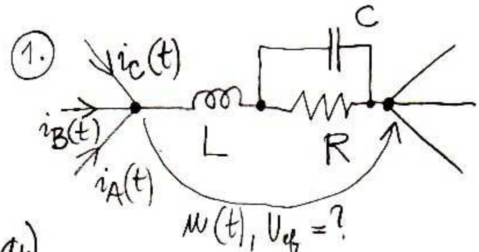
1.



$R = 20\Omega \quad L = 10mH \quad C = 50\mu F$

$i_A(t) = 2 + 2\cdot\cos(10^3 t) \Longleftrightarrow \mathcal{I}_A(t) = 2 + 2e^{j10^3 t}$

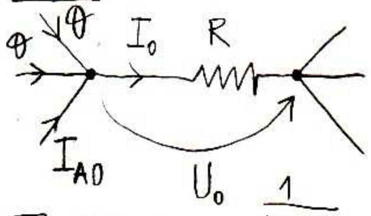
$i_B(t) = 3\sin(10^3 t) + 2\cos(2\cdot 10^3 t) \Leftrightarrow \mathcal{I}_B(t) = 3e^{j(10^3 t - \frac{\pi}{2})} + 2e^{j(2\cdot 10^3 t)}$

$i_C(t) = \cos(2\cdot 10^3 t + \frac{\pi}{4}) + 2\cos(4\cdot 10^3 t - \frac{\pi}{4}) \Leftrightarrow$

$\Leftrightarrow \mathcal{I}_C(t) = e^{j(2\cdot 10^3 t + \frac{\pi}{4})} + 2e^{j(4\cdot 10^3 t - \frac{\pi}{4})}$

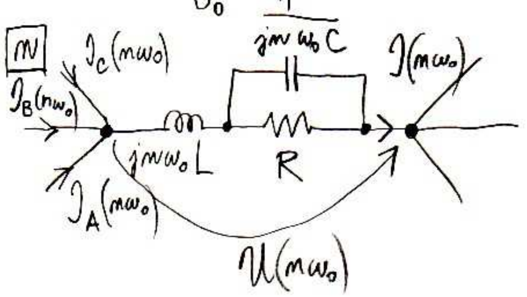
a)

$\boxed{n=0}$



$I_0 = I_{A0} = 2A \qquad U_0 = RI_0 = 20\cdot 2 = 40V$

$\boxed{n}$



$\mathcal{U}(n\omega_0) = \mathcal{Z}(n\omega_0)\cdot\mathcal{I}(n\omega_0) = (\mathcal{I}_{A_n} + \mathcal{I}_{B_n} + \mathcal{I}_{C_n})\cdot \mathcal{Z}_n$

$\mathcal{I}(n\omega_0) = \mathcal{I}_A(n\omega_0) + \mathcal{I}_B(n\omega_0) + \mathcal{I}_C(n\omega_0)$

$$\mathcal{Z}(n\omega_0) = jn\omega_0 L + \frac{R\cdot\frac{1}{jn\omega_0 C}}{R + \frac{1}{jn\omega_0 C}} = jn\omega_0 L + \frac{R}{1 + jn\omega_0 RC}$$

$\underline{n=1}$

$$\mathcal{U}(\omega_0) = \left(j\omega_0 L + \frac{R}{1+j\omega_0 RC}\right)\cdot(\mathcal{I}_A(\omega_0) + \mathcal{I}_B(\omega_0))$$

$$= \left(j\underbrace{10^3\cdot 10\cdot 10^{-3}}_{10} + \frac{20}{1+j\underbrace{10^3\cdot 20\cdot 50\cdot 10^{-6}}_{1}}\right)\cdot(2 + \underbrace{3e^{-j90^\circ}}_{-j3}) = \underbrace{\left[j10 + \frac{\overbrace{20}^{14,142\angle -45^\circ = 10 - j10}}{\sqrt{2}\angle 45^\circ}\right]}_{j10 + 10 - j10}(2-j3) = 10(2-j3) = 20 - j30 =$$

$$= 36,056\angle -56,31^\circ$$

$\underline{n=2}$

$$\mathcal{U}(2\omega_0) = \left(j2\omega_0 L + \frac{R}{1+j2\omega_0 RC}\right)\left[\mathcal{I}_B(2\omega_0) + \mathcal{I}_C(2\omega_0)\right] =$$

$$= \left(j\underbrace{2\cdot\cancel{10^3}\cdot 10\cdot\cancel{10^{-3}}}_{20} + \frac{20}{1+j\underbrace{2\cdot 10^3\cdot 20\cdot 50\cdot 10^{-6}}_{2}}\right)\left[2 + \underbrace{1e^{j45^\circ}}_{0,7071+j0,7071}\right] = \underbrace{\left[j20 + \frac{20}{2,236\angle 63,435^\circ}\right]}_{4+j12 = 12,649\angle 71,565^\circ}\underbrace{(2,7071 + j0,7071)}_{2,798\angle 14,639^\circ} =$$

$\frac{20}{2,236\angle 63,435^\circ} = 8,9445\angle -63,435^\circ = 4 - j8$

$= 35,392\angle 86,204^\circ$

$\underline{n=4}$

$$\mathcal{U}(4\omega_0) = \left(j4\omega_0 L + \frac{R}{1+j4\omega_0 RC}\right)\cdot\mathcal{I}_C(4\omega_0) = \left[j\underbrace{4\cdot\cancel{10^3}\cdot 10\cdot\cancel{10^{-3}}}_{40} + \frac{20}{1+j4}\right]\cdot 2\angle -45^\circ =$$

$$= \left(j40 + \frac{20}{4,123\angle 75,96^\circ}\right)\cdot 2\angle -45^\circ = \underbrace{\left(j40 + \underbrace{4,851\angle -75,96^\circ}_{1,1768 - j4,7059}\right)}_{1,1768 + j35,294 = 35,314\angle 88,09^\circ}\cdot 2\angle -45^\circ = 70,628\angle 43,09^\circ$$

$$u(t) = 40 + 36,056\cos(10^3 t \overbrace{- 56,31^\circ}^{-0,983\,rad}) + 35,392\cos(2\cdot 10^3 t \overbrace{+ 86,204^\circ}^{+1,505\,rad}) + 70,628\cos(4\cdot 10^3 t \overbrace{+ 43,09^\circ}^{+0,752\,rad})$$

$$U_{ef} = \sqrt{U_0^2 + \frac{U_{1m}^2 + U_{2m}^2 + U_{4m}^2}{2}} = \sqrt{40^2 + \frac{36,056^2 + 35,392^2 + 70,628^2}{2}} = \sqrt{5370,47} = 73,2835V$$

-

1.

$I_0 = 2A$

$I(\omega_0) = 10\ A$

$I(2\omega_0) = 12{,}649\ \underline{|71{,}565^\circ}$

$I(4\omega_0) = 2\ \underline{|-45^\circ}$

$I_{ef} = \sqrt{2^2 + 10^2 + 12{,}649^2 + 2^2} = $

$= 16{,}49234\ A$

b) $P = U_0 I_0 + \sum_{n=1}^{\infty} U_{n_{ef}} I_{n_{ef}} \cos(\varphi_U - \varphi_I) =$

$= 40 \cdot 2 + 36{,}056 \cdot 10 \cdot \overbrace{\cos(-56{,}31^\circ - 0)}^{0{,}5547} + 35{,}392 \cdot 12{,}649 \overbrace{\cos(86{,}204^\circ - 71{,}565^\circ)}^{0{,}96754} +$

$+ 70{,}628 \cdot 2 \overbrace{\cos(43{,}09^\circ + 45^\circ)}^{0{,}03333} = 80 + 200{,}00235 + 433{,}140751 +$

$+ 4{,}7080081 = 717{,}85111\ W$

$Q = \sum_{n=1}^{\infty} U_{nef} I_{nef} \sin(\varphi_U - \varphi_I) =$

$= 36{,}056 \cdot 10 \cdot \overbrace{\sin(-56{,}31^\circ)}^{-0{,}83205} + 35{,}392 \cdot 12{,}649 \overbrace{\sin(86{,}204^\circ - 71{,}565^\circ)}^{0{,}25273} + 70{,}628 \cdot 2 \cdot \sin(\overbrace{43{,}09^\circ + 45^\circ}^{0{,}999444}) =$

$= -300{,}00429 + 113{,}1396044 + 141{,}17752 = -45{,}68717\ VA$

(2.) $k = 0, 1, 2, \ldots, 7$

$$\overline{U}_1 = \frac{10}{8}\Big\{ \underset{\substack{\parallel \\ 0}}{u_0}\, \underbrace{e^{-j\frac{2\pi}{8}\cdot 0\cdot 1}}_{\substack{1 \\ k=0 \\ m=1}} + \underset{\substack{\parallel \\ \frac14}}{u_1}\cdot \underset{\substack{k=1 \\ m=1}}{e^{-j\frac{2\pi}{8}\cdot 1\cdot 1}} + \underset{\substack{\parallel \\ \frac12}}{u_2}\cdot \underset{\substack{k=2 \\ m=1}}{e^{-j\frac{2\pi}{8}\cdot 2\cdot 1}} +$$

$$+ \underset{\substack{\parallel \\ \frac34}}{u_3}\cdot \underset{\substack{k=3 \\ m=1}}{e^{-j\frac{2\pi}{8}\cdot 3\cdot 1}} + \underset{\substack{\parallel \\ 1}}{u_4}\cdot \underset{\substack{k=4 \\ m=1}}{e^{-j\frac{2\pi}{8}\cdot 4\cdot 1}} + \underset{\substack{\parallel \\ \frac34}}{u_5}\, \underset{\substack{k=5 \\ m=1}}{e^{-j\frac{2\pi}{8}\cdot 5\cdot 1}} + \underset{\substack{\parallel \\ \frac12}}{u_6}\, \underset{\substack{k=6 \\ m=1}}{e^{-j\frac{2\pi}{8}\cdot 6\cdot 1}} + \underset{\substack{\parallel \\ \frac14}}{u_7}\cdot \underset{\substack{k=7 \\ m=1}}{e^{-j\frac{2\pi}{8}\cdot 7\cdot 1}} \Big\} =$$

$$= \frac{10}{8}\Big\{ \frac14 e^{-j\frac{\pi}{4}} + \frac12 e^{-j\frac{\pi}{2}} + \frac34 e^{-j\frac34\pi} + e^{-j\pi} + \frac34 e^{-j\frac54\pi} + \frac12 e^{-j\frac32\pi} + \frac14 e^{-j\frac74\pi} \Big\} =$$

$$= \frac{10}{8}\Big\{ \frac14\left(\frac{\sqrt2}{2} - j\frac{\sqrt2}{2}\right) + \frac12(0 - j) + \frac34\left(-\frac{\sqrt2}{2} - j\frac{\sqrt2}{2}\right) - 1 + \frac34\left(-\frac{\sqrt2}{2} + j\frac{\sqrt2}{2}\right) + \frac12(0 + j) + \frac14\left(\frac{\sqrt2}{2} + j\frac{\sqrt2}{2}\right) \Big\} =$$

$$= \frac{10}{8}\Big\{ \frac12\cdot\frac{\sqrt2}{2} - \frac{\cancel{6}^{\,3}}{\cancel{4}_{\,2}}\cdot\frac{\sqrt2}{2} \ (-1) \Big\} = \frac{\cancel{10}^{\,5}}{8}\cdot\frac{\sqrt2 - 3\sqrt2}{2\,\cancel{4}}\left(-\frac{10}{8}\right) = \frac{5}{16}\cdot\sqrt2\,(-2)\left(-\frac{10}{8}\right) = -\frac{5\sqrt2}{8}\left(-\frac{10}{8}\right) \ \cancel{-0{,}883883} \ \ -2{,}133883$$

$$\overline{U}_3 = \frac{10}{8}\Big\{ u_1\cdot \underset{\substack{k=1 \\ m=3}}{e^{-j\frac{2\pi}{8}\cdot 1\cdot 3}} + u_2\, \underset{\substack{k=2 \\ m=3}}{e^{-j\frac{2\pi}{8}\cdot 2\cdot 3}} + u_3\, \underset{\substack{k=3 \\ m=3}}{e^{-j\frac{2\pi}{8}\cdot 3\cdot 3}} + u_4\cdot \underset{\substack{k=4 \\ m=3}}{e^{-j\frac{2\pi}{8}\cdot 4\cdot 3}} +$$

$$+ u_5\cdot \underset{\substack{k=5 \\ m=3}}{e^{-j\frac{2\pi}{8}\cdot 5\cdot 3}} + u_6\cdot \underset{\substack{k=6 \\ m=3}}{e^{-j\frac{2\pi}{8}\cdot 6\cdot 3}} + u_7\, \underset{\substack{k=7 \\ m=3}}{e^{-j\frac{2\pi}{8}\cdot 7\cdot 3}} \Big\} =$$

$$= \frac{10}{8}\Big\{ \frac14 e^{-j\frac34\pi} + \frac12 e^{-j\frac32\pi} + \frac34 e^{-j\frac94\pi} + 1\cdot e^{-j3\pi} + \frac34 e^{-j\frac{15}{4}\pi} + \frac12 e^{-j\frac{9\pi}{2}} + \frac14 e^{-j\frac{21\pi}{4}} \Big\} =$$

$$= \frac{10}{8}\Big\{ \frac14\left(-\frac{\sqrt2}{2} - j\frac{\sqrt2}{2}\right) + \frac12(0 + j) + \frac34\left(\frac{\sqrt2}{2} - j\frac{\sqrt2}{2}\right) - 1 + \frac34\left(\frac{\sqrt2}{2} + j\frac{\sqrt2}{2}\right) + \frac12(0 - j) + \frac14\left(-\frac{\sqrt2}{2} + j\frac{\sqrt2}{2}\right) \Big\} =$$

$$= \frac{10}{8}\Big\{ -\frac12\cdot\frac{\sqrt2}{2} + \frac32\cdot\frac{\sqrt2}{2} - 1 \Big\} = \frac{\cancel{2}\cdot 5}{\cancel{2}\cdot 2\cdot 2}\cdot\frac{(3-1)\cdot\sqrt2}{\cancel{2}\cdot 2} - \frac{\cancel{2}\cdot 5}{\cancel{2}\cdot 2\cdot 2} = \frac54\cdot\left(\frac{\sqrt2}{2} - 1\right) =$$

$$= \frac54\cdot\frac{\sqrt2 - 2}{2} = -0{,}3661165$$

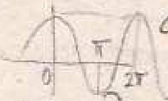
exaktne je to $U_m = \frac{4}{T}\int_0^{T/2} \frac{2U_m}{T}\, t\cos(m\omega_0 t)\,dt = \frac{2U_m}{m^2\pi^2}\left\{(-1)^m - 1\right\} = \begin{cases} 0; & m = 2, 4, 6, \ldots \\ -\frac{4U_m}{m^2\pi^2}; & m = 1, 3, 5, \ldots \end{cases}$

$$V_n = \frac{4}{T}\int_0^{T/2} \frac{2U_m}{T}\, t \cos(n\omega_0 t)\,dt = \frac{8U_m}{T^2}\left\{\underbrace{\left[\frac{\frac{T}{2}\sin\left(n\frac{2\pi}{T}\cdot\frac{T}{2}\right)}{n\frac{2\pi}{T}}\right]}_{0} - \frac{1}{n\frac{2\pi}{T}}\left[\frac{-\cos\left(n\frac{2\pi}{T}\cdot\frac{T}{2}\right)}{n\frac{2\pi}{T}} + \frac{1}{n\frac{2\pi}{T}}\right]\right\} =$$

2°

$u = t \qquad v' = \cos n\omega_0 t$

$u' = 1 \qquad v = \frac{\sin n\omega_0 t}{n\omega_0}$

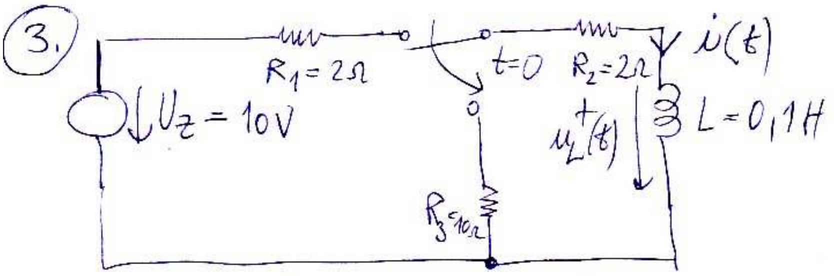


$\cos(n\pi) = (-1)^n$

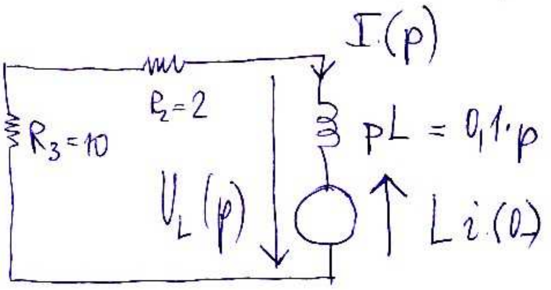
$$= \frac{8U_m}{\cancel{T^2}\cdot\frac{n^2 4\pi^2}{\cancel{T^2}}}\cdot\left[\frac{(-1)^n - 1}{1}\right] = \frac{2U_m}{n^2\pi^2}\left\{(-1)^n - 1\right\} = \begin{cases} 0, & n = 2, 4, 6, \ldots \\ -\frac{4U_m}{n^2\pi^2}, & n = 1, 3, 5 \end{cases}$$

-

3.



$$i(0_-) = \frac{U_Z}{R_1 + R_2} = \frac{10}{2+2} = \frac{2 \cdot 5}{2 \cdot 2} = 2{,}5A \ ; \ u_L(0_-) = 0\ V$$

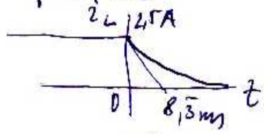


$$I(p) = \frac{L\,i(0_-)}{R_2 + R_3 + pL} =$$

$$= \frac{0{,}1 \cdot 2{,}5}{2 + 10 + 0{,}1p} = \frac{0{,}25}{0{,}1} \cdot \frac{1}{p + \frac{12}{0{,}1}} =$$

$$= 2{,}5 \frac{1}{p + 120} \Longleftrightarrow 2{,}5\, e^{-120t} \cdot 1(t) = i^+(t)$$

$$\tau = \frac{1}{120} = 8{,}\bar{3}\ ms$$



$$U_L(p) = I(p) \cdot pL - L\,i(0_-) =$$

$$= \frac{L\,i(0_-)}{R_2 + R_3 + pL} \cdot pL - L\,i(0_-) =$$

$$= \frac{0{,}1 \cdot 2{,}5 \cdot 0{,}1 \cdot p}{2 + 10 + 0{,}1 \cdot p} - 0{,}1 \cdot 2{,}5 = \frac{0{,}025p}{0{,}1p + 12} \underbrace{=}_{-0{,}25} \frac{0{,}025}{0{,}1} \cdot \frac{p}{p + 120} - 0{,}25 =$$

$$= 0{,}25 \left[1 - \frac{120}{p+120}\right] - 0{,}25 = 0{,}25 - 30 \cdot \frac{1}{p+120} - \cancel{0{,}25} =$$

$$= -30 \frac{1}{p+120} \Longleftrightarrow -30\, e^{-120t} \cdot 1(t) = u_L^+(t)$$

$$p : (p+120) = 1 - \frac{120}{p+120}$$

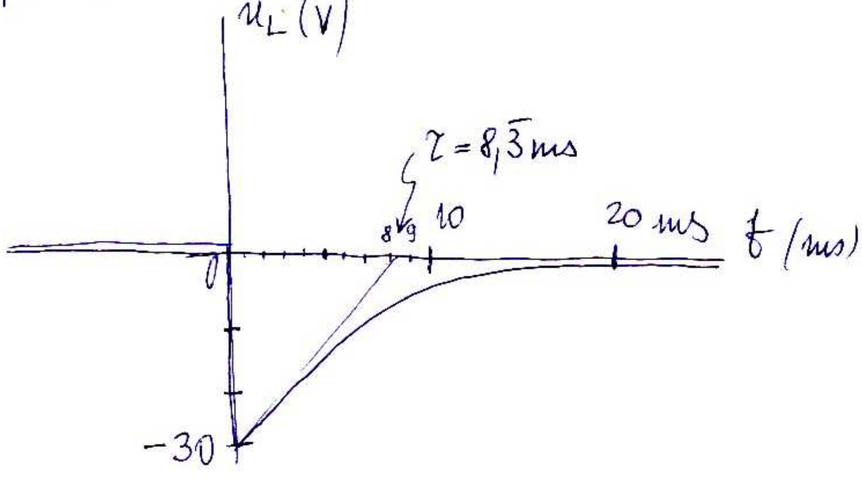
$$\frac{-(p+120)}{0 - 120}$$

$u_L(0_-) = 0V$

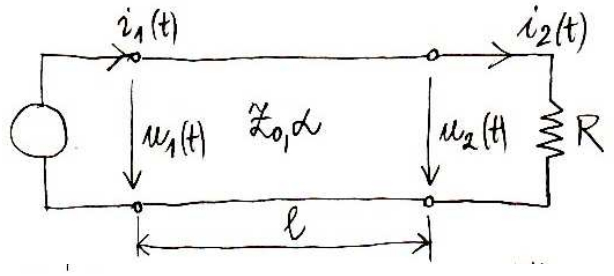
$u_L(0_+) = -30V$

$u_L(\infty) = 0\ V$

$\tau_L = 8{,}\bar{3}\ ms$



4.



prvá verzia skúšky

IHV: $\beta = 0$; $Z_0 = 200\,\Omega$, $\lambda = 2\,m$

$l = 9{,}25\,m$, $R = 100\,\Omega$

$u_1(t) = 100 \cdot \cos(10^6 \cdot t)$ (V)

c.)
$$Z_{vst} = \frac{U_1}{I_1} = \frac{U(\xi = l)}{I(\xi = l)} = \frac{U_2 \cosh(\gamma l) + I_2 Z_0 \sinh(\gamma l)}{I_2 \cosh(\gamma l) + \frac{U_2}{Z_0} \sinh(\gamma l)} \underset{\substack{\uparrow \\ U_2 = I_2 Z_2}}{=} Z_0 \cdot \frac{Z_2 \cosh(\gamma l) + Z_0 \sinh(\gamma l)}{Z_0 \cosh(\gamma l) + Z_2 \sinh(\gamma l)}$$

$$\gamma = \beta + j\alpha = j\alpha = j\frac{2\pi}{\lambda} = j\frac{2\pi}{2} = j\underbrace{\pi}_{\alpha} \ (m^{-1})$$

$$\alpha = \frac{2\pi}{\lambda}$$

$$\cosh(\gamma l) = \cosh(j\alpha l) = \cos(\alpha l) = \cos(\pi \cdot 9{,}25) = -0{,}7071 = -\frac{1}{\sqrt{2}}$$

$$\sinh(\gamma l) = \sinh(j\alpha l) = j\sin(\alpha l) = j\sin(\pi \cdot 9{,}25) = -j0{,}7071 = -j\frac{1}{\sqrt{2}}$$

$$Z_{vst} = 200 \cdot \frac{100 \cdot \left(-\frac{1}{\sqrt{2}}\right) + 200 \cdot \left(-j\frac{1}{\sqrt{2}}\right)}{200 \cdot \left(-\frac{1}{\sqrt{2}}\right) + 100 \cdot \left(-j\frac{1}{\sqrt{2}}\right)} = 200 \cdot \frac{1 + j2}{2 + j} = 200\,\frac{2{,}236\,\angle 63{,}435^\circ}{2{,}236\,\angle 26{,}565^\circ} = \underline{200\,\angle 36{,}87^\circ}$$

$$U(x = l) = U_2 = U_1 \cosh(\gamma l) - I_1 Z_0 \sinh(\gamma l) =$$

$$= 100 \cdot \left(-\frac{1}{\sqrt{2}}\right) - \underbrace{0{,}5\,\angle -36{,}87^\circ}_{(0{,}4 - j0{,}3)} \cdot 200 \cdot \left(-j\frac{1}{\sqrt{2}}\right) = -\frac{100}{\sqrt{2}} + j\frac{0{,}4 \cdot 200}{\sqrt{2}} + \frac{0{,}3 \cdot 200}{\sqrt{2}} =$$

$$= \frac{-40}{\sqrt{2}} + j\frac{80}{\sqrt{2}} = \frac{40}{\sqrt{2}}(-1 + j2) = \frac{40}{\sqrt{2}} \cdot 2{,}236\,\angle 116{,}565^\circ = \underline{63{,}2436\,\angle 116{,}565^\circ}$$

$$u_2(t) = \underline{63{,}2436} \cdot \cos(10^6 \cdot t + \underbrace{116{,}565^\circ}_{2{,}0344\,rad}) \ (V)$$

$$I_1 = \frac{U_1}{Z_{vst}} = \frac{100}{200\,\angle 36{,}87^\circ} = \underline{0{,}5\,\angle -36{,}87^\circ} = 0{,}4 - j0{,}3$$

a.)
$$\underset{\substack{\downarrow \\ \text{v mieste } \xi \\ \text{od konca}}}{\rho(\xi)} = \frac{U_s(\xi)}{U_p(\xi)} = \frac{Z_2 - Z_0}{Z_2 + Z_0} e^{-2\gamma\xi} \underset{\xi = 0}{=} \frac{U_{s2}}{U_{p2}} = \frac{Z_2 - Z_0}{Z_2 + Z_0} = \frac{R - Z_0}{R + Z_0} = \frac{100 - 200}{100 + 200} = \frac{-100}{300} = -0{,}\overline{3}$$

b.)
$$PSV \underset{\substack{\uparrow \\ \text{IHV, } \rho(\xi) = \text{konšt.}}}{=} \frac{1 + |\rho_2|}{1 - |\rho_2|} = \frac{1 + \frac{1}{3}}{1 - \frac{1}{3}} = \frac{\frac{4}{3}}{\frac{2}{3}} = \frac{4}{2} = \underline{\underline{2}} \left(= \frac{U_{max}^{(1)}}{U_{min}^{(1)}} \right)$$