① $|q_1| \cdot \cancel{q_0} \cdot (x-d)^2 - |q_2| \cdot q_0 \cdot x^2 = 0$

$|q_1| \cdot x^2 - |q_1| \cdot 2dx + |q_1| \cdot d^2 - |q_2| \cdot x^2 = 0$

$(|q_1| - |q_2|) \cdot x^2 - |q_1| \cdot 2dx + |q_1| \cdot d^2 = 0 \quad / : |q_1|$

$\left(1 - \left|\frac{q_2}{q_1}\right|\right) \cdot x^2 - 2dx + d^2 = 0$

kvadratická rovnica

1. ak $|q_2| > |q_1| \Rightarrow \sqrt{p} > 1 \Rightarrow \frac{d}{1-\sqrt{p}}$ ... je záporné ~~=0~~, pred q_1

$\frac{d}{1+\sqrt{p}}$ · je kladné (medzi q_1 a q_2)

2. ak $|q_2| < |q_1| \Rightarrow \sqrt{p} < 1 \Rightarrow \frac{d}{1-\sqrt{p}}$ .. je kladné (— " —)

$\frac{d}{1+\sqrt{p}}$ je kladné (za q_2)

3, ak $|q_2| = |q_1| \Rightarrow$ riešenie $\frac{d}{2}$

③ obežná rýchlosť → odstredivá sila = elektrickej

↳ $F = m \cdot a$

~~a =~~ ↓

~~uhlové~~ zrých. $\frac{\omega^2}{\cancel{r}} \cdot r$
dostredivé

$v = \omega \cdot r \Rightarrow$

$\frac{v^2}{r} = \frac{\omega^2 r^2}{r} = \frac{\omega^2}{r}$

$\Rightarrow$ treba si zapamätať

$a = \omega^2 \cdot r = \frac{v^2}{r}$

dostredivé zrýchlenie

$a = \frac{\omega^2}{r} \Rightarrow F = m \cdot \frac{\omega^2}{\cancel{r}} \cdot r \Rightarrow$

$F = m \cdot \frac{v^2}{r}$

⑥ ak $\boxed{\text{rot } E = 0} \Rightarrow$ daná funkcia môže predstavovať reálne elektrostat. pole

7, 12, 14, 15, 16, 17, 19, 20

8 $V(r) = q \cdot \frac{e^{-ar}}{4\pi\varepsilon_0 r}$ q, a - konšt.

9 $\rho(r) = -\frac{a \cdot e^{-\frac{2r}{a_0}}}{\pi a_0^3}$ a_0 - konšt.

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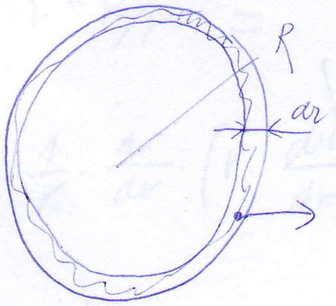
$$\boxed{Q = \int_{\mathcal{V}} \rho \, d\tau}$$



$E \, dS$ ← obsah kruh. elementu, nie kruhu ⇒

$\rightarrow 2\pi r \cdot dr$

$\frac{Q}{4\pi\varepsilon_0 \cdot r}$ $Q = \sigma \cdot S$ ⇒

$\frac{\sigma \cdot S}{4\pi\varepsilon_0 \cdot r}$

Prechod sférických súradníc na kartéz. | na cylindrické

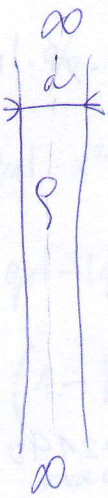
$x = r \cdot \sin \vartheta \cdot \cos \varphi$ | $\rho = r \cdot \sin \vartheta$

$y = r \cdot \sin \vartheta \cdot \sin \varphi$ | $\varphi' = \varphi$

$z = r \cdot \cos \vartheta$ | $z = r \cdot \cos \vartheta$

$$V = (\rho, \varphi, \vartheta) = \frac{a \cos \vartheta}{r^2} + \frac{b}{r}$$

$$E = -\operatorname{grad} V = -\frac{\partial V}{\partial r} - \frac{\partial V}{\partial \vartheta}$$



$E(r)$
$\varphi(r)$

(21) $\left(\frac{\alpha A}{4\pi\varepsilon_0} e^{-ar}(-a)\right)' = \frac{A}{4\pi\varepsilon_0}\cdot e^{-ar}(-a) \cancel{m} \cdot (-a) =$

$\alpha(xy) - \alpha z^2$

(23) $\alpha y \cdot \bar{i} + \alpha x \bar{j} - 2\alpha z \bar{k} = \bar{E} \quad /\cdot(-1)!$

priemet E: $E_a = \bar{E} \cdot \frac{\bar{a}}{|a|} =$

$(\alpha y \cdot \bar{i} + \alpha x \cdot \bar{j} - 2\alpha z \cdot \bar{k}) \cdot (\bar{i} + 3\bar{k}) =$

$\frac{\alpha y - 6\alpha z}{\sqrt{10}}$

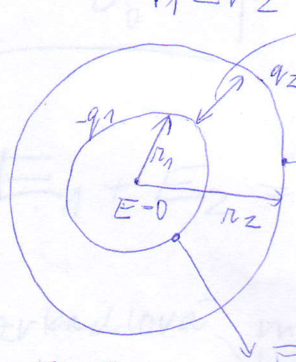
priemet vektora $\bar{X}$ do smeru vektora $\bar{y}$

⇒ počítame $X = \bar{X} \cdot \frac{\bar{y}}{|\bar{y}|}$

v bode? ↗ dosadíme

(24) $|-q_1| < |q_2|$

$r_1 < r_2$



$$E = \int dE = \frac{1}{4\pi\varepsilon_0}\cdot(-q_1)\cdot\int_{R_1}^{r}\frac{dr}{r^2} = \frac{-1}{4\pi\varepsilon_0}\cdot(-q_1)\cdot\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$r \in \langle R_1, R_2\rangle$

$$\hookrightarrow = \frac{1}{4\pi\varepsilon_0}\cdot q_1\left(\frac{1}{R_2}-\frac{1}{R_1}\right)$$

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q_2-q_1}{R^2}\right)$$

$R > R_2$

$$E \Rightarrow \left(\cancel{\iint E\cdot dS = \frac{Q_1}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\cdot\left\{\frac{-q_1+q_2}{R^2}\right\}}\right)$$

nerátam

$E = ?$

$V = ?$

potenciál:

(26) $\rho = k/r =$

$$\frac{1}{r}\cdot\frac{d}{dr}\left(r\cdot\frac{dV}{dr}\right) = -\frac{k}{\varepsilon_0 r} \Rightarrow \frac{d}{dr}\left(r\frac{dV}{dr}\right) = -\frac{k}{\varepsilon_0}$$

$$-\frac{k}{\varepsilon_0}r + C_1 = r\frac{dV}{dr} \Rightarrow \frac{dV}{dr} = \underbrace{-\frac{k}{\varepsilon_0} + \frac{C_1}{R}}$$

pre $r \le a \Rightarrow V = 0 \Rightarrow -\frac{k}{\varepsilon_0} + \frac{C_1}{R} = 0 \Rightarrow C_1 = \frac{k}{\varepsilon_0}\cdot a$

a

$$\left(V = \frac{A}{4\pi\varepsilon_0}\cdot e^{-\alpha r}\right)'' = \left\{\frac{A}{4\pi\varepsilon_0}\cdot e^{-\alpha r}\cdot(-\alpha)\right\}'$$

$$\frac{1}{r}\frac{d}{dr}\left(r\cdot\frac{dV}{dr}\right) = +\frac{1}{\varepsilon_0}\rho$$

$$\left((-1)r\cdot\frac{A\alpha}{4\pi\varepsilon_0}\cdot e^{-\alpha r}\right)' = \frac{(+1)A\alpha\cdot(+\alpha)}{4\pi\varepsilon_0}e^{-\alpha r}\cdot r + \frac{(-1)A\alpha}{4\pi\varepsilon_0}e^{-\alpha r} \Rightarrow$$

$$\hookrightarrow\left(\frac{A\alpha}{4\pi\varepsilon_0}e^{-\alpha r}\left(\alpha\cdot r - 1\right)\right)\cdot\frac{1}{r} = \boxed{\frac{A\alpha}{4\pi}\cdot e^{-\alpha r}\cdot\left(\alpha-\frac{1}{r}\right)}$$

(PR) $V = a\cdot r^2 + b$; $\rho = ?$

$$\frac{1}{r}\cdot\frac{d}{dr}\left(r\frac{dV}{dr}\right) = \frac{1}{r}\cdot\frac{d}{dr}\left(r\cdot 2ar\right) = \frac{1}{r}\cdot\frac{d}{dr}\left(2ar^2\right) = \frac{1}{r}\cdot\left(4ar\right) = 4a\cdot\varepsilon_0$$

$\boxed{4a}$

$E = -\,\mathrm{grad}\,V$

$\hookrightarrow \dfrac{a\cos\vartheta}{r^2} + \dfrac{b}{r}$

$5x$

ar^2 $\qquad \boxed{ar^2} + b$

$a\cdot(25\cdot r^2) =$

$\dfrac{1}{r^2}\cdot\dfrac{d}{dr}\left(r^2\cdot\dfrac{d}{dr}\right)$

$\dfrac{1}{r^2}\cdot\dfrac{d}{dr}\cdot\left(r^2\cdot 2ar\right) = \dfrac{1}{r^2}\dfrac{d}{dr}\left(2ar^3\right) = \dfrac{1}{r^2}\cdot 6ar^2 = 6a\varepsilon_0$

$V = \dfrac{A}{4\pi\varepsilon_0}\cdot e^{-\alpha r}$

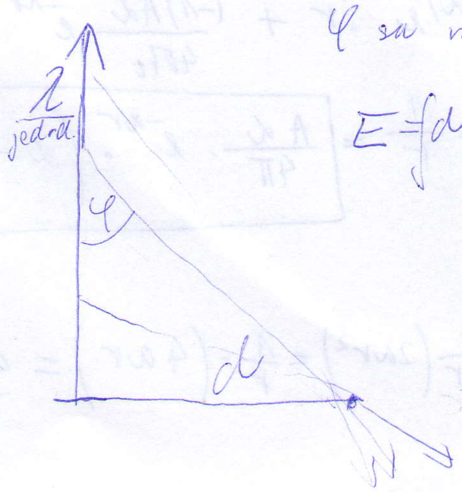
$\dfrac{1}{r^2}\cdot\dfrac{d}{dr}\cdot\left(r^2\cdot\dfrac{A(-\alpha)}{4\pi\varepsilon_0}\cdot e^{-\alpha r}\right) = \dfrac{1}{\cancel{r^2}}\left(\dfrac{(+\alpha)^2 A}{4\pi\varepsilon_0}\cdot e^{-\alpha r}\cdot \cancel{r^2} + \dfrac{(-\alpha)\cdot A}{4\pi\varepsilon_0}\cdot e^{-\alpha r}\cdot\dfrac{2\cancel{r}}{r}\right) =$

$= \dfrac{\alpha A}{4\pi}\cdot e^{-\alpha r}\left(-\alpha + \dfrac{2}{r}\right)$

ENERGIA DIPÓLU $\bar{p}$ v el. poli E je $\boxed{W = -\bar{p}\cdot\bar{E}}$

$\dfrac{Q}{r} \Rightarrow \dfrac{\bar{p}}{a}\cdot\dfrac{1}{r}$

$p = Q\cdot a$



φ sa meni od $0 \to \dfrac{\pi}{2}$

$E = \int dE = \dfrac{\lambda\cdot d}{2\pi\varepsilon_0}\cdot\int_0^{\frac{\pi}{2}} \dfrac{1}{L}\cos\varphi\; d\varphi$ $\quad$ $\cot g\,\varphi\; d\varphi$

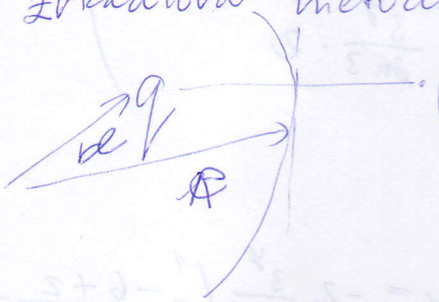
$l:\ \sin\varphi = \dfrac{d}{l} \Rightarrow l = \dfrac{d}{\sin\varphi}$

$E = ?$

$\boxed{V = \frac{\sigma}{\varepsilon_0} r}$ — rovina

$$E_1 + E_2 = \frac{\sigma}{\varepsilon_0}$$

zrkadlová metóda



$q' \longrightarrow q' = -\frac{l}{r} \cdot q$ → väčšie k menšiemu

vzdialenosť: $\delta = \frac{\cancel{R^2}\, l^2}{r}$

r v intenzite: $R - l$

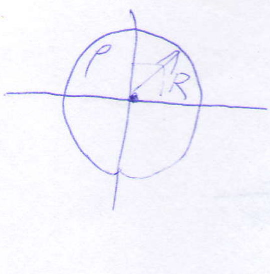
a $\delta - R$

ENERGIA el. poľa ~~s nábojom (Q)~~

$$W = \frac{\varepsilon_0}{2} \int E^2 \, d\tau$$

$$W = \frac{1}{2} \int_\tau \rho \cdot V \, d\tau$$

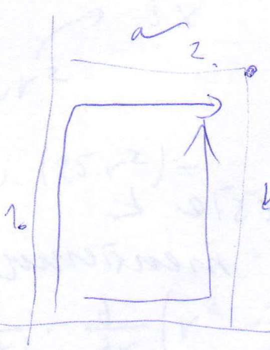
2 rovn. nabitá guľa R, ρ guli



$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \rho \cdot \frac{\frac{4}{3}\pi r^3}{r^2} = \frac{1}{\varepsilon_0} \rho \cdot \frac{1}{3} r$$

$$E = \frac{1}{\varepsilon_0} \cdot \frac{1}{3} \rho r \cdot E_r$$

① $E = (6xy,\ 3x^2 - 3^y \cdot 2,\ 0)$ → vyp. $\int E \cdot dl$



a)

1. $x = 0,\ y: 0 \to 1$:

$\int E \cdot dl = \int E_x \cdot dx + \int E_y \cdot dy = 6\frac{x^2}{2} \cdot y + 3x^2 y - \frac{3^y}{\ln 3} \cdot 2$

a) $x = 0$
$y: 0 \to 1$
$E_x = 0$
$E_y = -3^y \cdot 2$ ⇒ $\int E \cdot dl = \int_0^1 -2 \cdot 3^y\, dy = -2 \cdot \frac{3^y}{\ln 3}\Big|_0^1 = \frac{-6+2}{\ln 3} = $

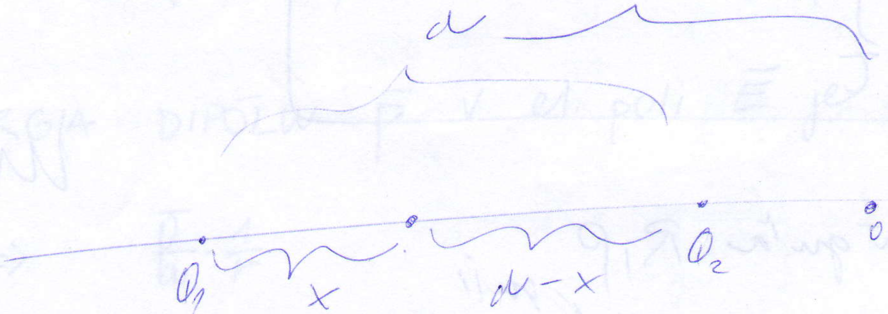
$= \frac{-4}{\ln 3}$

b) $y = 1$
$x: 0 \to 1$
$E_x = 6x \ldots$
$E_y = 3x^2 - 6$

$\int_0^1 6x^2\, dx = 6\frac{x^3}{3} = 2$

$dy = 0$ (nemení x ⇒ 0)

③



pre rovnako nabité náboje:

⇒ $E_1 = E_2$ ~~= 0~~

$\frac{Q_1}{x} = \frac{Q_2}{d - x}$ $\qquad$ $\frac{Q_1}{x} = \frac{Q_2}{d + x}$

$Q_1(d - x) = Q_2(x)$

$Q_1 d - Q_1 x - Q_2 x = 0$

$x(Q_1 + Q_2) = Q_1 \cdot d$

$x = \frac{Q_1}{Q_1 + Q_2} \cdot d$