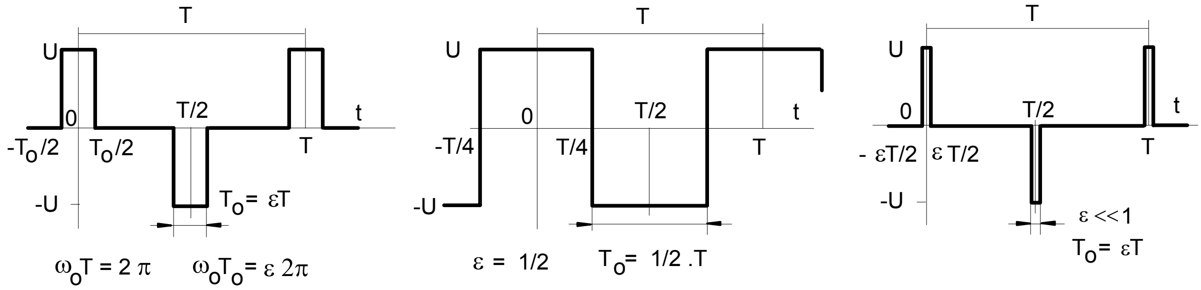
Harmonická analýza - Fourierov rad

- priebeh neobsahuje párne harmonické, nemá jednosmernú zložku a obsahuje len kosínové členy (párna funkcia).



$$\mathcal{U}_n = 2\frac{2}{T}\int_{-T/2}^{T/2} u(t)e^{-jn\omega_0 t}dt = \frac{4}{T}\int_{-T_0/2}^{T_0/2} Ue^{-jn\omega_0 t}dt = \frac{4U}{T}\int_{-T_0/2}^{T_0/2} e^{-jn\omega_0 t}dt = \frac{4U}{T}\left[\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right]_{-T_0/2}^{T_0/2} = \frac{4U}{T}\left[\frac{e^{-jn\omega_0 t}}{jn\omega_0}\right]_{T_0/2}^{-T_0/2} =$$

$$= \frac{4U}{jn\omega_0 T}\left[e^{jn\omega_0 T_0/2} - e^{-jn\omega_0 T_0/2}\right] = \frac{2U}{jn\pi}\left[e^{jn\pi\varepsilon} - e^{-jn\pi\varepsilon}\right] = \frac{4U}{n\pi}\left[\frac{e^{jn\pi\varepsilon} - e^{-jn\pi\varepsilon}}{2j}\right] = \frac{4U}{n\pi}\sin n\pi\varepsilon = 4\varepsilon U\frac{\sin n\pi\varepsilon}{n\pi\varepsilon}$$

Limitné prípady:

$\varepsilon = 1/2$, obdĺžnikový priebeh s nulovou jednosmernou zložkou

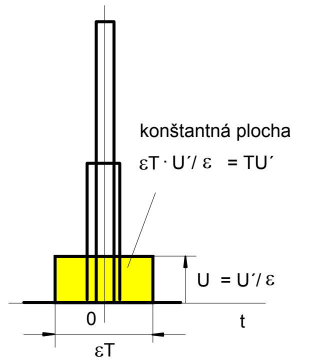
$$\mathcal{U}_n = 4\varepsilon U\frac{\sin n\pi\varepsilon}{n\pi\varepsilon} = 2U\frac{\sin n\pi\frac{1}{2}}{n\pi\frac{1}{2}} = \frac{4U}{n\pi}\sin\frac{n\pi}{2} = \frac{4U}{n\pi}(-1)^{\frac{n-1}{2}}\Big|_{n=1,3,5...}$$

$\varepsilon \to 0$, krátke impulzy

$$\mathcal{U}_n = 4\varepsilon U\frac{\sin n\pi\varepsilon}{n\pi\varepsilon}\Big|_{\varepsilon\to 0} = 4\varepsilon U\Big|_{\substack{n=1,3,5...\\ \varepsilon\to 0}}$$

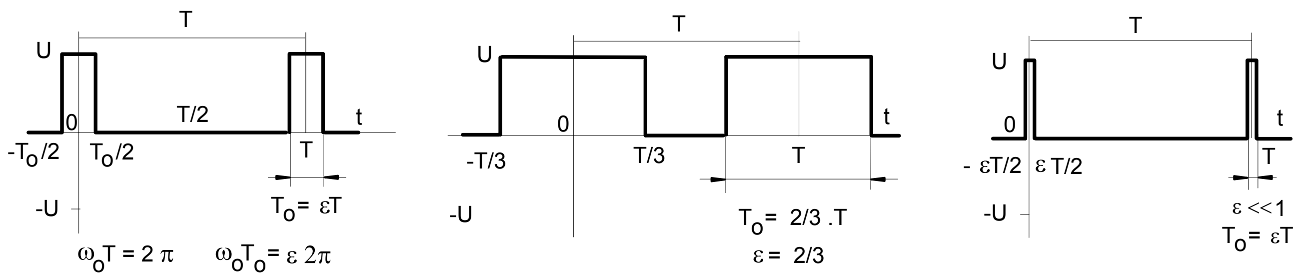
špeciálny prípad: $\mathcal{U}_n = 4\varepsilon U\big|_{U=U'/\varepsilon} = 4U'\big|_{n=1,3,5...}$

alternujúce $\delta(t)$ - impulzy a ich vyjadrenie Fourierovým radom:



$$TU'\cdot\frac{4}{T}\underbrace{\sum_{n=1,3,5}^{\infty}\cos(n\omega_0 t)}_{\text{rad nemá sučet pre ziadne } t} = TU'\cdot\frac{4}{T}\begin{cases}\sum\limits_{n=1,3,5}^{\infty}(+1), \text{ pre } t=\pm k\cdot T/2, \text{ kde: } k=0,2,4 \text{ - rad diverguje do } +\infty \\ \sum\limits_{n=1,3,5}^{\infty}(-1), \text{ pre } t=\pm k\cdot T/2, \text{ kde: } k=1,3,5 \text{ - rad diverguje do } -\infty\end{cases}$$

- priebeh môže obsahovať aj párne harmonické, má jednosmernú zložku, obsahuje však len kosinové členy (párna funkcia).



$$\mathcal{U}_n = \frac{2}{T}\int_{-T/2}^{T/2} u(t)e^{-jn\omega_0 t}dt = \frac{2}{T}\int_{-T_0/2}^{T_0/2} Ue^{-jn\omega_0 t}dt = \frac{2U}{T}\int_{-T_0/2}^{T_0/2} e^{-jn\omega_0 t}dt = \frac{2U}{T}\left[\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right]_{-T_0/2}^{T_0/2} = \frac{2U}{T}\left[\frac{e^{-jn\omega_0 t}}{jn\omega_0}\right]_{T_0/2}^{-T_0/2} =$$

$$= \frac{2U}{jn\omega_0 T}\left[e^{jn\omega_0 T_0/2} - e^{-jn\omega_0 T_0/2}\right] = \frac{U}{jn\pi}\left[e^{jn\pi\varepsilon} - e^{-jn\pi\varepsilon}\right] = \frac{2U}{n\pi}\left[\frac{e^{jn\pi\varepsilon} - e^{-jn\pi\varepsilon}}{2j}\right] = \frac{2U}{n\pi}\sin n\pi\varepsilon = 2\varepsilon U\frac{\sin n\pi\varepsilon}{n\pi\varepsilon}$$

jenosmerná zložka: $U_0 = \frac{1}{T}\int_{-T_0/2}^{T_0/2} U dt = \frac{U}{T}\int_{-T_0/2}^{T_0/2} dt = \frac{U}{T}T_0 = \varepsilon U$

Limitné prípady:

$\varepsilon = 1/2$, obdĺžnikový priebeh s jednosmernou superpozíciou (len nepárne harmonické)

$$\mathcal{U}_n = 2\varepsilon U\frac{\sin n\pi\varepsilon}{n\pi\varepsilon} = U\frac{\sin n\pi\frac{1}{2}}{n\pi\frac{1}{2}} = \frac{2U}{n\pi}\sin\frac{n\pi}{2} = \frac{2U}{n\pi}(-1)^{\frac{n-1}{2}}\Big|_{n=1,3,5...}, \quad U_0 = \frac{U}{2}$$

$\varepsilon = 1$, jednosmerný (konštantný) priebeh $\quad \mathcal{U}_n = 2U\frac{\sin n\pi}{n\pi}\Big|_{n=1,2,3} = 0, \quad U_0 = U$

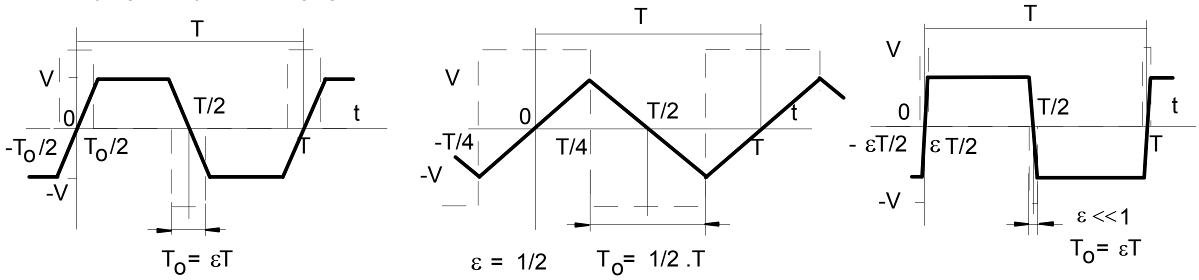
$\varepsilon \to 0$, krátke impulzy $\quad \mathcal{U}_n = 2\varepsilon U\frac{\sin n\pi\varepsilon}{n\pi\varepsilon}\Big|_{\varepsilon\to 0} = 2\varepsilon U\Big|_{\substack{n=1,2,3...\\ \varepsilon\to 0}}, \quad U_0 = \varepsilon U\Big|_{\varepsilon\to 0} \to 0$

špeciálny prípad: $\mathcal{U}_n = 2\varepsilon U\big|_{U=U'/\varepsilon} = 2U'\big|_{n=1,2,3...}, \quad U_0 = U'$ unipolárne d(t) - impulzy a ich vyjadrenie Fourierovým radom:

$$U'\cdot\left[1+2\underbrace{\sum_{n=1,2,3}^{\infty}\cos(n\omega_0 t)}_{\text{rad nemá sučet pre ziadne } t}\right] = U'\left[1+2\sum_{n=1,2,3}^{\infty}(+1)\right], \text{ pre } t=\pm k\cdot T, \text{ kde: } k=0,1,2,3 \text{ - rad diverguje do } +\infty$$

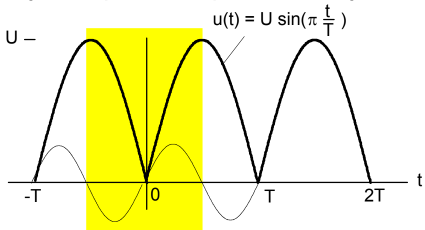
keďže n - je nepárne aj párne v $\omega_0 t = \pi$ rad nediverguje (nemá ale ani súčet).

Derivácia nasledujúcich priebehov (plné čiary) dáva predchádzajúce prípady (prerušované čiary), za predpokladu že: U = 2V/(εT) viete to využiť pri ich harmonickej analýze. Dajú sa podobne využiť prv uvedené priebehy aj vtedy keď majú jednosmernú zložku?



$\omega_0 T = 2\pi \quad \omega_0 T_0 = \varepsilon 2\pi$

A jeden príklad z praxe – dvojcestne usmernený priebeh



$$\mathcal{U}_n = \frac{2}{T}\int_0^T u(t)e^{-jn\omega_0 t}dt = \frac{2U}{T}\int_{-T/2}^{T/2}\underbrace{\left|\sin(\pi\frac{t}{T})\right|}_{párna}e^{-jn\omega_0 t}dt = \frac{2U}{T}\int_{-T/2}^{T/2}\underbrace{\left|\sin(\pi\frac{t}{T})\right|}_{párna}\underbrace{\cos n\omega_0}_{párna}dt - j\underbrace{\frac{2U}{T}\int_{-T/2}^{T/2}\overbrace{\underbrace{\left|\sin(\pi\frac{t}{T})\right|}_{párna}\underbrace{\sin n\omega_0}_{nepárna}}^{nepárna}dt}_{0} =$$

$$= \frac{4U}{T}\int_0^{T/2}\sin(\frac{\omega_0}{2}t)\cos n\omega_0 dt = \frac{4U}{T}\int_0^{T/2}\sin(\frac{\omega_0}{2}t)\cos n\omega_0 dt = \frac{2U}{T}\left\{\int_0^{T/2}\sin(\omega_0 t\frac{1+2n}{2})dt + \int_0^{T/2}\sin(\omega_0 t\frac{1-2n}{2})dt\right\} =$$

$$= \frac{2U}{\omega_0 T}\left\{\frac{2}{1+2n}\left[\cos(\omega_0 t\frac{1+2n}{2})\right]_{T/2}^{0} + \frac{2}{1-2n}\left[\cos(\omega_0 t\frac{1-2n}{2})\right]_{T/2}^{0}\right\} = \frac{2U}{\pi}\left\{\frac{1}{1+2n}\left[1-\underbrace{\cos(\pi\frac{1+2n}{2})}_{0}\right] + \frac{1}{1-2n}\left[1-\underbrace{\cos(\pi\frac{1-2n}{2})}_{0}\right]\right\}$$

$$= \frac{2U}{\pi}\left\{\frac{1}{1+2n}+\frac{1}{1-2n}\right\} = \frac{4U}{\pi}\left\{\frac{1}{1-(2n)^2}\right\}$$