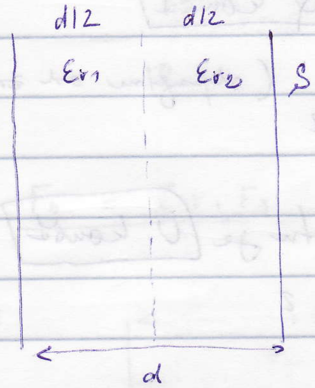
Cricenii

(PR)



S, d, ε_{r1}

$C = \frac{Q}{U}$

$D = E \cdot \varepsilon_0 \varepsilon_r \Rightarrow$

$\Rightarrow E = \frac{D}{\varepsilon_0 \varepsilon_r}$

$$U = \int \bar{E} \cdot d\bar{r} = \int \bar{E}_1 d\bar{r} + \int \bar{E}_2 d\bar{r} = E \uparrow\uparrow dr = \int E_1 dr + \int E_2 dr =$$

$$= \frac{D}{\varepsilon_0}\left[\int \frac{1}{\varepsilon_{r1}} + \int \frac{1}{\varepsilon_{r2}}\right] = \frac{D}{\varepsilon_0} \cdot \left(\frac{d}{2 \cdot \varepsilon_{r1}} + \frac{d}{2 \cdot \varepsilon_{r2}}\right) =$$

$$= \frac{\overset{S \cdot \sigma_v}{D} \cdot d}{2\varepsilon_0}\left(\frac{1}{\varepsilon_{r1}} + \frac{1}{\varepsilon_{r2}}\right)$$

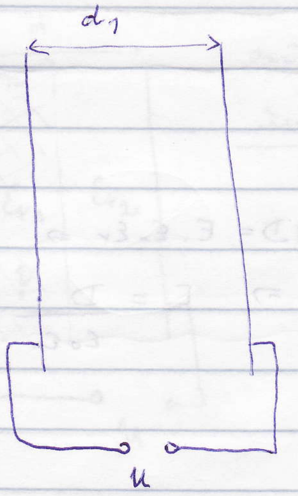
$$C = \frac{S \cdot \sigma_v \cdot 2\varepsilon_0}{\sigma_v \cdot d \cdot \left(\frac{1}{\varepsilon_{r1}} + \frac{1}{\varepsilon_{r2}}\right)}$$

altfel

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C} = \frac{1}{\varepsilon_0 \varepsilon_{r1} \cdot \frac{S}{d/2}} + \frac{1}{\varepsilon_0 \varepsilon_{r2} \cdot \frac{S}{d/2}}$$

PR. 35



U, S, d_1

tu je $\boxed{Q \text{ konst.}}$

a) $d_2 = 3d_1$ (odpojím od zdroje)

$U_1 = ?$, $Q_1 = ?$

b) $d_2 = 3d_1$ — tu je $\boxed{U \text{ konst.}}$

$U_2 = ?$, $Q_2 = ?$

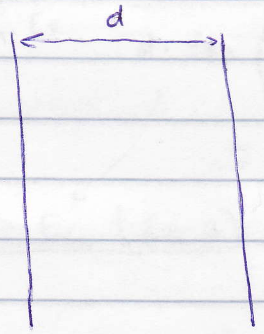
$U_1 = \int \bar{E} \cdot d\bar{r}$; $C = \frac{Q}{U}$; $C = \varepsilon_0 \cdot \frac{S}{d}$

$$\varepsilon_0 \cdot \frac{S}{d} = \frac{Q}{U} \Rightarrow \boxed{Q = \frac{\varepsilon_0 \cdot S \cdot U}{d}}$$

$$\varepsilon_0 \cdot \frac{S}{3d_1} = \frac{\frac{\varepsilon_0 \cdot S \cdot U}{d}}{U_1} \Rightarrow U_1 = \frac{\frac{\varepsilon_0 \cdot S \cdot U}{d_1}}{\varepsilon_0 \cdot \frac{S}{3d_1}} = \underline{3U}$$

$$Q_2 = \frac{U \cdot \varepsilon_0 \cdot S}{3d_1}$$

PR.



a) $F_1 = ?$

U zadané

b) $F_2 = ?$

ε_r vložíme

$F = \bar{E} \cdot q$; $E = \frac{\sigma_v}{2\varepsilon_0}$

$$F = \frac{\sigma_v}{2\varepsilon_0} \cdot \sigma_v \cdot S$$

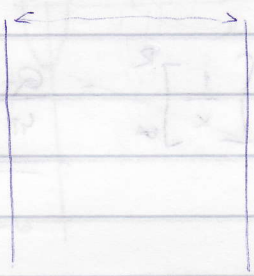
$$U = \int E d\bar{r} = \int_0^d \frac{D}{\varepsilon_0} \cdot dr =$$

$$= \frac{\sigma_v}{\varepsilon_0} \cdot d$$

$$F_1 = \frac{\frac{\varepsilon_0^2 \cdot u^2}{d^2}}{2\varepsilon_0} \cdot S = \boxed{\frac{\varepsilon_0 \cdot U^2}{2d^2} \cdot S}$$

b) $\underline{F_2 = \varepsilon_r \cdot F_1}$

Pr.



$W = ?$; C; U; $\underline{d \to 2d}$

$$C = \frac{Q}{U}$$

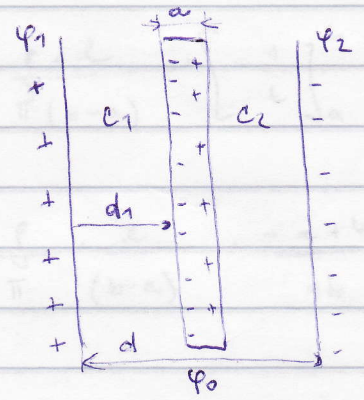
$$W_1 = \frac{1}{2} \cdot \frac{Q^2}{C} = \frac{C^2 \cdot U^2}{2C} = \frac{1}{2} CU^2$$

$$C = \varepsilon \cdot \frac{S}{d} \qquad C_2 = \varepsilon \cdot \frac{S}{2d} = \frac{1}{2} C$$

$$W_2 = \frac{1}{2} \cdot C_2 \cdot U^2 = \frac{1}{2} \cdot \frac{1}{2} \cdot C \cdot U^2$$

$$W = W_1 - W_2 = \frac{1}{2} CU^2 - \frac{1}{4} CU^2 = \boxed{\frac{1}{4} CU^2}$$

Pr.



$\varphi_1, \varphi_2, d_1, d, a, \varphi_0 = ?$

$$C = \frac{Q}{\varphi_1 - \varphi_2}$$

$$C_1 = \frac{Q}{\varphi_1 - \varphi_0} = \varepsilon_0 \cdot \frac{S}{d_1}$$

$$C_2 = \frac{Q}{\varphi_0 - \varphi_2} = \varepsilon_0 \cdot \frac{S}{d-(d_1+a)}$$

$$\varepsilon_0 \cdot \frac{S}{d_1} \cdot (\varphi_1 - \varphi_0) = \varepsilon_0 \cdot \frac{S(\varphi_0 - \varphi_2)}{d - d_1 - a}$$

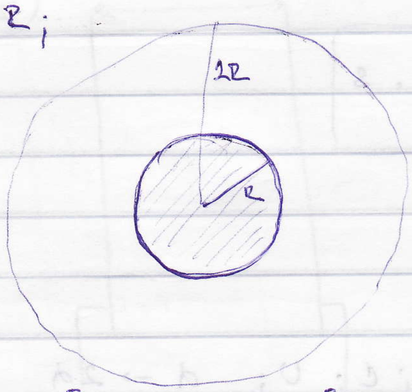
$$\varphi_1 d - \varphi_1 d_1 - \varphi_1 a - \varphi_0 d + \varphi_0 d_1 + \varphi_0 a = \varphi_0 d_1 - \varphi_2 d_1$$

$$\varphi_0 (a - d) = -\varphi_2 d_1 - \varphi_1 (d - d_1 - a)$$

$$\varphi_0 = \frac{-\varphi_2 \cdot d_1 - \varphi_1 (d - d_1 - a)}{a - d}$$

Pr. R;

$$\boxed{W = \frac{1}{2} Q \cdot \varphi}$$ energia gule



$$\varphi_{(R)} = -\int_{\infty}^{R} \bar{E} \cdot dr = -\int_{\infty}^{R} \frac{Q}{4\pi\varepsilon_0 \cdot x^2}\, dx = \frac{Q}{4\pi\varepsilon_0} \left[\frac{1}{x}\right]_{\infty}^{R} = \frac{Q}{4\pi\varepsilon_0 R}$$

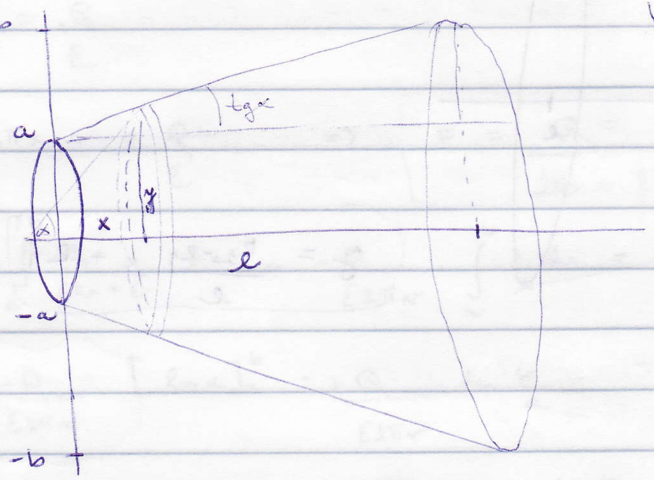
$$\varphi(2R) = \frac{Q}{4\pi\varepsilon_0 \cdot 2R}$$

$$W = \frac{1}{2} \cdot Q \cdot \frac{Q}{4\pi\varepsilon_0 R} \qquad W(2R) = \frac{1}{2} Q \cdot \frac{Q}{4\pi\varepsilon_0 2R}$$

$$W = \frac{1}{2} \cdot \frac{Q^2}{4\pi\varepsilon_0 R} - \frac{1}{2} \frac{Q^2}{4\pi\varepsilon_0 2R} = \frac{1}{2} \cdot \frac{Q^2}{8\pi\varepsilon_0 R}$$

Cvičenie 1.4.10

Pr. 3



$k = tg\alpha = \frac{b-a}{l}$ = smernica

$y = \frac{b-a}{l} \cdot x + a$

$R = \rho \cdot \frac{l}{S} \Rightarrow dR = \rho \cdot \frac{dx}{y^2 \cdot \pi} \Rightarrow R = \int_0^l \rho \cdot \frac{dx}{\pi y^2}$

$R = \int_0^l \frac{\rho \cdot dx}{\pi y^2} = \int_0^l \frac{\rho \cdot dx}{\pi \cdot \left(\frac{b-a}{l} \cdot x + a\right)^2} = \frac{\rho}{\pi} \int \frac{1}{t^2} \cdot \frac{l}{b-a}\, dt =$

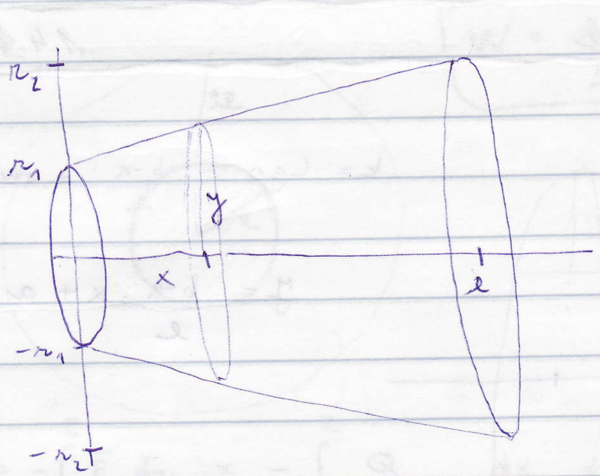
$$\left| \begin{array}{l} t = \frac{b-a}{l} \cdot x + a \\ \frac{dt}{dx} = \frac{b-a}{l} \Rightarrow dx = \frac{dt \cdot l}{b-a} \end{array} \right| =$$

$= \frac{\rho \cdot l}{\pi (b-a)} \cdot \left[-\frac{1}{t}\right]_a^b = \frac{\rho}{\pi} \frac{l}{(b-a)} \left[-\frac{1}{b} + \frac{1}{a}\right] =$

$= \frac{\rho}{\pi} \cdot \frac{l}{(b-a)} \cdot \frac{-a+b}{ab} = \frac{\rho \cdot l}{\pi \cdot a \cdot b}$

Pe. 8

$\rho, r_1, r_2, l, I \; ; \quad E(x) = ?$



$$\boxed{y = \frac{R_2 - R_1}{l} x + R_1}$$

$$\bar{E} = \rho \cdot \bar{J}$$

$$E = \rho \cdot \frac{I}{S} \Rightarrow E = \rho \cdot \frac{I}{\pi y^2} = \rho \cdot \frac{I}{\left(\frac{R_2 - R_1}{l} \cdot x + R_1\right)^2 \cdot \pi}$$

web 1

$I = 4 + 2t^2$

$Q = ?$

$t_1 = 5\,s$

$t_2 = 10\,s$

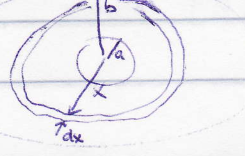
$$I = \frac{dQ}{dt}$$

$$dQ = I \cdot dt$$

$$Q = \int_5^{10} 4 + 2t^2\, dt = \left[4t + \frac{2t^3}{3}\right]_5^{10} =$$

$$=$$

w4 $\quad c \sim R; \; \rho, \varepsilon_r$



$$R = \rho \cdot \frac{l}{S}$$

$$dR = \rho \cdot \frac{l}{S}$$

$$R = \int_a^b \rho \cdot \frac{dx}{2\pi x r} = \frac{\rho}{2\pi r} \int_a^b \frac{1}{x}\, dx =$$

$$= \frac{\rho}{2\pi r} \left[\ln\right]_a^b = \frac{\rho}{2\pi r} \ln \frac{b}{a}$$

$$C = \frac{Q}{U} \qquad U = -\int \vec{E} \cdot d\vec{r} \qquad \oiint \vec{E} \cdot d\vec{S} = \frac{Q}{\varepsilon}$$

$$E \cdot dS = \frac{Q}{\varepsilon}$$

$$E \cdot 2\pi x \cdot r = \frac{Q}{\varepsilon} \Rightarrow \boxed{E = \frac{Q}{2\pi x\, r \cdot \varepsilon}}$$

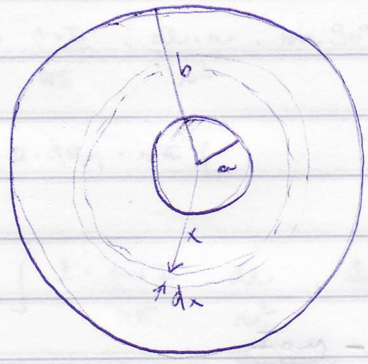
$$U = -\int_a^b \frac{Q}{2\pi x\, r\, \varepsilon} \cdot dx = -\frac{Q}{\varepsilon 2\pi r} \cdot \int_a^b \frac{1}{x}\, dx =$$

$$= -\frac{Q}{\varepsilon 2\pi r} \cdot \left[\ln x\right]_a^b = +\frac{Q}{\varepsilon 2\pi r} \ln \frac{b}{a}$$

$$C = \frac{\varepsilon 2\pi r}{\ln \frac{b}{a}} \qquad \ln \frac{b}{a} = \frac{\varepsilon \cdot 2\pi r}{C}$$

$$R = \frac{\rho}{2\pi r} \cdot \frac{\varepsilon \cdot 2\pi r}{C}$$

PR. $\varepsilon_r, \rho, \; C \sim R = ?$



$$R = \rho \cdot \frac{l}{S}$$

$$dR = \rho \cdot \frac{dx}{4\pi x^2}$$

$$R = \int_a^b \rho \cdot \frac{dx}{4\pi x^2} = \frac{\rho}{4\pi} \cdot \left[-\frac{1}{b} + \frac{1}{a}\right]$$

$$C = \frac{Q}{U} \qquad U = \int \vec{E} \cdot d\vec{r} \qquad E = \frac{Q}{4\pi x^2 \cdot \varepsilon_0}$$

$$U = \int_a^b \frac{Q}{4\pi x^2 \varepsilon_0} \cdot dx = \frac{Q}{4\pi \varepsilon_0} \cdot \left[\frac{1}{a} - \frac{1}{b}\right]$$

$$C = \frac{Q\, 4\pi\varepsilon}{Q\left[\frac{1}{a} - \frac{1}{b}\right]} = \frac{4\pi\varepsilon}{\left[\frac{1}{a} - \frac{1}{b}\right]}$$

$$\frac{C}{R} = \frac{4\pi\varepsilon \cancel{\left[\frac{1}{a} - \frac{1}{b}\right]}}{\cancel{\left[\frac{1}{a} - \frac{1}{b}\right]} \rho \cdot 4\pi}$$

$$\boxed{R = \frac{\rho \cdot \varepsilon}{C}}$$