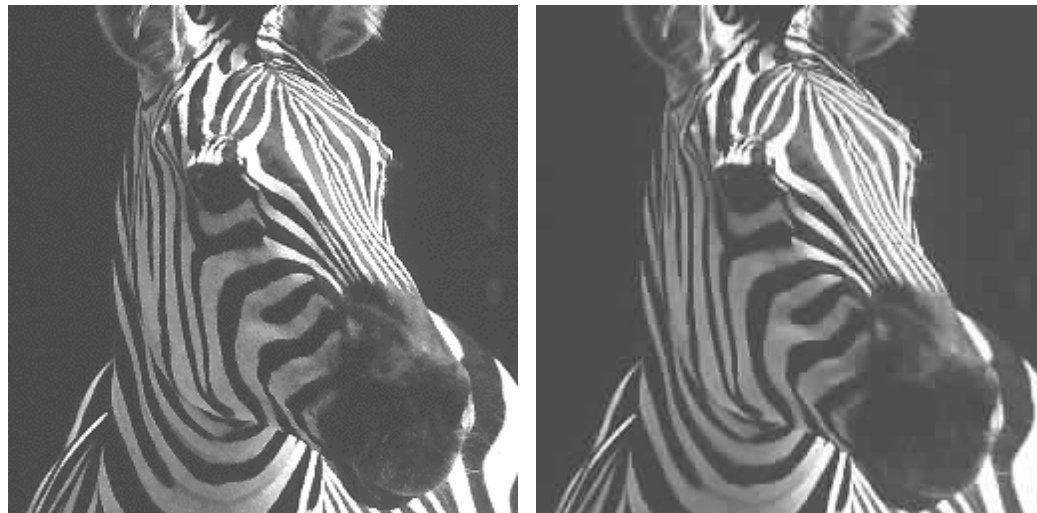
RMSE vs. súčet chýb  eliminácia chýb

Aproximácia
1 koeficientom
generátora

	10 vzoriek	chyba ($t - \tilde{t}$)	15 vzoriek	chyba ($t - \tilde{t}$)
<i>a</i>	0.980469	0.000316	0.980469	0.000316
<i>b</i>	0.875	-0.043531	0.816895	0.014574
<i>c</i>	0.53125	0.02432	0.53125	0.02432
<i>d</i>	0.25	-0.05491	0.142761	0.052329
<i>e</i>	0.953125	-0.29246	0.922638	0.001241
<i>g</i>	0.435547	-0.052864	0.354248	0.028435
<i>f</i>	0.6875	0.019606	0.6875	0.019606
<i>h</i>	0.6875	0.019606	0.6875	0.019606

Suma	-0.116703	Suma	0.160428
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Príklad ZEBRA, typ 1, jednokroková



Originál ZEBRA

DCT



ECT 40 vzoriek

ECT 3 vzorky

ECT 5 vzoriek

Príklad LENA, typ 2, dvojkroková



Original LENA 256x256



DCT



ECT 2 vzorky



ECT 20 vzoriek

Aproximácia DCT v JPEG

RMSE

ZEBRA, typ 1

DCT	11.8072
-----	---------

N=2	38.7521
N=3	37.2224
N=5	13.9402
N=10	12.6671
N=20	12.6001
N=30	12.0134
N=40	11.6145
N=50	11.8322
N=70	11.9923

LENA, typ 2

DCT	7.4283
-----	--------

N=2	14.7976
N=5	10.2274
N=8	7.9551
N=10	7.4027
N=12	7.4025
N=15	7.3825
N=20	7.3780

Aproximácia ICT v JPEG

ICT- Integer Cosine Transform

- nahrádza aritmetiku reálnych čísel celočíselnou aritmetikou
- znižuje výpočtovú náročnosť
- MICT (Modified Integer Cosine Transform) znižuje počet koeficientov DCT jadra

Aproximácia ICT v JPEG

Solution no:	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>K</i>
01	2	0	0	0	1	1	1	1	8
02	28	8	6	4	15	15	15	15	1800
03	164	42	40	4	87	87	87	87	60552
04	24	20	12	6	17	17	17	17	2312
05	56	16	12	8	30	30	30	30	7200
06	72	30	24	8	41	41	41	41	13448
07	84	24	18	12	45	45	45	45	16200
14	140	40	30	20	75	75	75	75	45000
15	180	130	104	20	123	123	123	123	121032
16	88	40	24	22	51	51	51	51	20808
29	176	80	48	44	102	102	102	102	83232
30	192	160	96	48	136	136	136	136	147968
31	216	180	108	54	153	153	153	153	187272
32	240	200	120	60	170	170	170	170	231200