Číslicové spracovanie signálov II

2D diskrétne ortogonálne transformácie

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Diskrétny Fourierov rad (1/2)

- DFS - "Discrete Fourier Series" - reprezentácia diskr. period. postupnosti vo frekv. oblasti

$\tilde{x}(n_1, n_2)$ - periodická postupnosť s periódou $N_1 x N_2$

$$0 \le n_1 \le N_1 - 1 \qquad 0 \le n_2 \le N_2 - 1$$

$\tilde{X}(k_1, k_2)$ - periodická postupnosť frekvenčných zložiek s periódou $N_1 x N_2$ - koeficienty DFS z postupnosti $\tilde{x}(n_1, n_2)$

$$0 \le k_1 \le N_1 - 1 \qquad 0 \le k_2 \le N_2 - 1$$

Diskrétny Fourierov rad (2/2)

$$\tilde{X}(k_1,k_2)=\sum_{n_1=0}^{N_1-1}\sum_{n_2=0}^{N_2-1}\tilde{x}(n_1,n_2)e^{-j\frac{2\pi}{N_1}k_1n_1}e^{-j\frac{2\pi}{N_2}k_2n_2}$$

$$0\le k_1\le N_1-1 \qquad 0\le k_2\le N_2-1$$

$$\tilde{x}(n_1,n_2)=\frac{1}{N_1N_2}\sum_{k_1=0}^{N_1-1}\sum_{k_2=0}^{N_2-1}\tilde{X}(k_1,k_2)e^{j\frac{2\pi}{N_1}k_1n_1}e^{j\frac{2\pi}{N_2}k_2n_2}$$

$$0\le n_1\le N_1-1 \qquad 0\le n_2\le N_2-1$$

Vlastnosti DFS (1/6)

$\tilde{x}(n_1,n_2), \tilde{y}(n_1,n_2)$ - periodické postupnosti s periódou N_1xN_2

$$\tilde{x}(n_1,n_2) \leftrightarrow \tilde{X}(k_1,k_2)$$
$$\tilde{y}(n_1,n_2) \leftrightarrow \tilde{Y}(k_1,k_2)$$

- Linearita

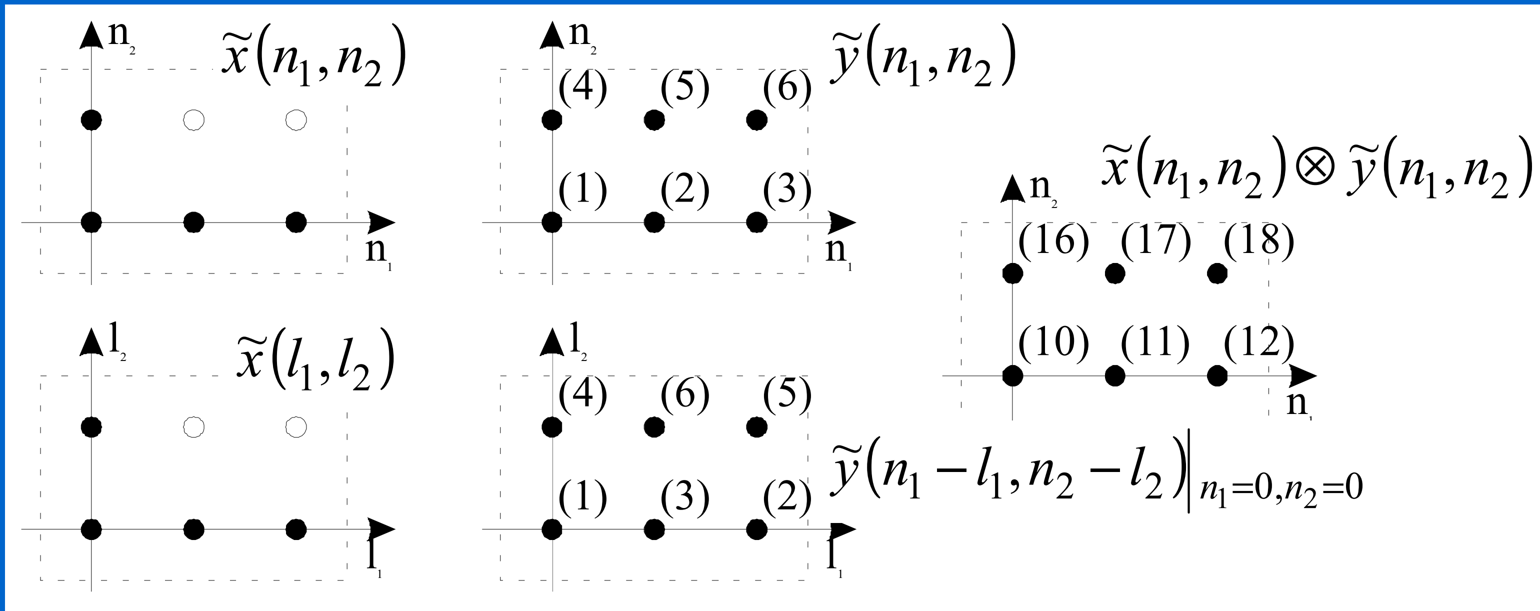
$$a\cdot\tilde{x}(n_1,n_2)+b\cdot\tilde{y}(n_1,n_2) \leftrightarrow a\cdot\tilde{X}(k_1,k_2)+b\cdot\tilde{Y}(k_1,k_2)$$

- Periodická konvolúcia

$$\tilde{x}(n_1,n_2)\otimes\tilde{y}(n_1,n_2)=\sum_{l_1=0}^{N_1-1}\sum_{l_2=0}^{N_2-1}\tilde{x}(l_1,l_2)\tilde{y}(n_1-l_1,n_2-l_2) \leftrightarrow \tilde{X}(k_1,k_2)\cdot\tilde{Y}(k_1,k_2)$$

Vlastnosti DFS (2/6)

- Periodická konvolúcia ($N_1 x N_2 = 3x2$) - príklad



$$\tilde{x}(n_1,n_2)\otimes\tilde{y}(n_1,n_2)=\sum_{l_1=0}^{2}\sum_{l_2=0}^{1}\tilde{x}(l_1,l_2)\tilde{y}(n_1-l_1,n_2-l_2)$$

Vlastnosti DFS (3/6)

- Násobenie

$$\tilde{x}(n_1,n_2)\tilde{y}(n_1,n_2) \quad \leftrightarrow \quad \frac{1}{N_1N_2}\tilde{X}(k_1,k_2)\otimes\tilde{Y}(k_1,k_2)=$$

$$=\frac{1}{N_1N_2}\sum_{l_1=0}^{N_1-1}\sum_{l_2=0}^{N_2-1}\tilde{X}(l_1,l_2)\tilde{Y}(k_1-l_1,k_2-l_2)$$

- Separovateľnosť

$$\tilde{x}(n_1,n_2)=\tilde{x}_1(n_1)\tilde{x}_2(n_2) \quad \leftrightarrow \quad \tilde{X}(k_1,k_2)=\tilde{X}_1(k_1)\tilde{X}_2(k_2)$$

$$\tilde{X}_1(k_1): \quad N_1-\text{bodová 1D DFS}$$

$$\tilde{X}_2(k_2): \quad N_2-\text{bodová 1D DFS}$$

Vlastnosti DFS (4/6)

- Veta o posunutí postupnosti

$$\tilde{x}(n_1 - m_1, n_2 - m_2) \quad \leftrightarrow \quad \tilde{X}(k_1, k_2) e^{-j\frac{2\pi}{N_1}k_1 m_1} e^{-j\frac{2\pi}{N_2}k_2 m_2}$$

- Veta o počiatočnej hodnote a DC zložke

$$\tilde{x}(0,0) = \frac{1}{N_1 N_2} \sum_{k_1=0}^{N_1-1} \sum_{k_2=0}^{N_2-1} \tilde{X}(k_1, k_2)$$

$$\tilde{X}(0,0) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} \tilde{x}(n_1, n_2)$$

Vlastnosti DFS (5/6)

- Parsevalov teorém

$$\sum_{n_1=0}^{N_1-1}\sum_{n_2=0}^{N_2-1}\tilde{x}(n_1,n_2)\tilde{y}^*(n_1,n_2)=\frac{1}{N_1N_2}\sum_{k_1=0}^{N_1-1}\sum_{k_2=0}^{N_2-1}\tilde{X}(k_1,k_2)\tilde{Y}^*(k_1,k_2)$$

$$\sum_{n_1=0}^{N_1-1}\sum_{n_2=0}^{N_2-1}\left|\tilde{x}(n_1,n_2)\right|^2=\frac{1}{N_1N_2}\sum_{k_1=0}^{N_1-1}\sum_{k_2=0}^{N_2-1}\left|\tilde{X}(k_1,k_2)\right|^2$$

- Symetria

$$\tilde{x}^*(n_1,n_2) \quad \leftrightarrow \quad \tilde{X}^*(-k_1,-k_2)$$

$$\tilde{x}(n_1,n_2): real \quad \leftrightarrow \quad \tilde{X}(k_1,k_2)=\tilde{X}^*(-k_1,-k_2)$$

Vlastnosti DFS (6/6)

- Symetria (pokrač.)

$$\tilde{X}_R(k_1,k_2)=\tilde{X}_R(-k_1,-k_2)$$
$$\tilde{X}_I(k_1,k_2)=-\tilde{X}_I(-k_1,-k_2)$$
$$\left|\tilde{X}(k_1,k_2)\right|=\left|\tilde{X}(-k_1,-k_2)\right|$$
$$\Theta_{\tilde{X}}(k_1,k_2)=-\Theta_{\tilde{X}}(-k_1,-k_2)$$

Diskrétna Fourierova transf. (1/5)

- DFT - "Discrete Fourier Transform" - reprezentácia diskr. periodifikovanej konečnej postupnosti vo frekvenčnej oblasti

$\tilde{x}(n_1,n_2)$ - periodická postupnosť s periódou $N_1 x N_2$

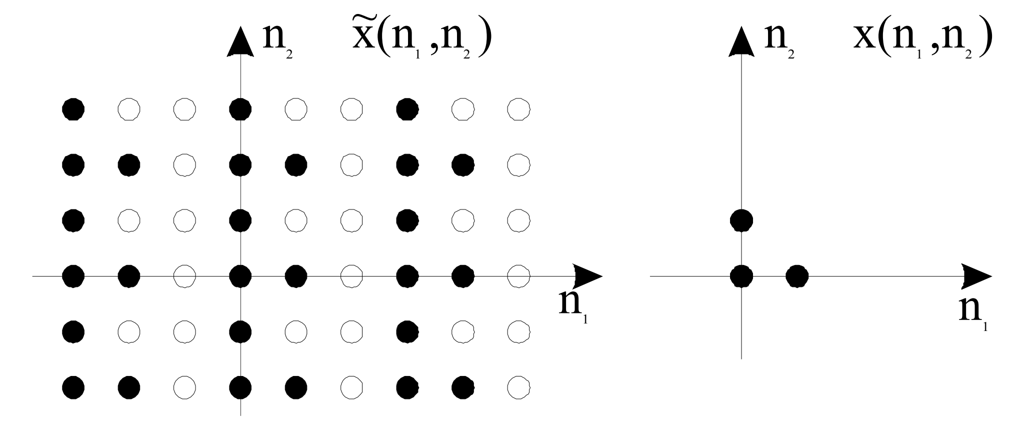
$x(n_1,n_2)$ - konečná postupnosť - 1 perióda z $\tilde{x}(n_1,n_2)$

$$x(n_1,n_2)=\tilde{x}(n_1,n_2)\cdot R_{N_1xN_2}(n_1,n_2)$$

$$R_{N_1xN_2}(n_1,n_2)=\begin{cases} 1 & 0\le n_1\le N_1-1;\quad 0\le n_2\le N_2-1 \\ 0 & inak \end{cases}$$

$$\tilde{x}(n_1,n_2)=\sum_{r_1=-\infty}^{\infty}\sum_{r_2=-\infty}^{\infty}x(n_1-r_1N_1,n_2-r_2N_2)$$

Diskrétna Fourierova transf. (2/5)



$\tilde{X}(k_1,k_2)$ - DFS koeficienty z $\tilde{x}(n_1,n_2)$

$X(k_1,k_2)$ - konečná postupnosť - 1 perióda z $\tilde{X}(k_1,k_2)$

$$X(k_1,k_2)=\tilde{X}(k_1,k_2)\cdot R_{N_1xN_2}(k_1,k_2)$$

$$\tilde{X}(k_1,k_2)=\sum_{r_1=-\infty}^{\infty}\sum_{r_2=-\infty}^{\infty}X(k_1-r_1N_1,k_2-r_2N_2)$$

$$x(n_1,n_2)\leftrightarrow\tilde{x}(n_1,n_2)\leftrightarrow\tilde{X}(k_1,k_2)\leftrightarrow X(k_1,k_2)$$

Diskrétna Fourierova transf. (3/5)

$$X(k_1,k_2)=\begin{cases}\sum_{n_1=0}^{N_1-1}\sum_{n_2=0}^{N_2-1} x(n_1,n_2)e^{-j\frac{2\pi}{N_1}k_1n_1}e^{-j\frac{2\pi}{N_2}k_2n_2} & 0\le k_1\le N_1-1;\ 0\le k_2\le N_2-1\\ 0 & inak\end{cases}$$

$$x(n_1,n_2)=\begin{cases}\frac{1}{N_1N_2}\sum_{k_1=0}^{N_1-1}\sum_{k_2=0}^{N_2-1} X(k_1,k_2)e^{j\frac{2\pi}{N_1}k_1n_1}e^{j\frac{2\pi}{N_2}k_2n_2} & 0\le n_1\le N_1-1;\ 0\le n_2\le N_2-1\\ 0 & inak\end{cases}$$

Postupnosť $x(n_1,n_2)$ rozmeru N_1xN_2 je reprezentovaná vo frekvenčnej oblasti postupnosťou $X(k_1,k_2)$ rozmeru N_1xN_2

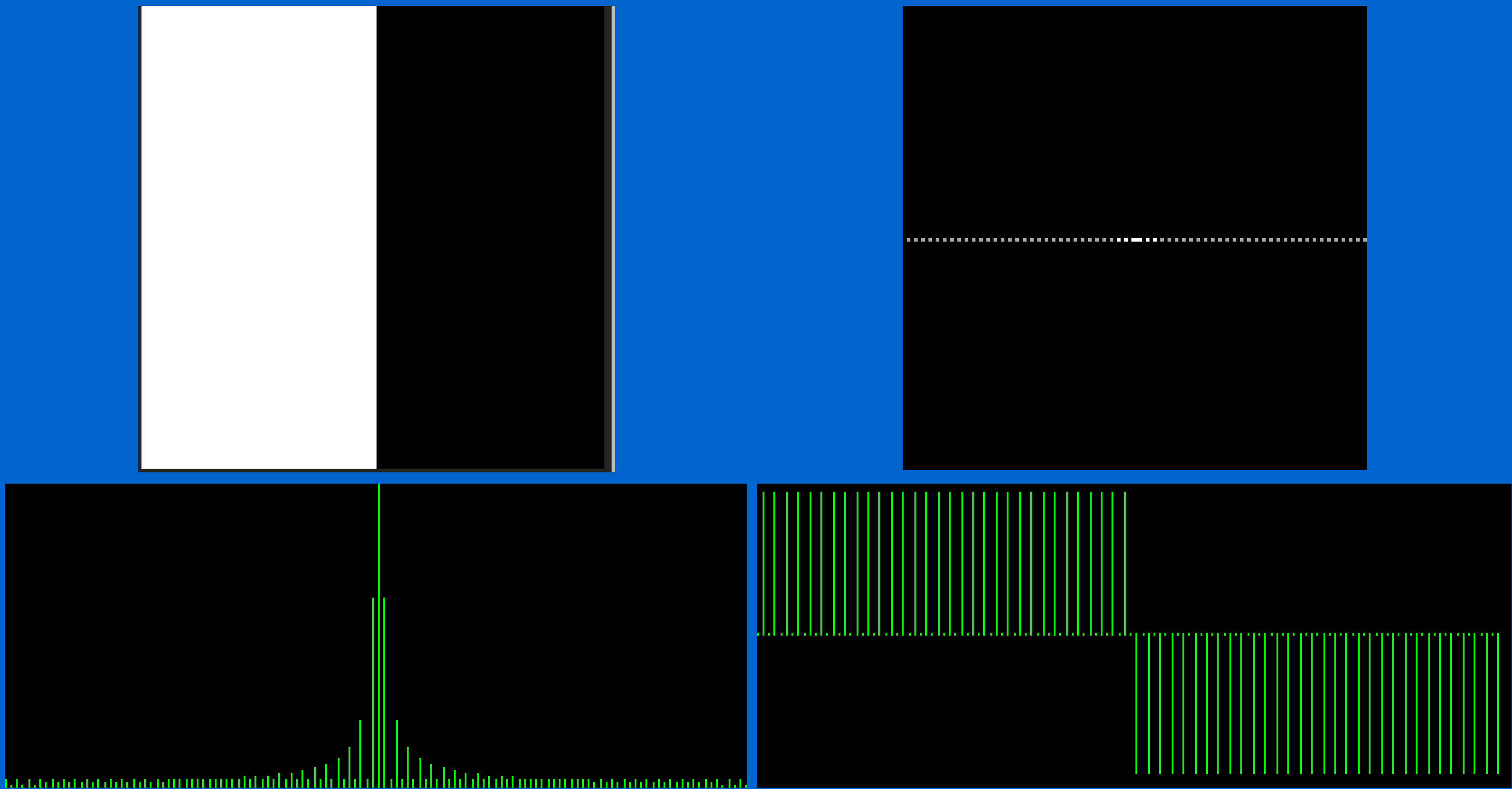
Diskrétna Fourierova transf. (4/5)

$$X(k_1,k_2) = X(\Omega_1,\Omega_2)\Big|_{\Omega_1=\frac{2\pi}{N_1}k_1;\ \Omega_2=\frac{2\pi}{N_2}k_2} \qquad 0 \le k_1 \le N_1 - 1;\ 0 \le k_2 \le N_2 - 1$$

DFT koeficienty z $x(n_1,n_2)$ *sú hodnoty* $X(\Omega_1, \Omega_2)$ *získané pre ekvidistantné hodnoty* Ω_1, Ω_2, *počnúc bodom* $\Omega_1=\Omega_2=0$

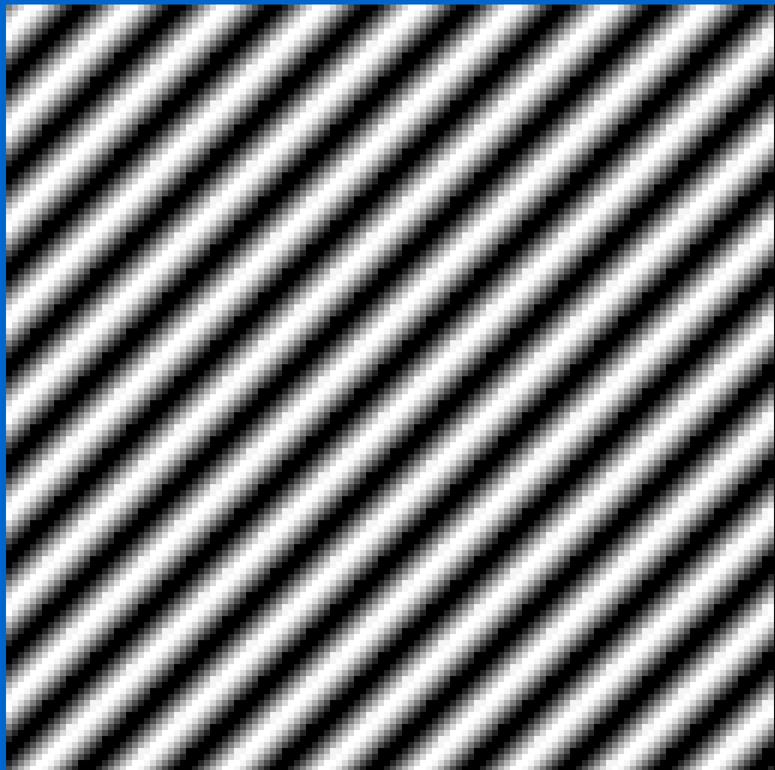
Diskrétna Fourierova transf. (5/5)

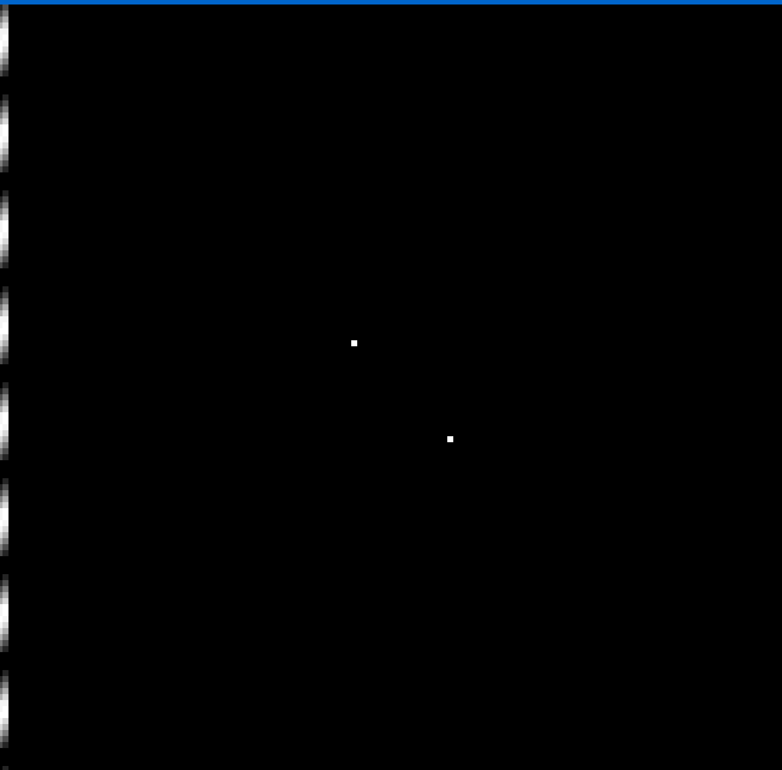
Pravouhlý impulz



DFT základných obrazov (1/3)

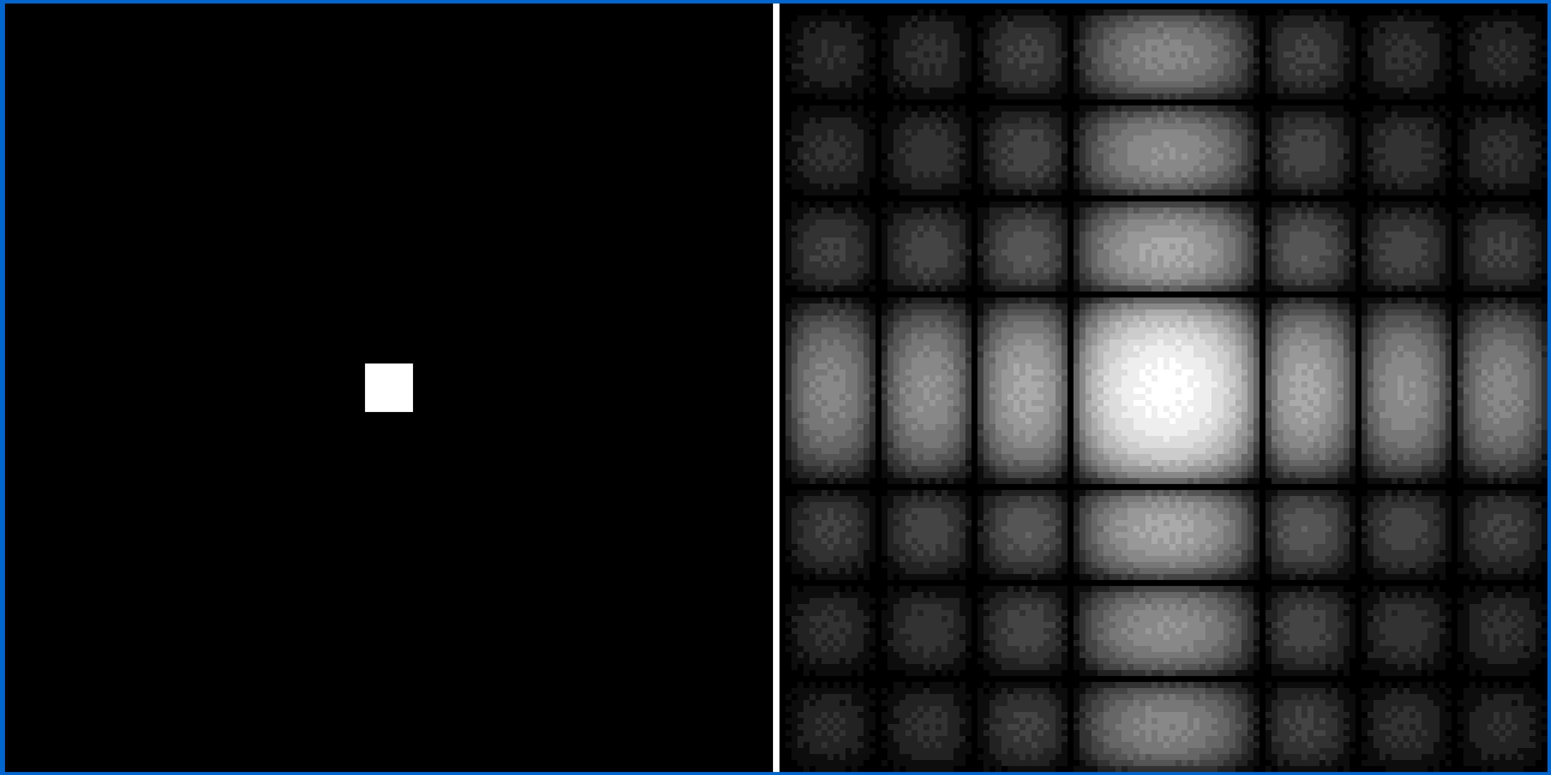
Sinusový signál





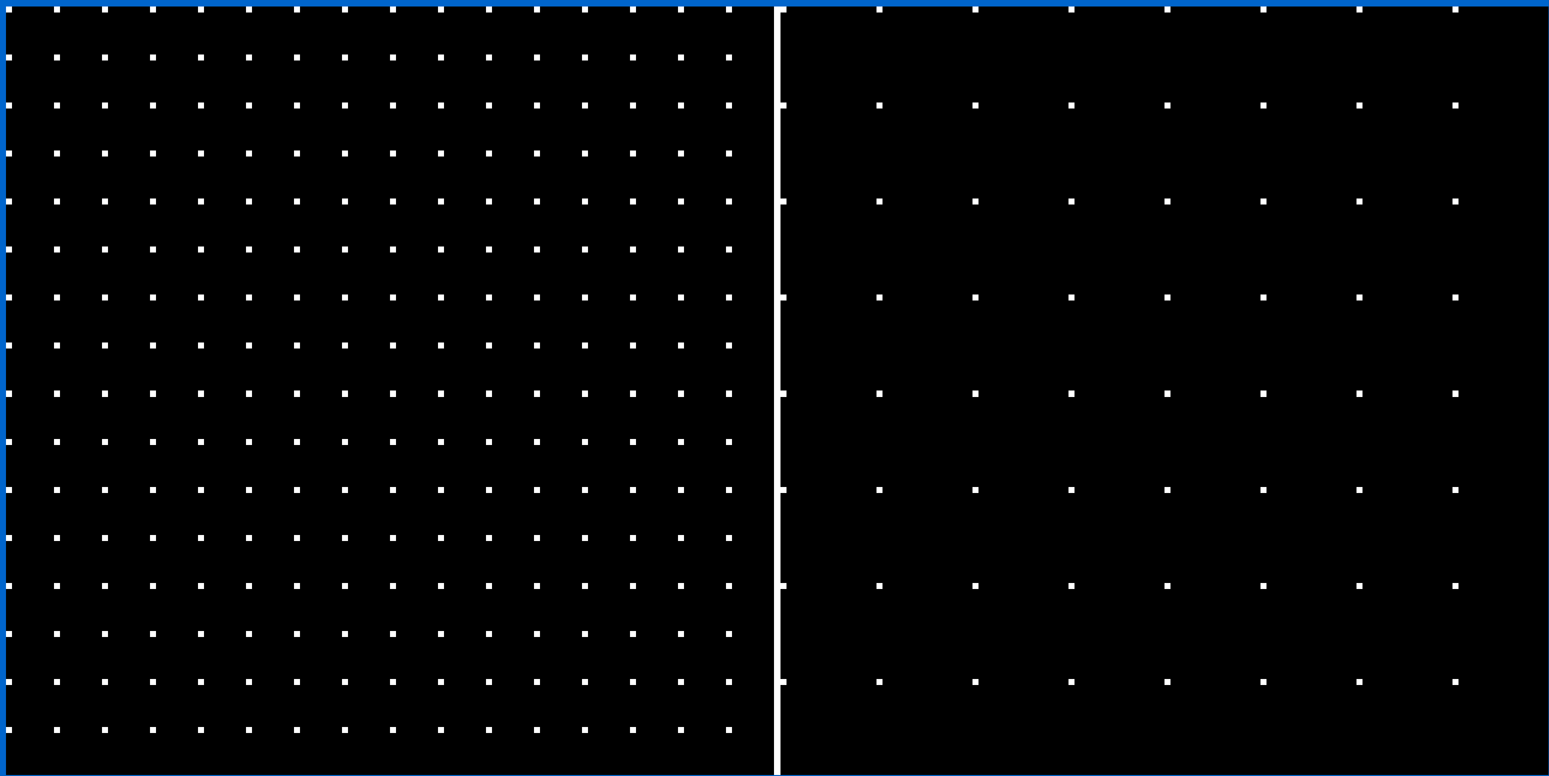
DFT základných obrazov (2/3)

2D pravouhlý impulz



DFT základných obrazov (3/3)

Kroneckerove impulzy



Výpočet DFT (1/4)

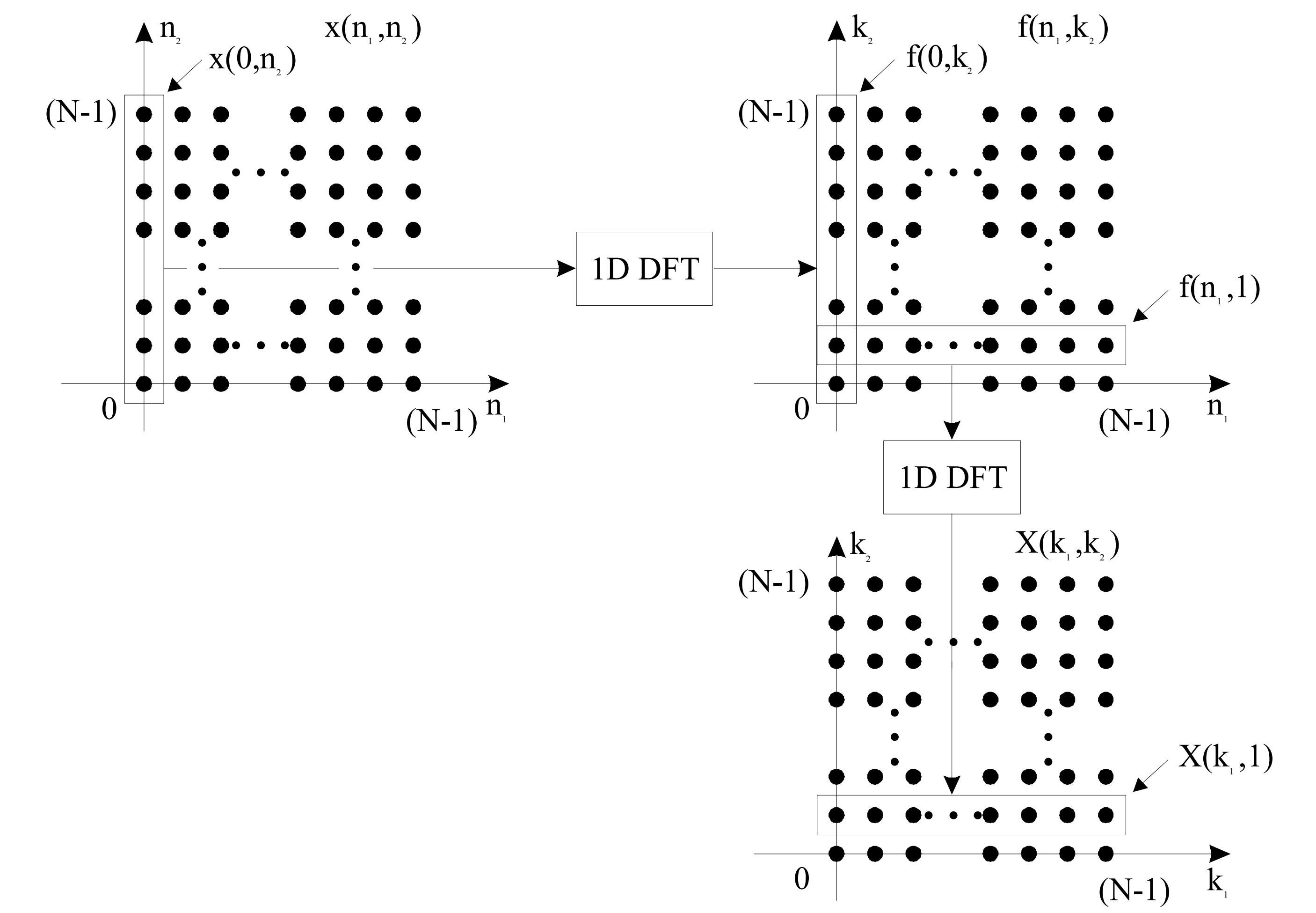
- Priamy výpočet - z definície 2D DFT
- 2 x 1D DFT - po riadkoch, po stĺpcoch

$$X(k_1,k_2)=\sum_{n_1=0}^{N_1-1}\sum_{n_2=0}^{N_2-1} x(n_1,n_2)e^{-j\frac{2\pi}{N_2}k_2n_2}e^{-j\frac{2\pi}{N_1}k_1n_1}$$

$$f(n_1,k_2)=\sum_{n_2=0}^{N_2-1} x(n_1,n_2)e^{-j\frac{2\pi}{N_2}k_2n_2} \qquad 0\le n_1\le N_1-1$$ - N_1 x 1D DFT rádu N_2

$$X(k_1,k_2)=\sum_{n_1=0}^{N_1-1} f(n_1,k_2)e^{-j\frac{2\pi}{N_1}k_1n_1}$$ - N_2 x 1D DFT rádu N_1

Výpočet DFT (2/4)



Výpočet DFT (3/4)

$$\mathbf{y} = \mathbf{T} \cdot \mathbf{x} \qquad \mathbf{x} = \frac{1}{K} \cdot \mathbf{T}^t \cdot \mathbf{y} \qquad \mathbf{T}^t \cdot \mathbf{T} = K \cdot \mathbf{I}$$

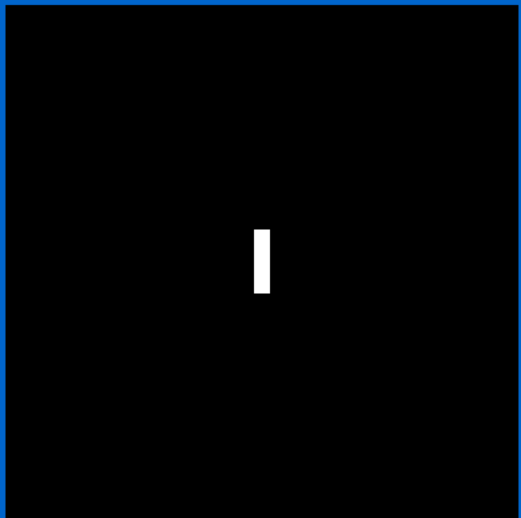
$$\mathbf{Y} = \mathbf{T} \cdot \mathbf{X} \cdot \mathbf{T}^t \qquad \mathbf{X} = \frac{1}{K^2} \cdot \mathbf{T}^t \cdot \mathbf{Y} \cdot \mathbf{T}$$

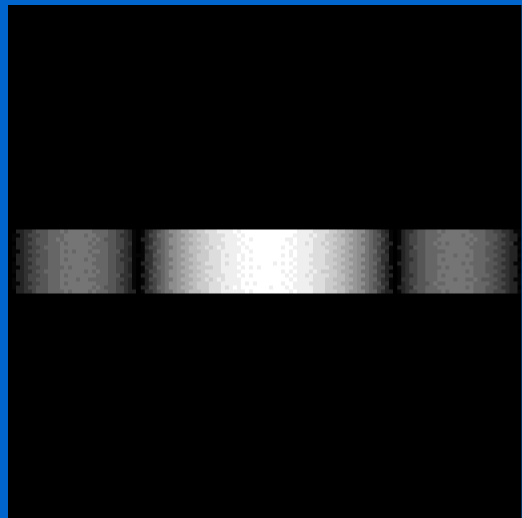
$\mathbf{T}_{k,n} = e^{-j \cdot \frac{2\pi}{N} \cdot k \cdot n}$ - transformačné jadro pre dopřednú DFT

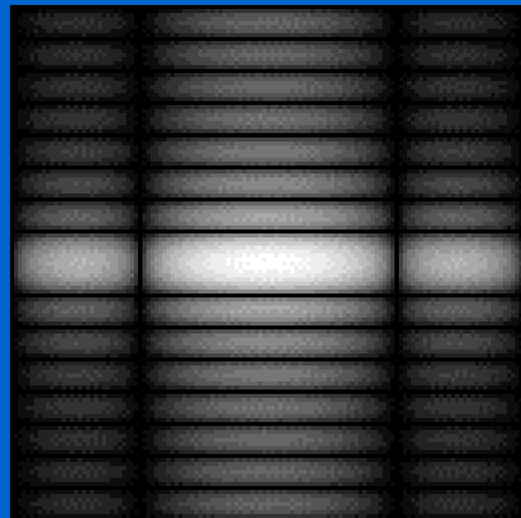
$\mathbf{IT}_{k,n} = e^{j \cdot \frac{2\pi}{N} \cdot k \cdot n}$ - transformačné jadro pre inverznú DFT

2D-DFT - Príklad

Výpočet DFT (4/4)







Výpocet 2D DFT (N_1=N_2=512)	Pocet násobení	Pocet scítaní
Priamy výpocet	$(N_1N_2)^2$ (100%)	$(N_1N_2)^2$ (100%)
2 x 1D DFT s priamym výpoctom 1D DFT	$N_1N_2(N_1+N_2)$ (0.4%)	$N_1N_2(N_1+N_2)$ (0.4%)
2 x 1D FFT	$N_1N_2/2 \log_2(N_1N_2)$ (0.0035%)	$N_1N_2 \log_2(N_1N_2)$ (0.007%)

Vlastnosti DFT (1/14)

$$x(n_1,n_2)=0,\ y(n_1,n_2)=0 \quad mimo \quad 0\le n_1\le N_1-1,\ 0\le n_2\le N_2-1$$

$$x(n_1,n_2) \quad \leftrightarrow \quad X(k_1,k_2)$$
$$y(n_1,n_2) \quad \leftrightarrow \quad Y(k_1,k_2)$$

- Linearita

$$a\cdot x(n_1,n_2)+b\cdot y(n_1,n_2) \quad \leftrightarrow \quad a\cdot X(k_1,k_2)+b\cdot Y(k_1,k_2)$$

- Kruhová konvolúcia

$$x(n_1,n_2)\otimes y(n_1,n_2)=[\tilde{x}(n_1,n_2)\otimes \tilde{y}(n_1,n_2)]\cdot R_{N_1xN_2} \quad \leftrightarrow \quad X(k_1,k_2)\cdot Y(k_1,k_2)$$

Vlastnosti DFT (2/14)

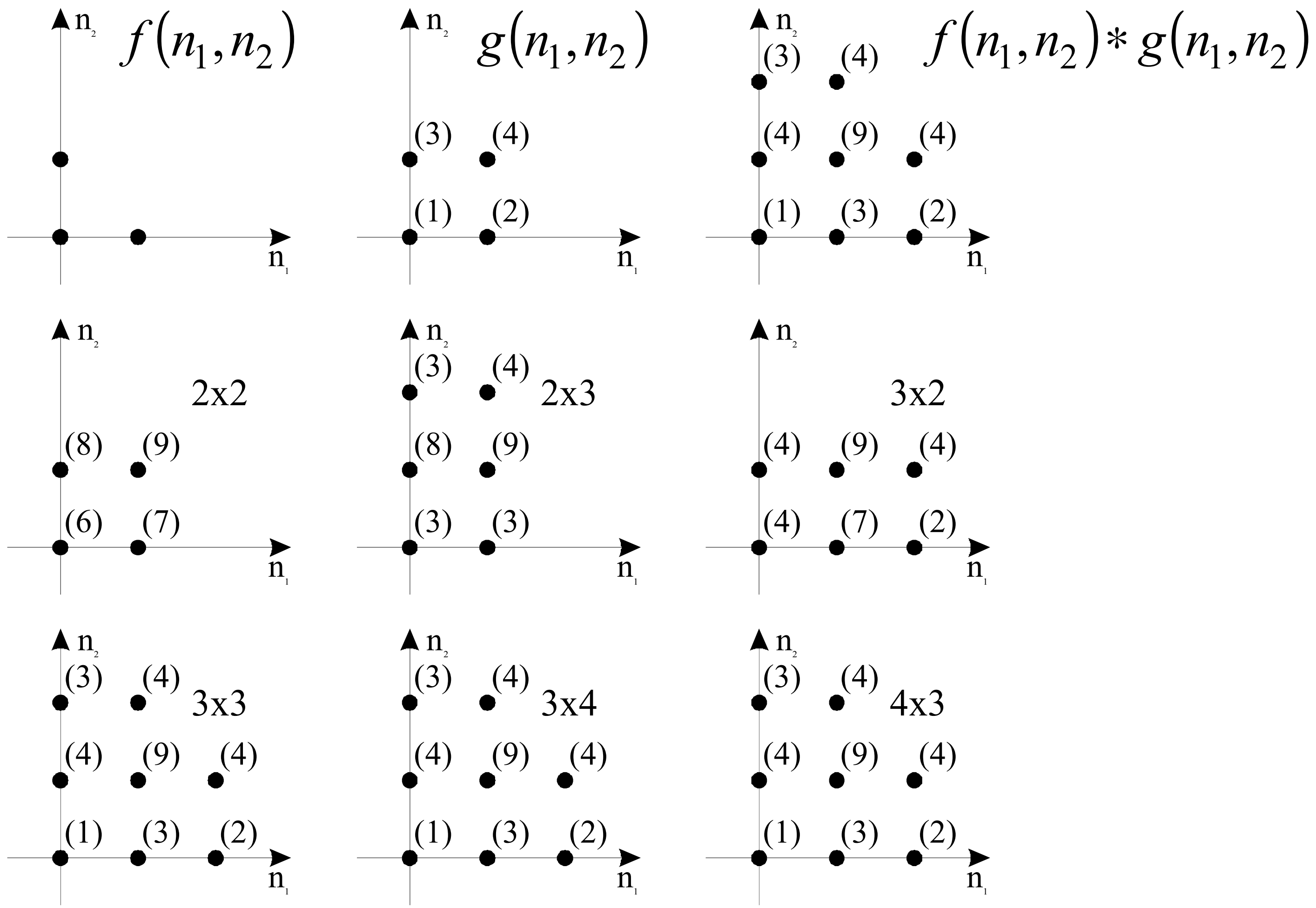
- Vzťah medzi kruhovou a lineárnou konvolúciou
  - využitie: výpočet lineárnej konv. pomocou DFT

$$f(n_1,n_2)=0 \quad mimo \quad 0\le n_1\le N_1'-1, \quad 0\le n_2\le N_2'-1$$
$$g(n_1,n_2)=0 \quad mimo \quad 0\le n_1\le N_1''-1, \quad 0\le n_2\le N_2''-1$$
$$f(n_1,n_2)*g(n_1,n_2)=f(n_1,n_2)\otimes g(n_1,n_2) \quad N_1\ge N_1'+N_1''-1, \quad N_2\ge N_2'+N_2''-1$$

- Násobenie

$$x(n_1,n_2)y(n_1,n_2) \quad \leftrightarrow \quad \frac{1}{N_1N_2}X(k_1,k_2)\otimes Y(k_1,k_2)=$$
$$=\frac{1}{N_1N_2}\left[\tilde{X}(k_1,k_2)\otimes \tilde{Y}(k_1,k_2)\right]\cdot R_{N_1xN_2}(k_1,k_2)$$

Vlastnosti DFT (3/14)



Vlastnosti DFT (4/14)

- Separovateľnosť

$$x(n_1, n_2) = x_1(n_1)x_2(n_2) \quad \leftrightarrow \quad X(k_1, k_2) = X_1(k_1)X_2(k_2)$$

$$X_1(k_1): \quad N_1 - \text{bodová 1D DFT}$$

$$X_2(k_2): \quad N_2 - \text{bodová 1D DFT}$$

- Kruhový (cyklický) posuv postupnosti

$$\tilde{x}(n_1 - m_1, n_2 - m_2) R_{N_1 x N_2}(n_1, n_2) = x\left((n_1 - m_1)_{N_1}, (n_2 - m_2)_{N_2}\right)$$

$$\updownarrow$$

$$X(k_1, k_2) e^{-j\frac{2\pi}{N_1}k_1 m_1} e^{-j\frac{2\pi}{N_2}k_2 m_2}$$

Vlastnosti DFT (5/14)

- Veta o počiatočnej hodnote a DC zložke

$$x(0,0)=\frac{1}{N_1N_2}\sum_{k_1=0}^{N_1-1}\sum_{k_2=0}^{N_2-1}X(k_1,k_2)$$

$$X(0,0)=\sum_{n_1=0}^{N_1-1}\sum_{n_2=0}^{N_2-1}x(n_1,n_2)$$

- Parsevalov teorém

$$\sum_{n_1=0}^{N_1-1}\sum_{n_2=0}^{N_2-1}x(n_1,n_2)y^*(n_1,n_2)=\frac{1}{N_1N_2}\sum_{k_1=0}^{N_1-1}\sum_{k_2=0}^{N_2-1}X(k_1,k_2)Y^*(k_1,k_2)$$

$$\sum_{n_1=0}^{N_1-1}\sum_{n_2=0}^{N_2-1}\left|x(n_1,n_2)\right|^2=\frac{1}{N_1N_2}\sum_{k_1=0}^{N_1-1}\sum_{k_2=0}^{N_2-1}\left|X(k_1,k_2)\right|^2$$

Vlastnosti DFT (6/14)

- Symetria

$$x^*(n_1,n_2) \quad \leftrightarrow \quad \tilde{X}^*(-k_1,-k_2)R_{N_1xN_2}(k_1,k_2) = X^*\left((-k_1)_{N_1},(-k_2)_{N_2}\right)$$

$$x(n_1,n_2): real \quad \leftrightarrow \quad X(k_1,k_2) = \tilde{X}^*(-k_1,-k_2)R_{N_1xN_2}(k_1,k_2)$$

$$X_R(k_1,k_2) = \tilde{X}_R(-k_1,-k_2)R_{N_1xN_2}(k_1,k_2)$$

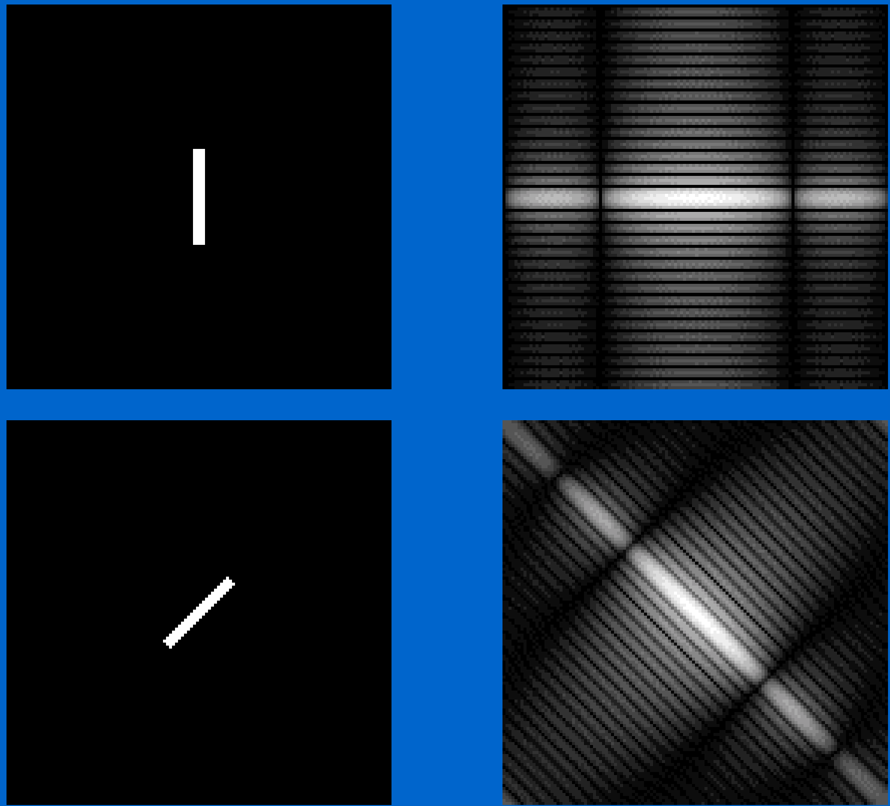
$$X_I(k_1,k_2) = -\tilde{X}_I(-k_1,-k_2)R_{N_1xN_2}(k_1,k_2)$$

$$|X(k_1,k_2)| = |\tilde{X}(-k_1,-k_2)|R_{N_1xN_2}(k_1,k_2)$$

$$\Theta_X(k_1,k_2) = -\Theta_{\tilde{X}}(-k_1,-k_2)R_{N_1xN_2}(k_1,k_2)$$

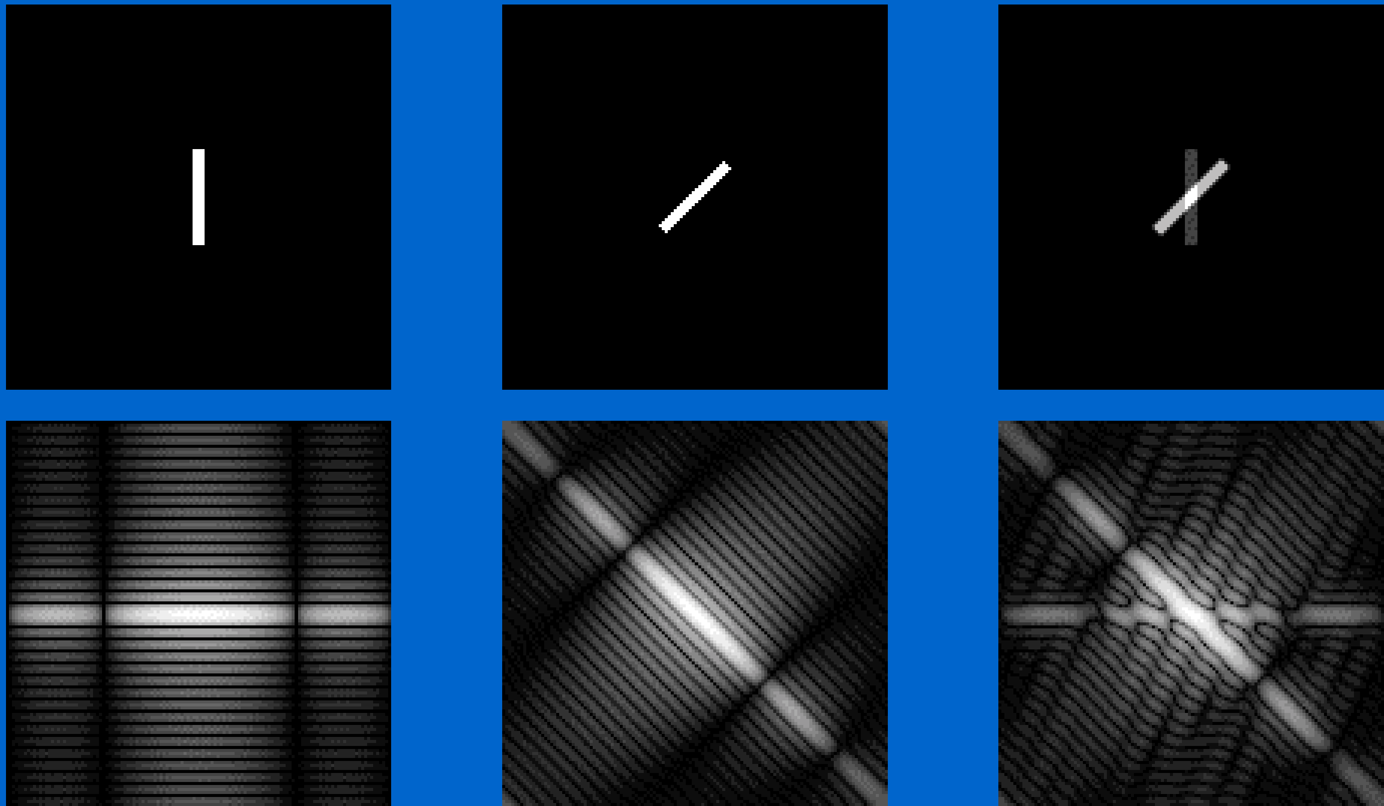
Vlastnosti DFT (7/14)

Rotácia



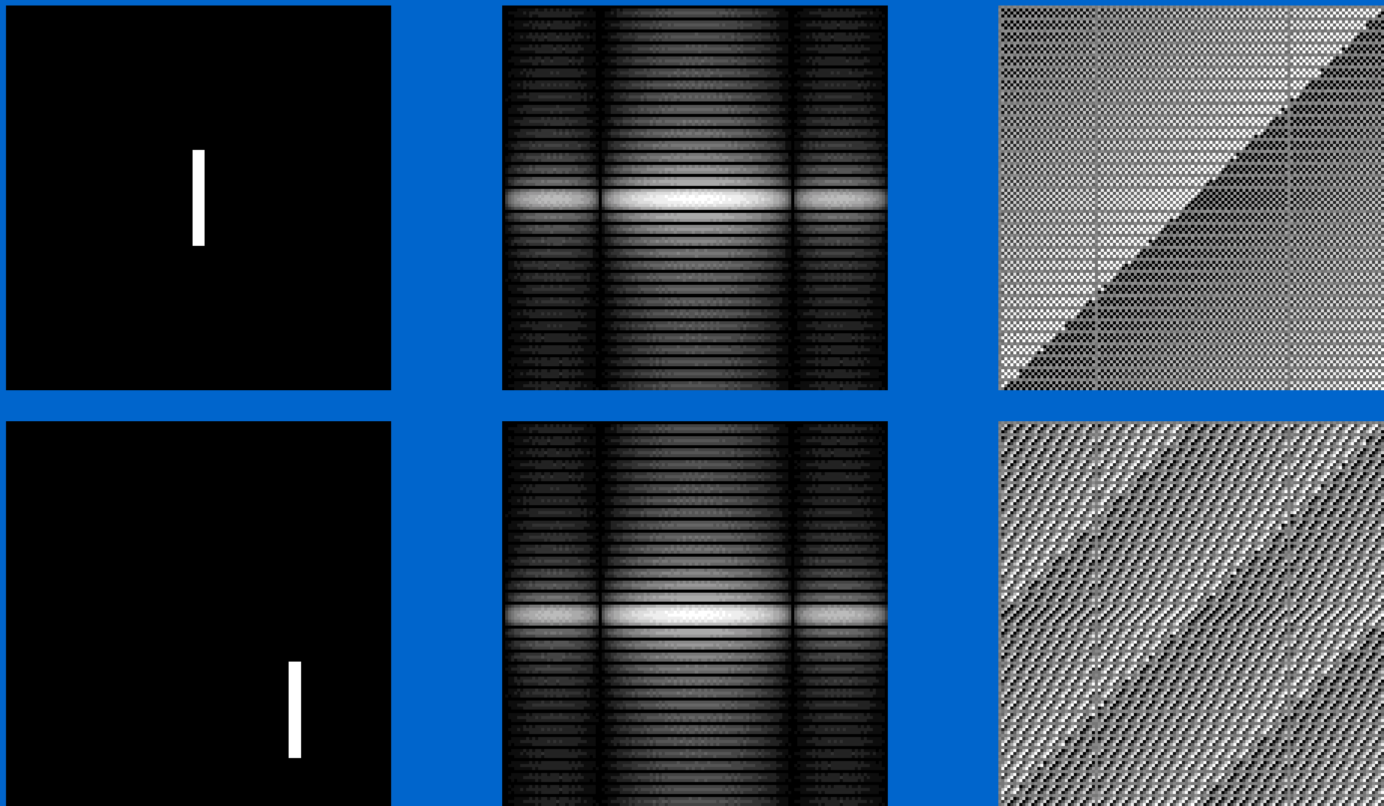
Vlastnosti DFT (8/14)

Linearita



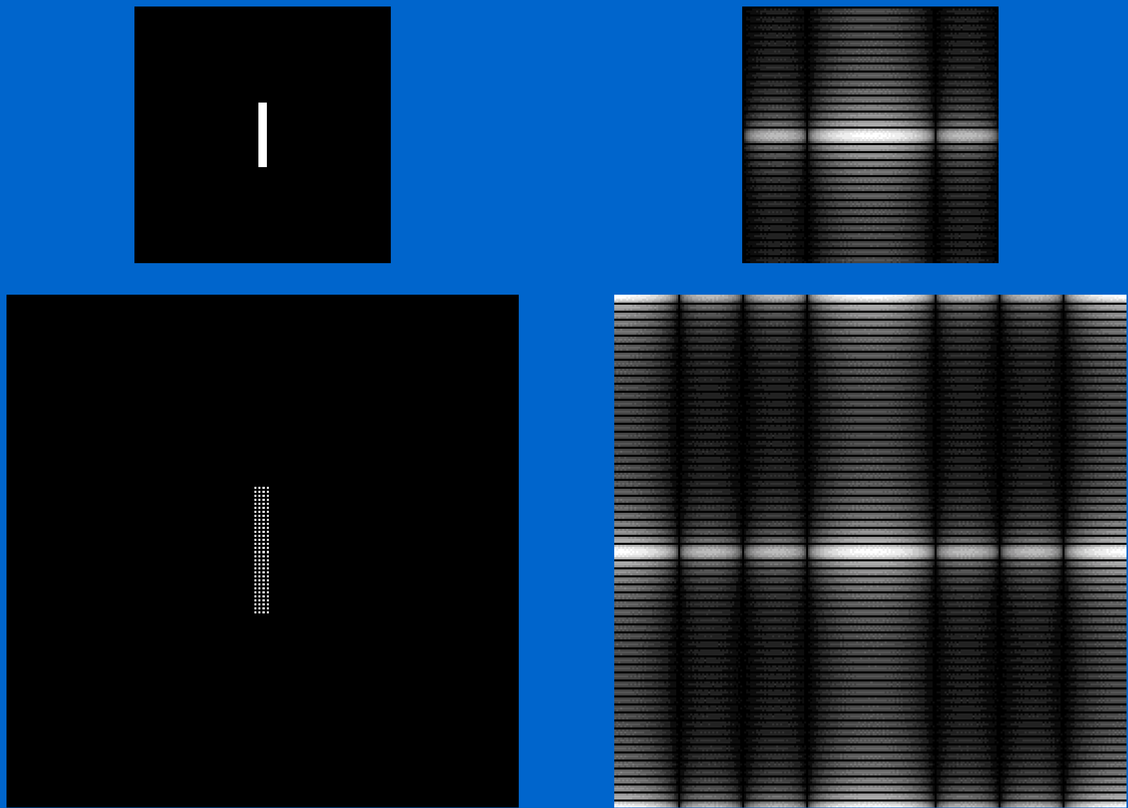
Vlastnosti DFT (9/14)

Posuv



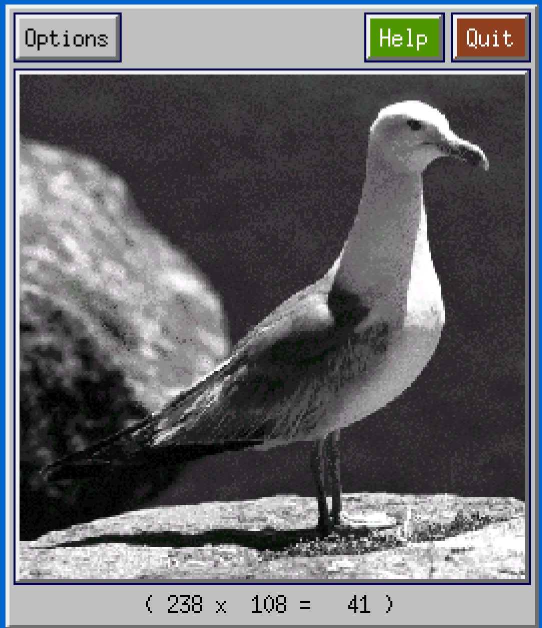
Vlastnosti DFT (10/14)

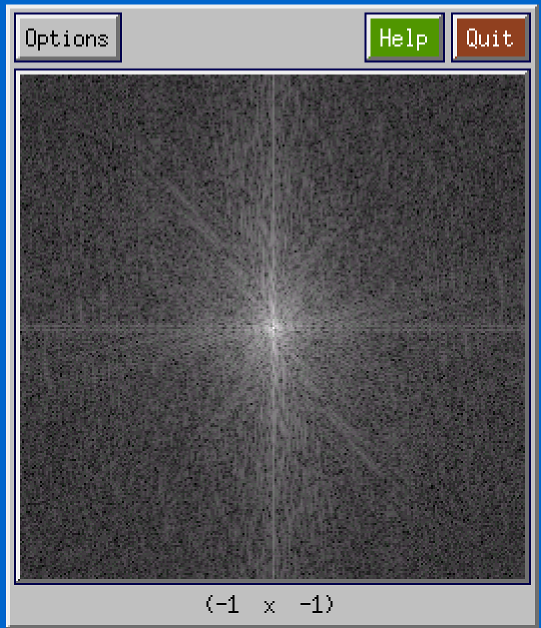
Interpolácia

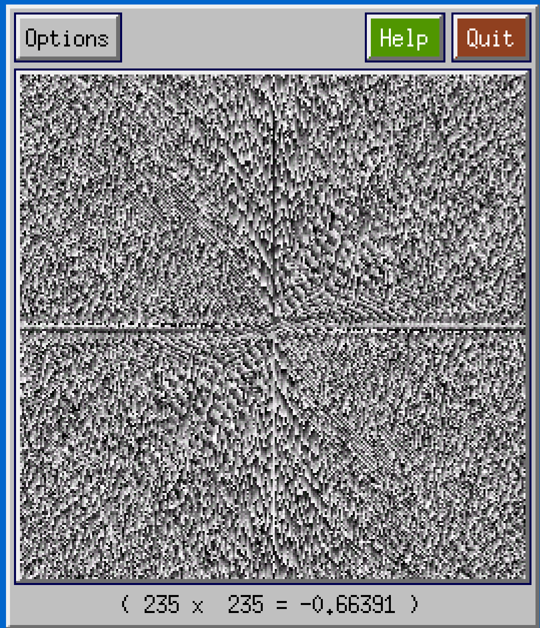


Vlastnosti DFT (11/14)

Originál a jeho spektrum - magnitúda a fáza

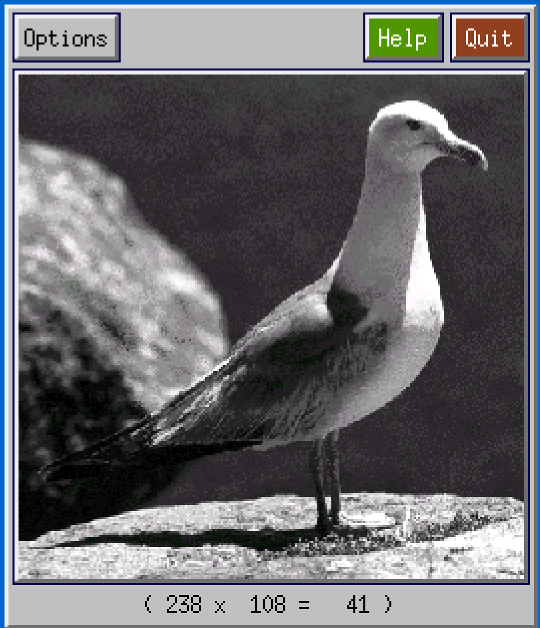






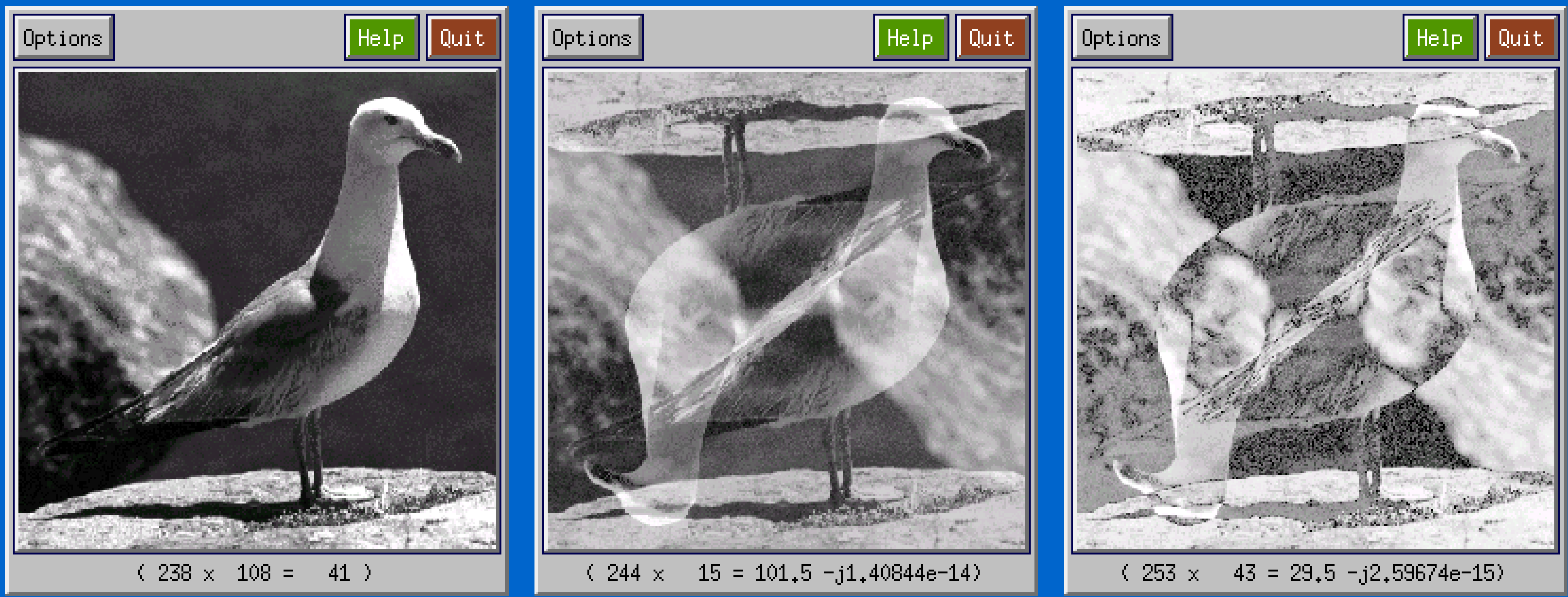
Vlastnosti DFT (12/14)

Rekonštrukcia len z fázy a len z magnitúdy



Vlastnosti DFT (13/14)

Rekonštrukcia len z reál. a len z imag. zložky



Vlastnosti DFT (14/14)

Rekonštrukcia po súčte reál. a imag. zložky (linearita)

DCT

- DCT - “Discrete Cosine Transform” - Diskrétna kosínusová transformácia
- 4 typy - DCT-I až DCT-IV
- DCT-II - párne symetrická DCT (najčastejšie používaná v oblasti kompresie a kódovania obrazu - medzinárodné štandardy JPEG, MPEG, H.261, H.263, …)

Párne symetrická 1D DCT (1/3)

$$C_x(k) = \begin{cases} \sum_{n=0}^{N-1} 2x(n)\cos\left(\frac{\pi}{2N}k(2n+1)\right) & 0 \le k \le N-1 \\ 0 & inak \end{cases}$$

$$x(n) = \begin{cases} \frac{1}{N}\sum_{k=0}^{N-1} w(k)C_x(k)\cos\left(\frac{\pi}{2N}k(2n+1)\right) & 0 \le n \le N-1 \\ 0 & inak \end{cases}$$

$$w(k) = \begin{cases} 1/2 & k=0 \\ 1 & 1 \le k \le N-1 \end{cases}$$

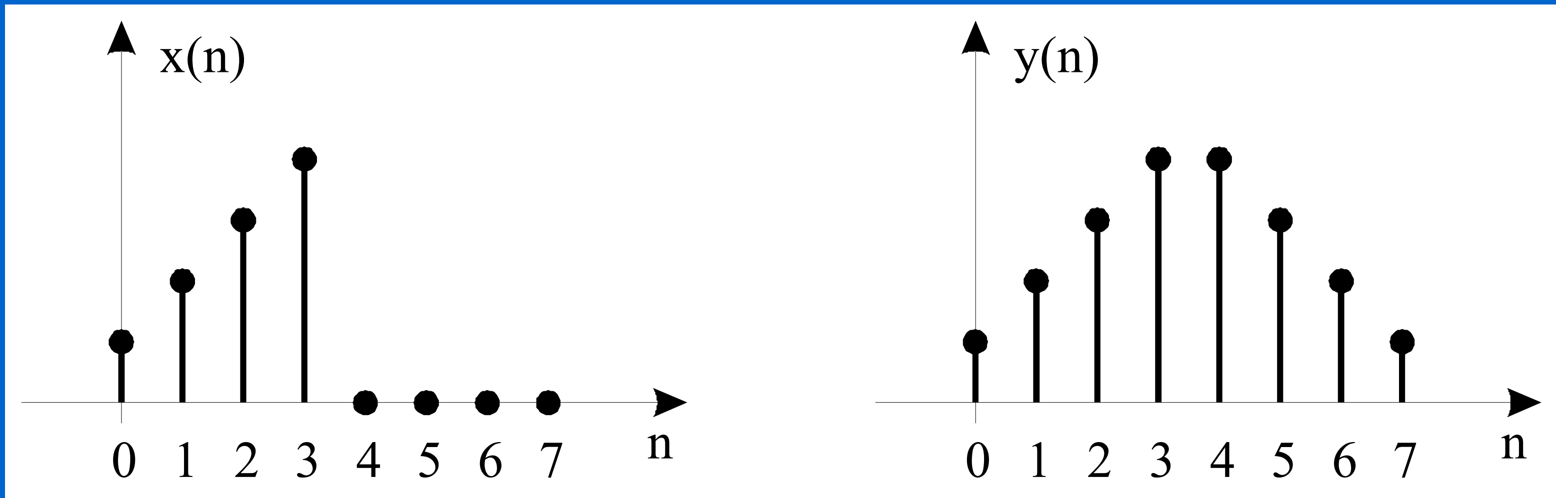
- x(n) - 1D postupnosť dĺžky N
- $C_x(k)$ - N koeficientov získaných po 1D DCT

Párne symetrická 1D DCT (2/3)

- Súvis medzi N-bodovou DCT a 2N bodovou DFT

$$\underset{x(n)}{N\text{–bodová}} \quad \leftrightarrow \quad \underset{y(n)}{2N\text{–bodová}} \quad \xleftrightarrow{DFT} \quad \underset{Y(k)}{2N\text{–bodová}} \quad \leftrightarrow \quad \underset{C_x(k)}{N\text{–bodová}}$$

$$y(n) = x(n) + x(2N-1-n) = \begin{cases} x(n) & 0 \le n \le N-1 \\ x(2N-1-n) & N \le n \le 2N-1 \end{cases}$$



Párne symetrická 1D DCT (3/3)

- Výpočet DCT na báze DFT

1. $y(n) = x(n) + x(2N-1-n) \quad 0 \le n \le 2N-1$
2. $Y(k) = DFT[y(n)] \qquad (2N - bodová\ DFT)$
3. $C_x(k) = \begin{cases} W_{2N}^{k/2} Y(k) & 0 \le k \le N-1 \\ 0 & inak \end{cases}$ $\quad W_{2N} = e^{-j\frac{2\pi}{2N}}$

- Výpočet IDCT na báze IDFT

1. $Y(k) = \begin{cases} W_{2N}^{-k/2} C_x(k) & 0 \le k \le N-1 \\ 0 & k = N \\ -W_{2N}^{-k/2} C_x(2N-k) & N+1 \le k \le 2N-1 \end{cases}$
2. $y(n) = IDFT[Y(k)] \qquad (2N - bodová\ IDFT)$
3. $x(n) = \begin{cases} y(n) & 0 \le n \le N-1 \\ 0 & inak \end{cases}$

DCT via DFT

Párne symetrická 2D DCT

$$C_x(k_1,k_2)=\begin{cases}\sum_{n_1=0}^{N_1-1}\sum_{n_2=0}^{N_2-1}4x(n_1,n_2)\cos\left(\frac{\pi}{2N_1}k_1(2n_1+1)\right)\cos\left(\frac{\pi}{2N_2}k_2(2n_2+1)\right) & 0\le k_1\le N_1-1;\quad 0\le k_2\le N_2-1\\ 0 & inak\end{cases}$$

$$x(n_1,n_2)=\begin{cases}\frac{1}{N_1N_2}\sum_{k_1=0}^{N_1-1}\sum_{k_2=0}^{N_2-1}w_1(k_1)w_2(k_2)C_x(k_1,k_2)\cos\left(\frac{\pi}{2N_1}k_1(2n_1+1)\right)\cos\left(\frac{\pi}{2N_2}k_2(2n_2+1)\right) & 0\le n_1\le N_1-1;\quad 0\le n_2\le N_2-1\\ 0 & inak\end{cases}$$

$$w_1(k_1)=\begin{cases}1/2 & k_1=0\\ 1 & 1\le k_1\le N_1-1\end{cases}$$

$$w_2(k_2)=\begin{cases}1/2 & k_2=0\\ 1 & 1\le k_2\le N_2-1\end{cases}$$

Porovnanie DCT a DFT

Vlastnosti DCT (1/2)

$$x(n_1,n_2)=0,\quad y(n_1,n_2)=0 \qquad mimo \qquad 0\le n_1\le N_1-1;\quad 0\le n_2\le N_2-1$$

$$x(n_1,n_2) \quad \leftrightarrow \quad C_x(k_1,k_2)$$
$$y(n_1,n_2) \quad \leftrightarrow \quad C_y(k_1,k_2)$$

- Linearita

$$a\cdot x(n_1,n_2)+b\cdot y(n_1,n_2) \quad \leftrightarrow \quad a\cdot C_x(k_1,k_2)+b\cdot C_y(k_1,k_2)$$

- Separovatel'nosť

$$x(n_1,n_2)=x_1(n_1)\cdot x_2(n_2) \quad \leftrightarrow \quad C_x(k_1,k_2)=C_{x1}(k_1)\cdot C_{x2}(k_2)$$

$$C_{x1}(k_1):\quad N_1-bodová \quad 1D\ DCT$$
$$C_{x2}(k_2):\quad N_2-bodová \quad 1D\ DCT$$

Vlastnosti DCT (2/2)

- Energetické pomery

$$\sum_{n_1=0}^{N_1-1}\sum_{n_2=0}^{N_2-1}\left|x(n_1,n_2)\right|^2 = \frac{1}{4N_1N_2}\sum_{k_1=0}^{N_1-1}\sum_{k_2=0}^{N_2-1} w_1(k_1)w_2(k_2)\left|C_x(k_1,k_2)\right|^2$$

$$w_1(k_1) = \begin{cases} 1/2 & k_1=0 \\ 1 & 1\le k_1\le N_1-1 \end{cases}$$

$$w_2(k_2) = \begin{cases} 1/2 & k_2=0 \\ 1 & 1\le k_2\le N_2-1 \end{cases}$$

DCT via DFT

- Symetria

$$\overline{x}(n_1,n_2) \leftrightarrow \overline{C}_x(k_1,k_2)$$

$$x(n_1,n_2): real \leftrightarrow C_x(k_1,k_2): real$$