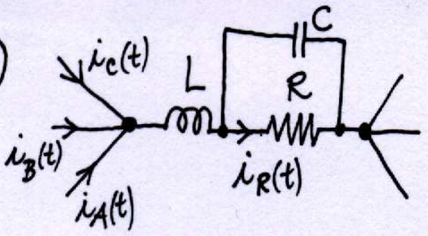
(1.)



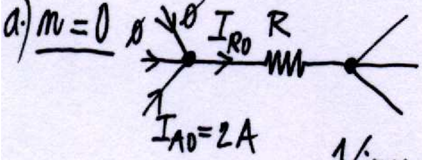
$R = 20\Omega \quad L = 10mH \quad C = 50\mu F$

$i_A(t) = 2 + 2\cdot\cos(10^3 t) \leftrightarrow \mathcal{I}_A(t) = 2 + 2e^{j10^3 t}$

$i_B(t) = 3\sin(10^3 t) + 2\cos(2\cdot 10^3 t) \leftrightarrow \mathcal{I}_B(t) = 3e^{j(10^3 t - \frac{\pi}{2})} + 2e^{j(2\cdot 10^3 t)}$

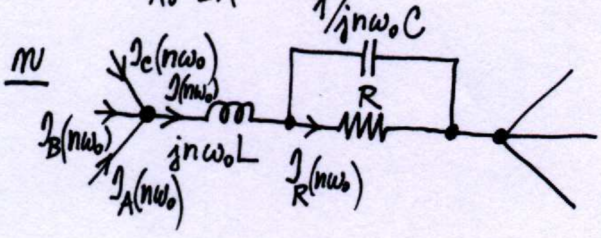
$i_C(t) = \cos(2\cdot 10^3 t + \frac{\pi}{4}) + 2\cos(4\cdot 10^3 t - \frac{\pi}{4}) \leftrightarrow \mathcal{I}_C(t) = e^{j(2\cdot 10^3 t + \frac{\pi}{4})} + 2e^{j(4\cdot 10^3 t - \pi/4)}$

a.) $n = 0$



$I_{R0} = I_{A0} = 2A$

n



I. Kirchhoffov zákon

$\mathcal{I}(n\omega_0) = \mathcal{I}_A(n\omega_0) + \mathcal{I}_B(n\omega_0) + \mathcal{I}_C(n\omega_0)$

$\mathcal{I}_R(n\omega_0) = \mathcal{I}(n\omega_0)\cdot\dfrac{\frac{1}{R}}{\frac{1}{R} + jn\omega_0 C}$

prúdový delič

$n=1$ $\mathcal{I}_R(\omega_0) = \mathcal{I}(\omega_0)\cdot\dfrac{G}{G + j\omega_0 C} = (2 - j3)\dfrac{\frac{1}{20}}{\frac{1}{20} + j10^3\cdot 50\cdot 10^{-6}} = 3{,}60555\angle{-56{,}31^\circ}\,\dfrac{0{,}05}{0{,}05 + j0{,}05} =$

$= 3{,}60555\angle{-56{,}31^\circ}\,\dfrac{\cancel{0{,}05}}{\sqrt{2}\,\cancel{0{,}05}\angle 45^\circ} = \underbrace{2{,}5495\angle{-101{,}31^\circ}}_{-0{,}5 - j2{,}5} \Rightarrow I^{(1)}_{R,ef} = \dfrac{2{,}5495}{\sqrt{2}} = \underline{\underline{1{,}802768\,A}}$

$n=2$ $\mathcal{I}_R(2\omega_0) = \mathcal{I}(2\omega_0)\dfrac{G}{G + j2\omega_0 C} = \Big(\underbrace{2{,}7071 + j0{,}7071}_{2{,}798\angle 14{,}639^\circ}\Big)\dfrac{0{,}05}{0{,}05 + j\,2\cdot 10^3\cdot 50\cdot 10^{-6}} =$

$= \dfrac{0{,}1399\angle 14{,}639^\circ}{0{,}05 + j0{,}1} = \dfrac{0{,}1399\angle 14{,}639^\circ}{0{,}1118\angle 63{,}435^\circ} = \underbrace{1{,}25134\angle{-48{,}796^\circ}}_{0{,}8243 - j0{,}94147} \Rightarrow I^{(2)}_{R,ef} = \dfrac{1{,}25134}{\sqrt{2}} = \underline{\underline{0{,}88483A}}$

$n=4$ $\mathcal{I}_R(4\omega_0) = \mathcal{I}(4\omega_0)\cdot\dfrac{G}{G + j4\omega_0 C} = 2\angle{-45^\circ}\cdot\dfrac{0{,}05}{0{,}05 + j\underbrace{4\cdot 10^3\cdot 50\cdot 10^{-6}}_{0{,}2}} = \dfrac{0{,}1\angle{-45^\circ}}{0{,}20616\angle 75{,}964^\circ} =$

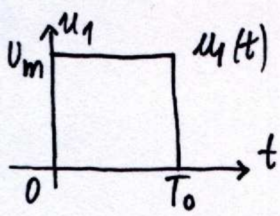
$= \underbrace{0{,}48506\angle{-120{,}964^\circ}}_{-0{,}2496 - j0{,}41593} \Rightarrow I^{(4)}_{R,ef} = \dfrac{0{,}48506}{\sqrt{2}} = 0{,}\underline{\underline{34299}}\,A$

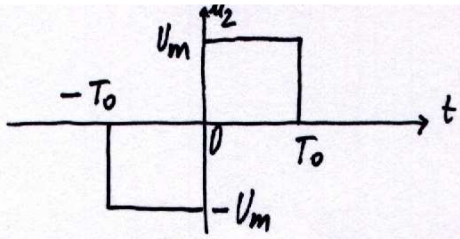
$I_{R,ef} = \sqrt{\frac{1}{T}\int_0^T i_R^2(t)dt} = \sqrt{I_{R0}^2 + \sum_{n=1}^{\infty} I_{R,ef}^2} \approx \sqrt{2^2 + 1{,}802768^2 + 0{,}88483^2 + 0{,}34299^2} =$

$= \sqrt{8{,}150538} = \underline{\underline{2{,}854915\,A}}$

b.) $P_{\check{c}} = R\cdot I_{R,ef}^2 = 20\cdot(2{,}854915)^2 \doteq \underline{163\,W}$

(2.)





snahou je vyjadriť výsledok cez funkciu sínus

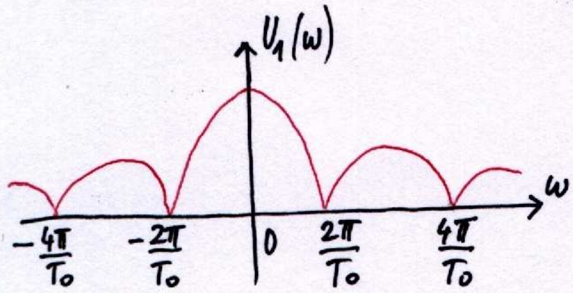
a.) $$U_1(\omega)=\int_0^{T_0} U_m e^{-j\omega t}\,dt = U_m\left[\frac{e^{-j\omega t}}{-j\omega}\right]_0^{T_0} = U_m\,\frac{1-e^{-j\omega T_0}}{j\omega}=$$

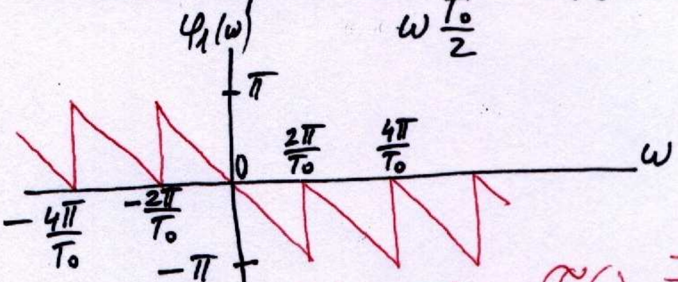
sínus

$$=U_m\cdot T_0\,\frac{e^{j\omega\frac{T_0}{2}}e^{-j\omega\frac{T_0}{2}}-e^{-j\omega\frac{T_0}{2}}e^{-j\omega\frac{T_0}{2}}}{2\cdot j\omega\cdot\frac{T_0}{2}}=U_m\cdot T_0\,\frac{e^{j\omega\frac{T_0}{2}}-e^{-j\omega\frac{T_0}{2}}}{2j\cdot\omega\frac{T_0}{2}}\,e^{-j\omega\frac{T_0}{2}}=$$

$$=U_m T_0\,\frac{\sin\left(\omega\frac{T_0}{2}\right)}{\omega\frac{T_0}{2}}\,e^{-j\omega\frac{T_0}{2}} \Rightarrow$$ modulové spektrum $$U_1(\omega)=\left|U_1(\omega)\right|=U_m T_0\left|\frac{\sin\left(\omega\frac{T_0}{2}\right)}{\omega\frac{T_0}{2}}\right|$$

fázové spektrum $$\varphi_1(\omega)=\arg\{U_1(\omega)\}=\begin{cases}-\omega\frac{T_0}{2} & \text{pre } \dfrac{\sin\left(\omega\frac{T_0}{2}\right)}{\omega\frac{T_0}{2}}>0\\[2ex] -\omega\frac{T_0}{2}+\pi & \text{pre } \dfrac{\sin\left(\omega\frac{T_0}{2}\right)}{\omega\frac{T_0}{2}}<0\end{cases}$$





b.) z vety o posune funkcie na osi t vyplýva: $f(t\pm T_0)\leftrightarrow F(\omega)e^{\pm j\omega T_0}$

potom bude $u_2(t)=u_1(t)-u_1(t+T_0)\longleftrightarrow U_2(\omega)=U_1(\omega)-U_1(\omega)e^{j\omega T_0}=$

$$=U_m\frac{1-e^{-j\omega T_0}}{j\omega}-U_m\frac{1-e^{-j\omega T_0}}{j\omega}e^{j\omega T_0}=U_m\frac{1-e^{-j\omega T_0}}{j\omega}-U_m\frac{e^{j\omega T_0}-1}{j\omega}=$$

$$=U_m\frac{1-e^{-j\omega T_0}}{j\omega}+U_m\frac{1-e^{j\omega T_0}}{j\omega}=U_m\cdot T_0\,\frac{e^{j\omega\frac{T_0}{2}}e^{-j\omega\frac{T_0}{2}}-e^{-j\omega\frac{T_0}{2}}e^{-j\omega\frac{T_0}{2}}}{2\cdot j\omega\cdot\frac{T_0}{2}}+$$

sínus

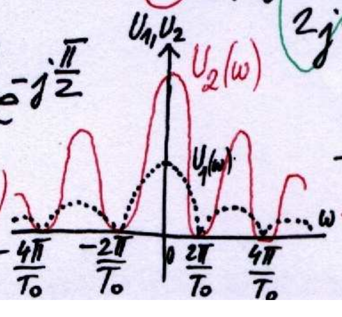
treba vyňať

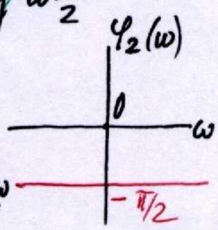
sínus

$$+U_m\cdot T_0\,\frac{e^{j\omega\frac{T_0}{2}}e^{-j\omega\frac{T_0}{2}}-e^{j\omega\frac{T_0}{2}}e^{j\omega\frac{T_0}{2}}}{2\cdot j\omega\cdot\frac{T_0}{2}}=U_m\cdot T_0\,\frac{e^{j\omega\frac{T_0}{2}}-e^{-j\omega\frac{T_0}{2}}}{2j\cdot\omega\frac{T_0}{2}}\,e^{-j\omega\frac{T_0}{2}}+U_m\cdot T_0\,\frac{e^{-j\omega\frac{T_0}{2}}-e^{j\omega\frac{T_0}{2}}}{2j\cdot\omega\frac{T_0}{2}}\,e^{j\omega\frac{T_0}{2}}=$$

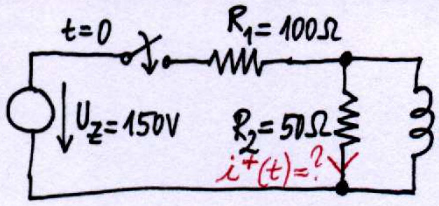
treba vyňať

$$=U_m T_0\cdot\frac{\sin\left(\omega\frac{T_0}{2}\right)}{\omega\frac{T_0}{2}}\Big[\underbrace{e^{-j\omega\frac{T_0}{2}}-e^{j\omega\frac{T_0}{2}}}_{\cancel{\cos(\omega\frac{T_0}{2})}-j\sin(\omega\frac{T_0}{2})-\cancel{\cos(\omega\frac{T_0}{2})}-j\sin(\omega\frac{T_0}{2})=-j2\sin(\omega\frac{T_0}{2})}\Big]=2U_m T_0\cdot\frac{\sin^2\left(\omega\frac{T_0}{2}\right)}{\omega\frac{T_0}{2}}\,e^{-j\frac{\pi}{2}}$$

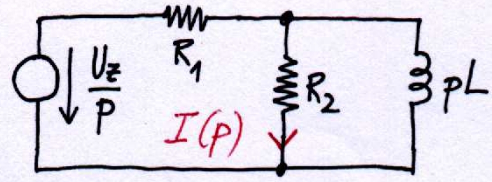




(3.)



→



$$I(p) = \frac{\frac{U_z}{\cancel{p}} \cdot \cancel{p}L}{R_1R_2 + R_1pL + R_2pL} = \frac{U_z \cdot L}{R_1R_2 + p(R_1+R_2)L} = \frac{U_z \cdot \cancel{L}}{(R_1+R_2)\cancel{L}} \cdot \frac{1}{p + \frac{R_1R_2}{(R_1+R_2)L}} \Leftrightarrow$$

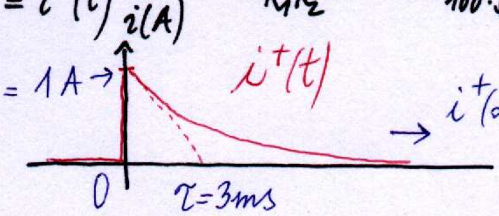
sérioparalelný obvod

$$\Leftrightarrow \frac{U_z}{R_1+R_2} e^{-\frac{R_1R_2}{(R_1+R_2)L}t} \cdot 1(t) = i^+(t) \qquad \tau = \frac{(R_1+R_2)L}{R_1R_2} = \frac{(100+50)0{,}1}{100 \cdot 50} = 3 \cdot 10^{-3} s = 3ms$$

$i(0_-) = 0A$

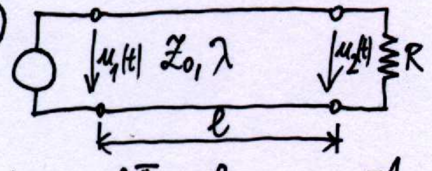
$i(0_+) = i^+(0) = \frac{U_z}{R_1+R_2} = 1A$

$i^+(0) = \frac{U_z}{R_1+R_2} = 1A$ →



→ $i^+(\infty) = 0A$

(4.)



$\beta = 0$ (IHV), $Z_0 = 300\Omega$, $\lambda = 3m$, $\ell = 6.75m$, $R = 100\Omega$

$u_1(t) = 300 \cdot \cos(10^8 t)$ (V)

a.) $\alpha = \frac{2\pi}{\lambda} = \underbrace{\frac{2}{3}\pi}_{2{,}094395} \text{ rad}\,m^{-1}$

b.) $v_f = \frac{\omega}{\alpha} = \frac{10^8}{\frac{2}{3}\pi} = \frac{3 \cdot 10^8}{2\pi} ms^{-1} = 4{,}7746 \cdot 10^7 ms^{-1}$

c.) $\rho_2 = \frac{Z_2 - Z_0}{Z_2 + Z_0} = \frac{R - Z_0}{R + Z_0} = \frac{100-300}{100+300} = \frac{-200}{400} = -0{,}5$

d.) $PSV = \frac{U_{max}}{U_{min}} = \frac{1+|\rho_2|}{1-|\rho_2|} = \frac{1+|-0{,}5|}{1-|-0{,}5|} = \frac{1{,}5}{0{,}5} = 3$

e.) $\gamma = \beta + j\alpha = j\alpha = j\frac{2\pi}{\lambda} = j\frac{2}{3}\pi\ (m^{-1})$

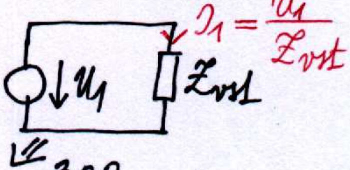
$U_2 = I_2 Z_2$

$$Z_{vst} = \frac{U(\xi=\ell)}{I(\xi=\ell)} = \frac{U_2 \cosh(\gamma\ell) + I_2 Z_0 \sinh(\gamma\ell)}{I_2 \cosh(\gamma\ell) + U_2/Z_0 \sinh(\gamma\ell)} = Z_0 \frac{Z_2 \cosh(\gamma\ell) + Z_0 \sinh(\gamma\ell)}{Z_0 \cosh(\gamma\ell) + Z_2 \sinh(\gamma\ell)}$$

vzťahy od konca

$\cosh(\gamma\ell) = \cosh(j\alpha\ell) = \cos(\alpha\ell) = \cos\left(\frac{2}{3}\pi \cdot 6{,}75\right) = 0$

$\sinh(\gamma\ell) = \sinh(j\alpha\ell) = j\sin(\alpha\ell) = j\sin\left(\frac{2}{3}\pi \cdot 6{,}75\right) = j$



$$Z_{vst} = 300 \cdot \frac{100 \cdot 0 + j\,300 \cdot j}{300 \cdot 0 + j\,100 \cdot j} = 900\Omega \qquad I_1 = \frac{U_1}{Z_{vst}} = \frac{300}{900} = \frac{1}{3} A$$

$$U_2 = U(x=\ell) = U_1 \cosh(\gamma\ell) - I_1 Z_0 \sinh(\gamma\ell) = 300 \cdot 0 - \frac{1}{3} \cdot 300 \cdot j = -j100$$

vzťah od začiatku

$$U_2 = 100 \angle{-90^\circ} \Rightarrow \underline{U_{2m} = 100V}$$