NADPIS:

Digitálne komunikácie

ZAPAMÄTAJTE SI:

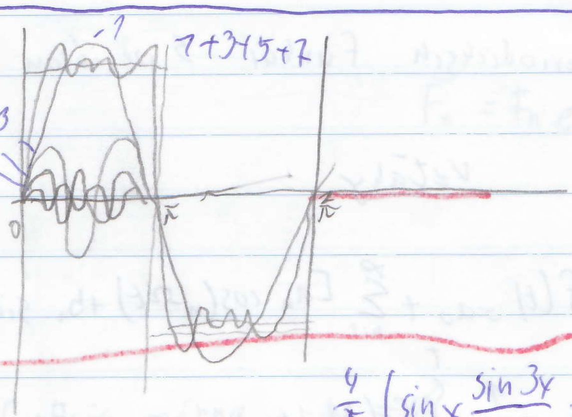
Matúš Turcsány

matus.turcsany@ericsson.com

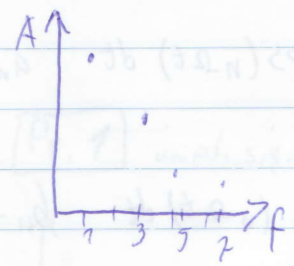
30+70 b.

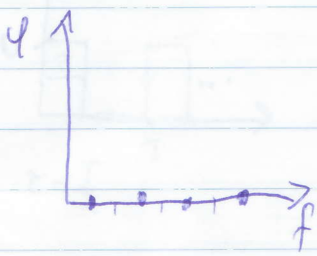
B. Sklar : Digital Comunications

www.complextoreal.com



$$\frac{4}{\pi}\left(\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \frac{\sin 7x}{7}\right)$$





ÚLOHY:

DÁTUM:

PODNADPISY: NADPIS: ZAPAMÄTAJTE SI

signály { náhodné (stokastické) ; nenáhodné (determinické) { periodické FR ; neperiodické FT ; kvázi periodické FR

Fourierove rady periodických funkcií s periódou $T\ (\Omega = 2\pi/T)$

<u>fourierov rad</u> <u>vzťahy</u>

<u>Trigonometrický</u>

$$f(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\Omega t) + b_n \sin(n\Omega t)]$$

$$a_0 = \frac{1}{T} \int_0^T f(t)\, dt \qquad a_0 = F_0$$

$$a_n = \frac{2}{T} \int_0^T f(t) \cos(n\Omega t)\, dt \qquad a_n = \bar{F}_n + \bar{F}_{-n}$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin(n\Omega t)\, dt \qquad b_n = \bar{F}_n - \bar{F}_{-n}$$

<u>Zlúčený</u>

$$f(t) = a_0 + \sum_{n=1}^{\infty} A_n \cos(n\Omega t + \varphi_n)$$

$$A_n = \sqrt{a_n^2 + b_n^2} \qquad A_n = 2|\bar{F}_n|$$

$$\varphi_n = -\mathrm{arctg}\, \frac{b_n}{a_n}$$

ÚLOHY: DÁTUM:

NADPIS: ZAPAMÄTAJTE SI:

Exponenciálny

(komplexný)

$$f(t) = \sum_{n=-\infty}^{\infty} \bar{F}_n e^{jn\Omega t}$$

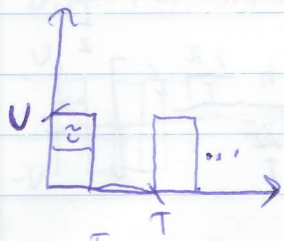
$$\bar{F}_n = \frac{1}{T}\int_0^T f(t)\, e^{-jn\Omega t}\, dt \qquad F_0 = a_0 = A_0$$

$$\bar{F}_n = F_n e^{j\varphi_n} \qquad \bar{F}_n = \frac{1}{2}(a_n - jb_n)$$

funkcia párna $- b_n = \emptyset$

funkcia nepárna $- a_n = \emptyset$

$\boxed{\text{Pr. 1}}$ unipol. signál



$\tau = \frac{T}{2}$

$$f(t) = U \qquad t(0, T/2]$$
$$= 0 \qquad t\left(\tfrac{T}{2}, T\right)$$

$$a_0 = \frac{1}{T}\int_0^T f(t)\, dt = \frac{1}{T}\left[\int_0^{T/2} U\, dt + \int_{T/2}^{T} 0\, dt\right]$$

$$a_0 = \frac{1}{T}\int_0^{T/2} U\, dt = \frac{U}{T}\left[t\right]_0^{T/2} = \frac{U}{T}\left(\frac{T}{2} - 0\right) = \underline{\underline{\frac{U}{2}}}$$

$\cancel{a_0 = \frac{1}{T}\int_0}$

ÚLOHY: DÁTUM:

PODNADPISY: NADPIS: ZAPAMÄTAJTE SI:

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(n\Omega t)\,dt = \frac{2}{T}\int_0^{T/2} U\cos(n\Omega t)\,dt =$$

$$= \frac{2U}{T}\int_0^{T/2}\cos(n\Omega t)\,dt = \frac{2U}{T}\left[\frac{\sin(n\Omega t)}{n\Omega}\right]_0^{T/2} =$$

$$= \frac{2U}{T}\left[\frac{\sin\left(\frac{n2\pi t}{T}\right)}{\frac{n2\pi}{T}}\right]_0^{T/2} = \frac{2U}{T}\cdot\left(\frac{\sin\left(\frac{n2\pi}{T}\frac{T}{2}\right)}{\frac{n2\pi}{T}}\right.$$

$$\frac{2U}{T}\cdot\left(\frac{\sin\left(\frac{n2\pi}{T}\frac{T}{2}\right)}{\frac{n2\pi}{T}} - \frac{\overset{=0}{\sin 0}}{\frac{n2\pi}{T}}\right) = \frac{2U}{T}\left(\frac{\sin n\pi}{\frac{n2\pi}{T}}\right) = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(n\Omega t)\,dt =$$

$$= \frac{2}{T}\int_0^{T/2} U\sin(n\Omega t)\,dt = \frac{2U}{T}\left[-\frac{\cos n\frac{2\pi}{T}t}{n\frac{2\pi}{T}}\right]_0^{T/2} =$$

$$= \frac{2U}{T}\left[-\frac{\cos n\frac{2\pi}{T}\cdot\frac{T}{2}}{n\frac{2\pi}{T}} - \left(-\frac{\cos n\frac{2\pi}{T}\cdot 0}{n\frac{2\pi}{T}}\right)\right] =$$

$$= U\cdot\left(-\frac{\cos n\pi}{n\pi} + \frac{\cos n\frac{2\pi}{T}\cdot 0}{n\pi}\right) =$$

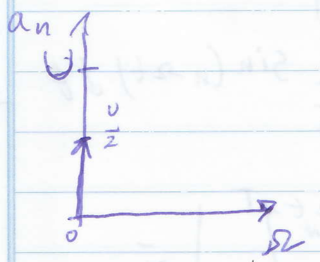
$$= \frac{U}{n\pi}\left(1-(-1)^n\right) = \begin{cases} n - \text{párne} = 0 \\ n - \text{nepárne} = \frac{2U}{n\pi}\end{cases}$$

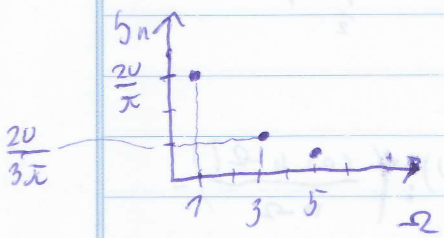
ÚLOHY: DÁTUM:

NADPIS: ZAPAMÄTAJTE SI:

$$f(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\Omega t) + b_n \sin(n\Omega t)]$$

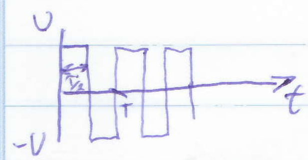
$$f(t) = \frac{U}{2} + \sum_{n=1;3;5}^{\infty} \frac{2U}{n\pi} \sin n\Omega t$$





Pr. 2

Bipolárny signál



$$f(t) = -U; \; t \in (0, \tfrac{T}{2})$$
$$U; \; t \in (\tfrac{T}{2}; T)$$

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt = \frac{1}{T}\int_0^{\frac{T}{2}} U\,dt + \frac{1}{T}\int_{\frac{T}{2}}^{T} -U\,dt =$$

$$= \frac{U}{T}\frac{T}{2} + \left(-\frac{U}{T}\frac{T}{2}\right) = \frac{U}{2} - \frac{U}{2} = 0$$

$$a_n = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(n\Omega t)\,dt = \frac{2}{T}\int_0^{T/2} U\sin(n\Omega t)\,dt =$$

$$= \frac{2U}{T}\left[\frac{\cos(n\Omega t)}{n\Omega}\right]_0^{\frac{T}{2}} = \frac{2U}{T}\;\frac{\cos n\frac{2\pi}{T}\frac{T}{2}}{n\frac{2\pi}{T}} - \frac{\cos n\Omega 0}{n\frac{2\pi}{T}} = \frac{2U}{T}$$

ÚLOHY: DÁTUM:

PODNADPISY: NADPIS: ZAPAMÄTAJTE SI:

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(n\Omega t)\,dt =$$

$$= \frac{2}{T}\left(\int_0^{\frac{T}{2}} U \sin(n\Omega t)\,dt + \int_{\frac{T}{2}}^{T}(-U)\cdot\sin(n\Omega t)\,dt\right) =$$

$$= \frac{2}{T}\left[U\cdot\int_0^{\frac{T}{2}}\sin(n\Omega t)\,dt + (-U)\cdot\int_{\frac{T}{2}}^{T}\sin(n\Omega t)\,dt\right] =$$

$$= \frac{2}{T}\left(U\cdot\left[\frac{\cos n\Omega t}{n\Omega}\right]_0^{\frac{T}{2}} + (-U)\cdot\left[\frac{\cos n\Omega t}{n\Omega}\right]_{\frac{T}{2}}^{T}\right) =$$

$$= \frac{2}{T}\left[U\cdot\left(\frac{\cos n\Omega\frac{T}{2}}{n\Omega} - \frac{\cos n\Omega 0}{n\Omega}\right) + (-U)\cdot\left(\frac{\cos n\Omega T}{n\Omega} - \frac{\cos n\Omega\frac{T}{2}}{n\Omega}\right)\right] = \frac{2}{T}\left[U\cdot\left(\frac{\cos n\pi}{n\frac{2\pi}{T}} - \frac{1}{n\frac{2\pi}{T}}\right) + (-U)\cdot\left(\frac{\cos n\frac{2\pi}{T}\cdot T}{n\frac{2\pi}{T}} - \frac{\cos n\frac{2\pi}{T}\cdot\frac{T}{2}}{n\frac{2\pi}{T}}\right)\right] =$$

$$= \frac{2}{T}\left[U\cdot\frac{\cos n\pi - 1}{n\frac{2\pi}{T}} + (-U)\cdot\frac{\cos n2\pi - \cos n\pi}{n\frac{2\pi}{T}}\right] =$$

$$= \frac{2}{T}\,\frac{U}{n\frac{2\pi}{T}}\left[(\cos n\pi - 1) - (\cos n2\pi - \cos n\pi)\right]$$

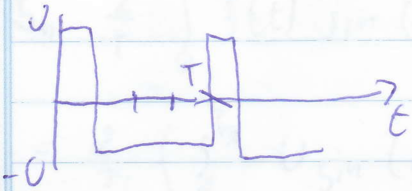
je to asi zle ale na konec má vyzerať takto

$$\frac{2U}{\cancel{T}\,n\frac{2\pi}{\cancel{T}}}\cdot\left(-\cos n\pi + \cos 0 + \cos n2\pi - \cos n\pi\right) =$$

$$= \frac{U}{n\pi}\left(-(-1)^n + 1 + 1 - (-1)^n\right) = \frac{2U}{n\pi}\left(1-(-1)^n\right)\begin{cases} n \text{ párne} = 0 \\ n \text{ nepárne } \frac{4U}{n\pi} \end{cases}$$

ÚLOHY: DÁTUM:

PODNADPISY: DÚ: | NADPIS: | ZAPAMÄTAJTE SI:



$$f(t) = \begin{cases} U & t \in (0, T/4) \\ -U & t \in (T/4, T) \end{cases}$$

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt = \frac{1}{T}\int_0^{T/4} U\,dt + \frac{1}{T}\int_{T/4}^{T} -U\,dt = \frac{U}{T}\frac{T}{4} - \frac{U}{T}\frac{3T}{4} =$$

$$= \frac{U}{4} - \frac{3U}{4} = -\frac{2U}{4} = -\frac{U}{2}$$

$$f(t) = \begin{cases} U, & t \in (0, T/4) \\ -U, & t \in (T/4, T) \end{cases}$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(n\Omega t)\,dt = \frac{2}{T}\int_0^{T/4} U\cos(n\Omega t)\,dt +$$

$$+ \frac{2}{T}\int_{T/4}^{T} (-U)\cos(n\Omega t)\,dt = \frac{2U}{T}\int_0^{T/4}\cos(n\Omega t)\,dt + \frac{2U}{T}\int_{T/4}^{T}\cos(n\Omega t)$$

$$\frac{2U}{T}\left[\frac{\sin n\Omega t}{n\Omega}\right]_0^{T/4} - \frac{2U}{T}\left[\frac{\sin n\Omega t}{n\Omega}\right]_{T/4}^{T} =$$

$$= \frac{2U}{T}\left[\frac{\sin n^2 \frac{2\pi}{T}\frac{T}{4}}{n\frac{2\pi}{T}}\right] - \frac{2U}{T}\left[\frac{\sin n\frac{2\pi}{T}T}{n\frac{2\pi}{T}} - \frac{\sin \frac{2\pi}{T}\cdot\frac{T}{4}}{n\frac{2\pi}{T}}\right] =$$

$$= \frac{U}{n\pi}\left(\sin\frac{\pi}{2}\right) - \frac{U}{n\pi}\left(\sin n\,2\pi - \sin n\frac{\pi}{2}\right) =$$

$$= \cancel{\frac{4U}{n\pi}}\ \frac{2U}{n\pi}\left(\sin n\frac{\pi}{2}\right)$$

ÚLOHY: | DÁTUM:

PODNADPISY:

NADPIS:

$$b_n = \frac{2}{T}\int_0^{T/4} U \sin(n\omega t)\, dt = ? \quad \ldots \quad \underline{b_n = \frac{2U}{n\pi}\left(1 - \cos\frac{n\pi}{2}\right)}$$

ZAPAMÄTAJTE SI:

ÚLOHY:

DÁTUM:

27.02.09 | PODNADPISY:

NADPIS: DK

ZAPAMÄTAJTE SI:

$$f(t) = \sum_{n=-\infty}^{\infty} \bar{F}_n e^{jn\Omega t} \qquad a_n = \cancel{\frac{2U}{n\pi}}$$

$$\bar{F}_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-jn\Omega t}\, dt$$

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-jn\Omega t}\, dt\; e^{jn\Omega t}$$

$T = \frac{2\pi}{\Omega}$

$$f(t) = \sum_{n=-\infty}^{\infty} e^{jn\Omega t}\, \frac{\Omega}{2\pi}\int_{-T/2}^{T/2} f(t)\, e^{-jn\Omega t}\, dt$$

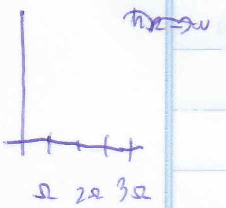
$\boxed{T \to \infty}$

$\Omega = \frac{2\pi}{T} \to 0$

1, $\Omega \to d\omega$

2, $n\Omega \to \omega$

3, $\sum \to \int$



$$f(t) = \int_{-\infty}^{\infty} e^{j\omega t}\, \frac{d\omega}{2\pi}\int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\underbrace{\left[\int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt\right]}_{F(\omega)} e^{j\omega t}\, d\omega$$

^ priam furierove transf.

$\cancel{f(t) = \int_{-\infty}^{\infty} e^{j\omega t}}$

priama f. transformácia

$$F_\omega = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

spetná f. transformácia

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega$$

ÚLOHY:

DÁTUM:

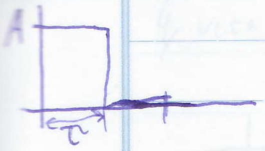
PODNADPISY: NADPIS: ZAPAMÄTAJTE SI:

$$\sin x = \frac{e^{jx}-e^{-jx}}{2j}$$

$$\cos x = \frac{e^{jx}+e^{-jx}}{2}$$

$\frac{\sin x}{x}$ - si(x)

Si(x)



$f(t) = A \quad t \langle 0, \tau \rangle$

$= 0$ inak

$$F(\omega) = \int_{-\infty}^{\infty} A \cdot e^{-j\omega t}\, dt = A \int_0^{\tau} e^{-j\omega t}\, dt = A\left[\frac{e^{-j\omega t}}{-j\omega}\right]_0^{\tau} =$$

$$= A\left(\frac{e^{-j\omega\tau}}{-j\omega} - \frac{e^{-j\omega 0}}{-j\omega}\right) = \frac{A}{j\omega}\left(1 - e^{-j\omega\tau}\right) \quad / \cdot \frac{e^{-j\omega\tau/2}}{e^{-j\omega\tau/2}}$$

$$e^{-j\omega\tau/2}\ \frac{A}{j\omega}\left(\frac{1}{e^{-j\omega\tau/2}} - \frac{e^{-j\omega\tau}}{e^{-j\omega\tau/2}}\right) e^{-j\omega\tau/2} =$$

$$= \frac{A}{j\omega}\left(e^{j\omega\tau/2} - e^{-j\omega\tau/2}\right) e^{-j\omega\tau/2} \cdot \frac{2}{2} =$$

$$2 = \frac{1}{\frac{1}{2}}$$

$$= \frac{2A}{\omega}\left(\sin\left(\omega\frac{\tau}{2}\right)\right) e^{-j\omega\tau/2} \cdot \frac{\tau}{\tau} =$$

$$= A\tau \frac{\sin \omega\tau/2}{\omega\tau/2} e^{-j\omega\tau/2} = F(\omega)$$

$$|F(\omega)| = A\tau\, \mathrm{si}\left(\omega\frac{\tau}{2}\right)$$

ÚLOHY: DÁTUM:

PODNADPISY: NADPIS: Vlastnosti F. transformácie ZAPAMÄTAJTE SI

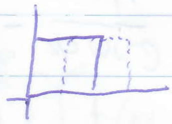
1/ Posunutie vo frekvencii

$$x(t)\, e^{j2\pi f_0 t} \xrightarrow{FT} F_x(\omega-\omega_0)$$

$\omega_0 = 2\pi f_0$

2/ posunutie v čase

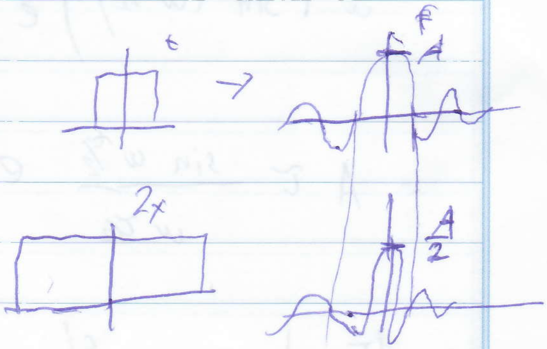
$$x(t-t_0) \to F(\omega)\, e^{-j\omega t_0}$$



3/ Zmena mierky $\quad x(at) \to \frac{1}{a} F\left(\frac{\omega}{a}\right)$

$a > 1 \to$ trvanie je dlhšie

$a < 1 \to$ trvanie je kratšie

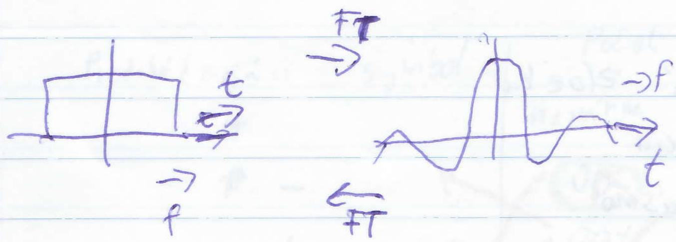


ÚLOHY: DÁTUM:

NADPIS:

ZAPAMÄTAJTE SI:

4 veta o dualite signálov spektier



ÚLOHY:

DÁTUM:

06.03.09.

PODNADPISY: NADPIS: Traslačné kódy ZAPAMÄTAJTE SI:

- kódy pre kanál
- minimalizujú jednosmernú zložku
- podporujú synchronizáciu
- upravujú frekvenčné pásmo

charakteristyky - Disparita - (rozdiel medzi počtom 1 a 0 v rámci slova)
- RDS (Running Digital Sum) - okamžitá disparita od zač. prenosu
- DSV (Digital sum Value) - súčet úrovní celej postupnosti

Empirické translačné kódy

- AMI Alternate Mark Inversion
- Manchester
- HDB_n (High Densiti Bipolar)
- B_nZS (Bipolar with n zeros substitution)

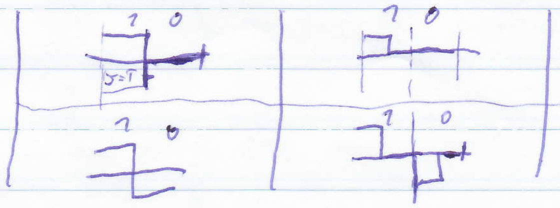
Zákl char < unipolar / bipolar

NRZ | RZ - ret to zero

1 0

charakteristický interval

	NRZ	RZ
unipola	1 0	1 0
bipolar	1 0	1 0



ÚLOHY: DÁTUM:

NADPIS: ZAPAMÄTAJTE SI:

B_3ZS

Predchádzajúci symbol	Počet impulzov nepárne	párne
#		
# −	00−	+0+
+	00+	−0−

B3ZS

1 0 1 000 1 1 000 000 001 0001

výst $\boxed{+0-00- + - +0+ -0- 00+ 00+-}$

0. 1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11.

prijm. 101 000 1 1 000 000 001 0001

Teória translačných kódov

Uvažujeme kanál bez šumu, pamäti s obmedzením

$\Downarrow$

$P_e = 0$

beh = run = 2   0011 0101   (max 2x 0 alebo 2x 1)

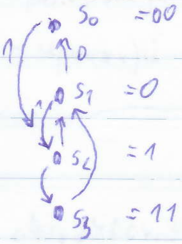
spôsob špecifikácie
- slovne
- stavový diagram
- mriežku
- maticu susednosti

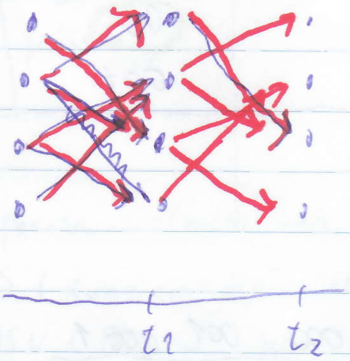
ÚLOHY: DÁTUM:

PODNADPISY: NADPIS: ZAPAMÄTAJTE SI:

Príklad

Zostrojte translačný kód kde max. beh = 2





$$B^1 = \begin{array}{c|cccc} & s_0 & s_1 & s_2 & s_3 \\ \hline s_0 & 0 & 0 & 1 & 0 \\ s_1 & 1 & 0 & 1 & 0 \\ s_2 & 0 & 1 & 0 & 1 \\ s_3 & 0 & 1 & 0 & 0 \end{array} \quad \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$B^2. \quad \begin{pmatrix} 2 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

A: $C = \lim_{\tau \to \infty} \frac{\log_2 M(\tau)}{\tau}$ [bit/s]

B: $C' = \lim_{n \to \infty} \frac{\log_2 M(n)}{n}$ [bit/symbol]

ÚLOHY: DÁTUM:

C: výpočet kapacity kanála – maximum koreňov vlastnej rovnice

$C' = \log_2 \max(\mathrm{root}([B - I \cdot \lambda]))$

A: $M(\tau)$ => počet dovolených sledov (signálov/s prototypov) ktorí trvajú čas τ -> doba za ktorú systém sekvenciu alebo signál

B: $M(n)$ => počet dovolených kódových postupností

n -> dĺžka kódového slova (počet symbolov v slove)

Blokový translačný kód

k symbolov → kóder → n symbolov

$(n-k)$

rýchlosť symbolov $= \frac{k}{n}$

$$R_k = \frac{k}{n} \qquad (n,k)$$

C: B - matica

I - jednotková matica

B1 - násobiť tarže

$$\cancel{B \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{pmatrix}}$$

$$B_1 \begin{pmatrix} -\lambda & 0 & 1 & 0 \\ 1 & -\lambda & 1 & 0 \\ 0 & 1 & -\lambda & 1 \\ 0 & 1 & 0 & -\lambda \end{pmatrix}$$

$[B - \lambda]$

$$[B - I\lambda]$$

$$\lambda^4 - \lambda^2 - 2\lambda - 1 = 0$$

$$\max \lambda = 1{,}618$$

$$C' = \log_2 1{,}618 = 0{,}6942 \; [\text{bit/symbol}]$$

PODNADPISY: NADPIS: ZAPAMÄTAJTE SI:

$k \to \boxed{k} \to R_k \quad \frac{k}{n} = \frac{6}{10}; \quad \frac{69}{100}$

$(10,6)$

Blokový (n,k) kód pre kanál s obmedzením opísaný pre maticou B neexistuje dovtedy kým suma každého riadku B^n nenadobudne hodnotu $\geq 2^k$, alebo kým neobsahuje podmaticu získanú vyčiarknutím jej ľubovoľného radu a príslušného stĺpca (odstránením určitých stavov), ktorá by spĺňala uvedenú vlastnosť.

Dôsledok: ak hľadáme konkrétny kód špecifikovaný rýchlosťou $R_k = \frac{k}{n}$ a maticou susednosti B vypočítame takú maticu B^n, pre ktorú podľa predchádzajúcej vety platí:

$$\sum_{j}^{M} b_{i,j}^{(n)} \geq 2^k, \forall i \in M$$

$\boxed{PR}$

Zostrojte $(3,2)$ kód s obmedzením nie viac ako 2×0 za sebou

$R_k = \frac{k}{n} = \frac{2}{3}$

$\xrightarrow{2} \boxed{k} \xrightarrow{3} 2^3$

2^2

00	→ 010	~~000~~
01	→ 011	~~001~~
10	→ 101	010
11	→ 110	011
		~~100~~
		101
		110
		111

ostal voľný

ÚLOHY: DÁTUM:

NADPIS: ZAPAMÄTAJTE SI:

Freiman Wyner metóda

ak je n		prefix	sufix
	párne	$\frac{n}{2}$	$\frac{n}{2}$
	nepárne	$\frac{n-1}{2}$	$\frac{n+1}{2}$

Pr: Zostrojte kód s max sekom 3

vstupné dvojice	000	00	0	1	11	111
	počiatočný stav					
00	–	–	000	00	00	00
01	–	1	1	1	1	1
10	0	0	0	0	0	–
11	11	11	11	111	–	–

Terminálny stav

ešo prefix

Pr.

~~000~~
|01, 001|
010
011
100
101
110
~~111~~

pref. sufix
|10, 001|
010
011
100
101
110

$k=3 \to \boxed{k} \to n=5$

2^3

000 → 01.001
001 → 01.010
010 → 01.011
011 → 01.100
100 → 01.101
101 → 01.110
110 → 10.001
111 → 10.010

ÚLOHY: DÁTUM:

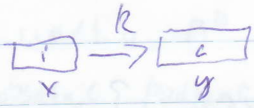
Pi. 13. 03. 2009

Samoopravné kódy

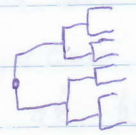
Lineárne Blokové Kódy

kódy < lin < stromové, blokové; nelin

Blokové



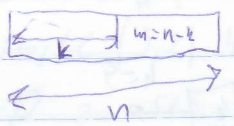
Stromové



Def. lin. blok. kódu

- je k-rozmerný podpriestor n-rozmerného lin. priestoru zostrojeného nad konečným poľom GF(q)

GF(2) < 0, 1



n=3 k=1 < |0|, |1| - dĺžka

000 → 0
001
010
011
100
101
110
111 → 1

PODNADPISY: NADPIS: ZAPAMÄTAJTE SI:

G - generujúca matica

$$G = \begin{pmatrix} g_1 \\ \vdots \\ g_k \end{pmatrix} \qquad (g_k) = n$$

$k \times n$

$$G = \left(\begin{array}{cccc|cccc} 1 & 0 & \dots & 0 & p_{11} & p_{12} & \dots & p_{1m} \\ 0 & 1 & \dots & 0 & k_{21} & p_{22} & \dots & p_{2m} \\ \vdots & & & & & & & \\ 0 & 0 & \dots & 1 & p_{k1} & p_{k2} & \dots & p_{km} \end{array}\right)$$

I_k $\qquad p = k \times m$

$(1 \times n) \quad (1 \times k)$

$c = i \cdot G$

H - kontrolná matica

$$H = p^T \mid I_{m=n-k}$$

$G \cdot H^T = 0$

$c = i \cdot G$

$c \cdot H^T = i \cdot G \cdot H^T = \underline{0}$

c	0	0	0
e	0	1	0
v	0	1	0

$v = c + e$

$$V \cdot H^T = (c+e) \cdot H^T = cH^T + eH^T = 0 + \underset{\text{sindróm (S)}}{\underline{S}}$$

e	$S = eH^T$
100	xxx
010	444
001	727

ÚLOHY: DÁTUM:

LB4 - lin. blok. kódy

$i_1 \longrightarrow c_1$

$i_2 \longrightarrow \underset{\sim}{\overset{+}{c_2}}$

$i_3 = i_1 + i_2 \to c_3$

w - váha kódového slova (počet nenulových prvkov)

$\boxed{0111\ 00\ 11}$ $w(c_1) = 5$

vieme určiť vzdialenosť 2 slov.

$d(u,v) = w(u+v)$

$d_{min} = 2t+1 \qquad d_{min} \leq n-k+1$

$t = \frac{d_{min}-1}{2}$

(n, k, d_{min})

$(7,4,3)$

$$G \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

písm.	$( )_{10}$	i (bin)	$c = i \cdot G$	w
A	1	0001	0101001	3
B	2	0010	0001110	3
C	3	0011	0100111	4
D	4	0100	0111010	4
E	5	0101	0010011	3
F	6	0110	0110101	3
G	7	0111	0011101	4
H	8	1000	1100010	3
I	9	1001	1001011	4
J	10	1010	1101100	4
K	11	1011	1000011	3
L	12	1100	1011000	3
M	13	1101		
N	14	1110		
O	15	1111		
		0000	0000000	

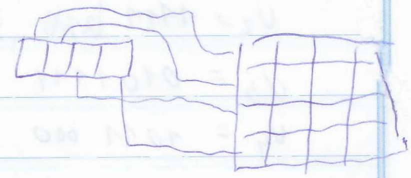
NADPIS:

ZAPAMÄTAJTE SI:

LB4

$$\begin{pmatrix} 1100010 \\ 0111010 \\ 0001110 \\ 0101001 \end{pmatrix}$$

0101 001



$$\begin{array}{c} P \\ G = \begin{pmatrix} 1000 & 011 \\ 0100 & 111 \\ 0010 & 110 \\ 0001 & 101 \end{pmatrix} \end{array}$$

	desiatkovo	i	c = i . G	w
A	1	0001	0001 101	3
B	2	0010	0010 110	3
C	3	0011	0011 011	4
D	4	0100	0100 111	4
E	5	0101	0101 010	3
F	6	0110	0110 001	3
G	7	0111	0111 100	4
H	8	1000	1000 011	3
I	9	1001	1001 110	4
J	10	1010	1010 101	4
K	11	1011	1011 000	3
L	12	1100	1100 100	3
M	13	1101	1101 001	4
N	14	1110	1110 010	4
O	15	1111	1111 111	7

v_1 = 0110110 →

v_2 = 1111 011

ÚLOHY: v_3 = 010 1111

DÁTUM:

v_4 = 1011000

v_5 = 1001101

PODNADPISY: NADPIS:

$G = I/P$

$v_1 = 0110110$

$v_2 = 1111\ 011$

$v_3 = 0101111$

$v_4 = 1011\ 000$

$v_5 = 1001101$

$$P = \begin{pmatrix} 0&1&1 \\ 1&1&1 \\ 1&1&0 \\ 1&0&1 \end{pmatrix}$$

$$P^T = \begin{pmatrix} 0&1&1&1 \\ 1&1&1&0 \\ 1&1&0&1 \end{pmatrix}$$

$$H = \left(\begin{array}{cccc|ccc} 0&1&1&1&1&0&0 \\ 1&1&1&0&0&1&0 \\ 1&1&0&1&0&0&1 \end{array}\right)$$

$$H^T = \left(\begin{array}{ccc} 0&1&1 \\ 1&1&1 \\ 1&1&0 \\ 1&0&1 \\ \hline 1&0&0 \\ 0&1&0 \\ 0&0&1 \end{array}\right) \begin{array}{l} P \\ \\ I_3 \end{array}$$

Sindrómová tabuľka

e	$S = C.H^T$
1 000 000	011
0100 000	111
0010 000	110
0001 000	101
0000100	100
0000010	010
0000001	001

$v_1 H^T = (c_1 + e).H^T = S$

$v_1 H^T = 111$

$\hat{c}_1 = v_1 + e_1$

$\hat{c}_1 = 0010110$ $\boxed{B}$

$S_2 = v_2 H^T = 100$

$S_2 = 1111\ 111$ $\boxed{O}$

$S_3 = 101$

ÚLOHY: DÁTUM:

$S_3 = 0100\ 111$ $\boxed{D}$

NADPIS: Dě. CW. 20.03.09

Cyklické kódy (n, k, d_{min})

$c^1 = c_1\ c_2\ c_3\ c_4$

$c^2 = c_4\ c_1\ c_2\ c_3$

$c = 0110$

$c = x^2 + x$

$c(x) = c_{n-1} \cdot x^{n-1} + c_{n-2} \cdot x^{n-2} + \dots + c_1 x^1 + c_0 x^0$

$i(x) = i_{k-1} x^{k-1} + \dots + i_1 x + i_0$

$g(x)$

$\oplus$ mod 2

$\boxdot$ mod x^{n-1}

nesyst.

$c(x) = i(x) g(x)$

Syst.

$\boxed{1011}\ 011 = \hat{v} = v + e = c + e$

$\to i$

$c = i/h$

ÚLOHY: DÁTUM:

nesyst. $c(x) = i(x)\, g(x)$

syst $(7,4,3)$

$i = 1001 \Rightarrow i(x) = x^3+1$

$g(x) = x^3+x+1$

$c(x)$

1	0	0	1			
x^6	x^5	x^4	x^3	x^2	x^1	x^0

mod - zvyšok po delení

$i(x) \cdot x^{3} \rightarrow \deg(g(x))$

$= x^6 + x^3$

$i(x) \cdot x^{\deg(g(x))} : g(x)$

$i(x) \cdot x^{\deg(g(x))} \bmod g(x) = Q(x)$

$c(x) = i(x) + Q(x)$

1, $i(x)\, x^{\deg(g(x))}$

2, $i(x) \cdot x^{\deg(f(x))}$

3 $c(x) = i(x)\, x^{\deg(g(x))} + Q(x)$

$(7,4,3)$

$g(x) = x^3 + x + 1$

	i bin	i(x)	c(x) nesyst.	c(x) syst.	
A	0001	1	x^3+x+1	x^3+x+1	0001 011
B	0010	x	x^4+x^2+x	x^4+x^2+x	0010 110
C	0011	$x+1$	x^4+x^3+x+1	$x^4+x^3+x^2+1$	0011101
D	0100	x^2	$x^5+x^3+x^2$	x^5+x^2+x+1	0100111
E	0101	x^2+1	x^5+x^2+x+1	$x^5+x^3+x^2$	0101100
F	0110	x^2+x	x^5+x^4	x^5+x^4+1	0110001
G	0111	x^2+x+1	x^5+x^4+1		
H					
9 I					
10 J					
1 K					
2 L					
3 M					
4 N					
5 O					

1 $\quad dg(g(x)) = 3$

$1 \cdot x^3 = x^3$

$x^3 : (x^3 + x + 1) = 1$

$-(x^3 + x + 1)$

$x + 1 \; - Q(x)$

$c(x) = x^3 + x + 1$

$x \cdot x^3 = x^4$

$x^4 : (x^3 + x + 1) = x$

$x^4 + x^2 + x$

$x^2 + x = Q(x)$

$c(x) = x^4 + x^2 + x$

C -> $(x^3 + x + 1) \cdot (x + 1) = x^4 + x^2 + x + x^3 + x + 1 = x^4 + x^3 + x^2 + 2x + 1$
$\quad 0$

$x + 1 \cdot x^3 = \boxed{x^4 + x^3} : x^3 + x + 1 = x + 1$

$(x^4 + x^2 + x)$

$x^3 + x^2 + x$

$(x^3 + x + 1)$

$(x^2 + 1) = Q(x)$

$c(x) = x^4 + x^3 + \cancel{x^2} + 1$

PODNADPISY: NADPIS: ZAPAMÄTAJTE SI

$6 \Rightarrow (x^2+x+1)(x^3+x+1) = x^5+x^3+x^2+x^4+x^2+x+x^3+x+1 = \boxed{x^5+x^4+1}$

$(x^2+x+1)\cdot x^3 = x^5+x^4+x^3 = x^5+x^4+x^5$

$(x^5+x^4+x^3) : (x^3+x+1) = x^2+x$

$x^5+x^3+x^2$

x^4+x^2

x^4+x^2+x

$x = Q(x)$

$c(x) = x^5+x^4+x^3+x$

prijaté

$\cancel{x} \quad V(x) = c(x) + e(x)$

$V(x) \bmod g(x) = (c(x)+e(x)) \bmod g(x) =$

$= \underbrace{c(x) \bmod g(x)}_{0} + \underbrace{e(x) \bmod g(x)}_{s(x)}$ → syndróm

$[i(x)\, g(x)] \bmod g(x) = 0$

ÚLOHY: DÁTUM:

NADPIS: Pr.

$(7,4,3)$ $g(x) = x^3+x+1$

$v_1(x) = x^5+x^3+1$

$v_2(x) = x^5+x^4+x^3+x^2$

$v_3(x) = x^6+x^5+x^4$

$v_4(x) = x^5+x^3+x+1$

Úloha: Dekódovať...

e(x)	s(x)
x^6	x^2+1
x^5	x^2+x+1
x^4	x^2+x
x^3	$x+1$
x^2	x^2
x	x
1	1

$x^6 : x^3+x+1 = x^3+x+1$

x^4+x^3

x^3+x^2+x

x^2+1

$x^5 : (x^3+x+1) = x^2+1$

x^3+x^2

x^2+x+1

$x^4 : x^3+x+1 = x$

x^2+x

$x^3 : x^3+x+1$

$v_1(x) \bmod g(x)$

$x^5+x^3+1 : x^3+x+1 = x^2$

$\underline{x^5+x^3+x^2}$

$x^2+1 \rightarrow e(x) = x^6$

$\hat{v}_1 = v_1(x)+x^6$

$\hat{v}_1(x) = x^6+x^5+x^3+1$

$\underline{1101}|001$

M

$v_2(x) \bmod g(x)$

$(x^5+x^4+x^3+x^2) : (x^3+x+1) = x^2+x$

$x^5+x^3+x^2$

x^4

$x^4+(x^2+x) \Rightarrow e(x) = x^4$

$\hat{v}_2(x) = v_2(x)+x^4$

$\underline{0101}|100$

E

ÚLOHY: ZAPAMÄTAJTE SI: DÁTUM:

PODNADPISY: NADPIS: ZAPAMÄTAJTE SI

V_3: $(x^6+x^5+x^4) : (x^3+x+1) = x^3+x^2$

x^5+x^3

$x^2 \to e(x) = x^2$

$\underline{1110}|\boxed{1}00$

(N)

$V_4(x)$ $(x^5+x^3+x+1) : (x^3+x+1) = x^2$

$x^2+1+x \to e(x) = x^5$

$0001|011$

A

E

syst $C(x) = x^5+x^3+x^2$

$g(x) = x^3+x+1$

$(7,4,3)$

nesyst $C(x) = x^5+x^2+x+1$

$\underline{0101}|100$

$\downarrow$

5. E

$C(x) : g(x)$

$(x^5+x^3+x^2) : (x^3+x+1) = x^2$

0

$\downarrow$

D

$x^5+x^2+x+1 : x^3+x+1 = x^2+1$

x^3+x+1

$\searrow$ E

ÚLOHY: DÁTUM:

27.03.2009

NADPIS: konečné polia

ZAPAMÄTAJTE SI:

PODNADPISY:

GF(2)

Galois

+	0	1
0	0	1
1	1	0

·	0	1
0	0	0
1	0	1

GF(q) — q - prvočíslo, mocnina prvočísla

GF(4)		GF(q)
$\alpha^{-\infty} = 0$		$\alpha^{-\infty}$
$\alpha^0 = 1$		α^0
α^1		α^1
α^2		$\vdots$
		α^{q-2}

primit. prvok: α

$\alpha^n = 1$

$n = q-1$

generujúci polynóm polá

$p(x)$ ireducibilný (x^3+x+1)

$(x^n - 1) \bmod p(x) = 0$

$n = 2^r - 1$ $\quad 2^r$ $\quad GF(2^r)$

PR1

GF(4) $\quad p(x) = x^2+x+1$

bin	α^x	polyn
00	0	0
01	1	1
10	α	x
11	α^2	$x^2 = x+1$

$x^2 \bmod p(x)$

$x^2 : (x^2+x+1) = 1$

x^2+x+1

$\overline{x+1}$

ÚLOHY:

DÁTUM:

PODNADPISY: NADPIS:

alebo $p(x) = x^2 + x + 1 \Rightarrow x^2 = x + 1$

bin	α^x	polyn
00	0	0
01	1	1
10	α	x
11	α^2	$x+1$
	α^3	$x(x+1) = x^2 + x = 1$

GF(8) $p(x) = x^3 + x + 1 \Rightarrow x^3 = x + 1$

Bin	α	polyn.
000	0	0
001	1	1
010	α	x
100	α^2	x^2
011	α^3	$x^3 = x + 1$
110	α^4	$x^4 \to x^2 + x$
111	α^5	$x^5 \to x^3 + x^2 \to x^2 + x + 1$
101	α^6	$x^6 \to x^3 + x^2 + x \to x^2 + 1$

$\alpha^6 + \alpha^4 = \begin{array}{r} 110 \\ +101 \\ \hline 011 \end{array} = \alpha^3$

$\cancel{x^2} + x + \cancel{x^2} + 1 = x + 1 = \alpha^3$

$\alpha^3 - \alpha^{11} = \alpha^3 - \alpha^4 = \begin{array}{r} 011 \\ 110 \\ \hline 101 \end{array} = \alpha^6$

ÚLOHY: DÁTUM:

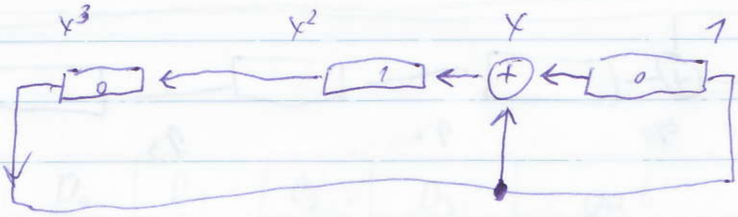
$\alpha^x = \alpha^{x \bmod (q-1)}$

SI: PODNADPISY: NADPIS: ZAPAMÄTAJTE SI:

$\alpha^8 \cdot \alpha^3 = \alpha^{11} = \alpha^4$

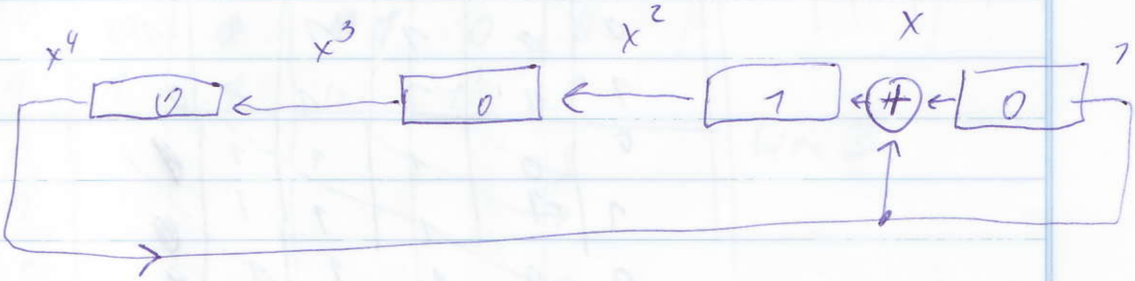
$\alpha^3 : \alpha^{18} = \alpha^{-15} = \alpha^{(-1+7)} = \alpha^6$

nakreslite hardwareovú realizáciu



krok	prvok
0	010
1	100
2	011
3	110

GF(16) $\quad p(x) = x^4 + x + 1$



krok	prvok
$-\infty$	0000
0	0010
1	0100
2	1000
3	0011
4	0110
5	1100
6	1011
7	
8	
9	
10	

11	
12	
13	
14	
15	
16	

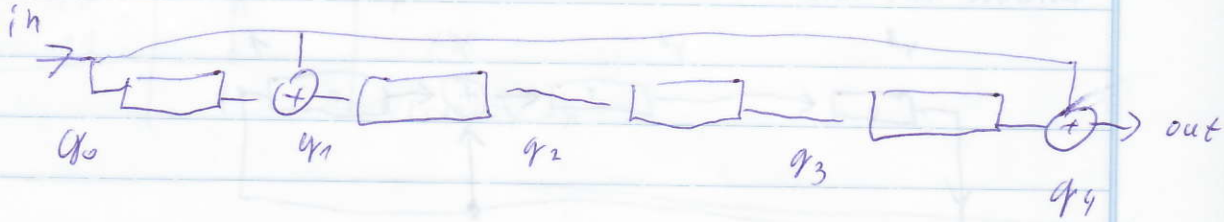
ÚLOHY: DÁTUM:

$c^{(1)}$

CK — nesyst. $c(x) = i(x)\,g(x)$

— syst. $c(x) = i(x) \cdot x^{\deg(g(x))} + i(x)\, x^{\deg(g(x))} \mod g(x)$

① $(15, 11, 3)$ $\quad g(x) = x^4 + x + 1$



n | k | d min
15, 11, 3

(1)

x^{10} 1

x^8 001

pr. $i(x) = x^{10} + x^8 + x^6 + x^4 + x^2 + 1$

1 0 1 0 1 0 1 0 1 0 1

~~$D_0 = in$~~
$D_1 = In + D_0$

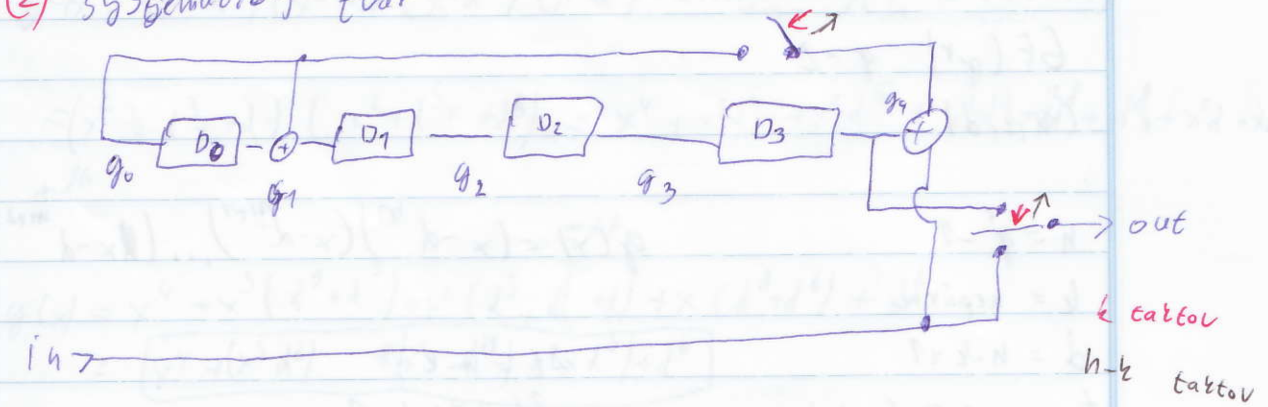
$D_0 = in$
$D_1 = in + D_0$
$D_2 = D_1^{-1}$
$D_3 = D_2^{-1}$
$out = in + D_3^{-1}$

in	D_0	D_1	D_2	D_3	out
0	0	1	1	1	1
0	0	0	1	1	1
0	0	0	0	1	1
0	0	0	0	0	1

in	D_0	D_1	D_2	D_3	out	
~~1~~	0	0	0	0		
1	1	1	0	0	1	x^{14}
0	0	1	1	0	0	x^{13}
1	1	1	1	1	1	x^{12}
0	0	1	1	1	1	
1	1	1	1	1	0	
0	0	1	1	1	0	
1	1	1	1	1	0	
0	0	1	1	1	0	
1	1	1	1	1	0	

NADPIS:

(2) systematický tvar



in	D_0	D_1	D_2	D_3	out
	0	0	0	0	
1	1	1	0	0	1
0	0	1	1	0	0
1	1	1	1	1	1
0	1	0	1	1	0
1	0	1	0	1	1
0	1	1	1	0	0
1	1	0	1	1	1
0	1	0	0	1	0
1	0	1	0	0	1
0	0	0	1	0	0
1	1	1	0	1	1
0					
0					
0					
0					

fáza 2

$D_0 = in + D_3^{-1}$

$D_1 = in + D_3^{-1} + D_0^{-1}$

$D_2 = D_1^{-1}$

$D_3 = D_2^{-1}$

out =

ÚLOHY:

ZAPAMÄTAJTE SI:

DÁTUM:

PODNADPISY: NADPIS: kodovanie $\boxed{\text{RS-kódy}}$

ZAPAMÄTAJTE SI:

$GF(q^r)$ $q=2$

(n,k,d)

$n = q^r - 1$

$k =$ nepárne

$d = n-k+1$

t - počet chýb $\left(\frac{n-k}{2}\right)$

$g(x) = (x-\alpha^m)(x-\alpha^{m+1}) \dots (x-\alpha^{m+2t-1})$

$2t = n-k$

$2t-1 = n-k-1$

$m=0$

nesyst. $c(x) = i(x)\, g(x)$

Syst. $c(x) = i(x)\, x^{\deg(i(x))} + Q(x)$ mod $g(x)$

Príklad

$(\overset{n}{7},\overset{k}{3},5)$, $GF(8)$, $p(x) = x^3+x+1$

$$i_1 \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \qquad i_2 \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

$x^2\ x^1\ x^0$

bin	pol x	α
000	0	0
001	1	1
010	x	α^1
100	x^2	α^2
011	$x+1$	α^3
110	x^2+x	α^4
111	x^2+x+1	α^5
101	x^2+1	α^6

$i(x) = \alpha^3 x^2 + \alpha^5 x^1 + 1$

$i_2(x)\ \alpha^6 x^2 + \alpha^4 x + \alpha^3$

$\cancel{g(x) = (x-1)\cdot(x-\alpha^1) =}$

$\cancel{= x^2 + x\alpha + x + \alpha^1 =}$

DÁTUM:

ÚLOHY:

$g(x) = (x-\alpha^0)(x-\alpha^1)(x-\alpha^2)(x-\alpha^3) = (x^2 + x\alpha + x + \alpha)(x^2\ x\alpha^3 + x\alpha^2 + \alpha^5) =$

$= (x^2 + \alpha^3 x + \alpha)(x^2 + \alpha^5 x + \alpha^5) = x^4 + x^3\alpha^5 + x^2\alpha^5 + x^3\alpha^3 + x\alpha^8 + \alpha^8 x + x^2\alpha + x\alpha^6 +$
$+ \alpha^6$

$\overset{\text{mod } 7}{=}$

$g(x) = x^4 + x^3(\alpha^5 + \alpha^3) + x^2(\alpha^5 + \alpha^8 + \alpha) + x(\alpha^8 + \alpha^6) + \alpha^6$
$= \boxed{x^4 + (x^3\alpha^2) + (x^2\ \alpha^5) + (x\alpha^5) + \alpha^6}$

$c_1(x) = i_1(x) \cdot g(x) = (x^2\alpha^3 + x^1\alpha^5 + 1) \cdot (x^4 + x^3\alpha^2 + x^2\alpha^5 + x\alpha^5 + \alpha^6 =$
$= x^6\alpha^3 + x^5\alpha^5 + x^4\alpha^8 + x^3\alpha^8 + x^2\alpha^9 + x^5\alpha^5 + x^4\alpha^7 + x^3\alpha^{10} + x^2\alpha^{10} + x\alpha^{11} +$
$x^4 + x^3\alpha^2 + x^2\alpha^5 + x\alpha^5 + \alpha^6 = x^6(\alpha^3) + x^5(\alpha^5 + \alpha^5) + x^4(\alpha^1 + \alpha^7 + 1) + x^3(\alpha^1 + \alpha^3 + \alpha^2) +$
$+ x^2(\alpha^2 + \alpha^3 + \alpha^5) + x(\alpha^5) + \alpha^6 = (x^6\alpha^3) + \cancel{(x^5)} + (x^4\alpha^4) + (x^3\ \alpha^6) + x^2 + x\alpha + \alpha^6$

$c_1(\alpha^1) = \alpha^9 + \alpha^5 + \alpha^3 + \alpha + \alpha^6 = 0$

$g(x) = x^4 + x^3\alpha^2 + x^2\alpha^5 + x\alpha^5 + \alpha^6$

$\deg g(x) = 4$

$i_1(x) \cdot x^{\deg g(x)} = \alpha^3 x^6 + \alpha^5 x^5 + x^4$

$\alpha^3 x^6 + \alpha^5 x^5 + x^4 : x^4 + x^3\alpha^2 + x^2\alpha^5 + x\alpha^5 + \alpha^6$

$\alpha^3 x^6 + x^5\alpha^5 + x^4\alpha + x^3\alpha + x^2\alpha \quad = x^2\alpha^3 + \alpha^3 + \alpha^3$

$x^4\alpha^3 + x^3\alpha + x^2\alpha^2$

$x^4\alpha^3 + x^3\alpha^5 + x^2\alpha + x\alpha + \alpha^2$

$x^3\alpha^6 + x^2\alpha^4 + x\alpha + \alpha^2$

$c_1(x) = i_1(x) \cdot x^{\deg(g(x))} + Q(x)$

$= x^6\alpha^3 + x^5\alpha^5 + x^4 + x^3\alpha^6 + x^2\alpha^4 + x\alpha + \alpha^2$

$c_1(\alpha^0) = (x+1) + ($

PODNADPISY: NADPIS: ZAPAMÄTAJTE SI

$GF(4)$

$(3,1,3)$ $n=3$ $g(x)=(x-\alpha^0)\cdot(x-\alpha^1)=x^2+x\alpha^1+x\cancel{1}+\alpha^1=$

$k=1$

$t=1$ gf

bin	x	α
00	0	0
01	1	1
10	x	α
11	x+1	α^2

$=x^2+\alpha^2x+\alpha$

$(3,2,2)$

$t=\cancel{0}$

$t=\lfloor \frac{n-k}{2} \rfloor$ $g(x)=(x-\alpha^0)=x+1$

ÚLOHY: DÁTUM:

NADPIS: DÚ:

1. $(15, 11, 5)$

$GF(16)$

gen. pol. $p(x)$ $x^4 + x + 1$

nájdite $g(x)$

výsledok

$$\boxed{g(x) = x^4 + \alpha^{12} x^3 + \alpha^4 x^2 + x + \alpha^6}$$

vygenerovať pole

určiť stupeň 6x

vynásobiť počítať

2. $(7, 5, 3)$

$GF(8)$

$p(x) = x^3 + x + 1$

kódovať budeme $i_1 \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$

$i_2 \begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$

$[g(x) = x^2 + x^4 + x + \alpha]$

skúška ...

syst/nesyst.

ÚLOHY:

DÁTUM:

ZAPAMÄTAJTE SI:

PODNADPISY: NADPIS: 09.04.2009. Dekódovanie RS-kódov

ZAPAMÄTAJTE SI

1,

$S_0 = v(\alpha^0)$

$S_1 = v(\alpha^1)$

$\vdots$

$S_{2t-1} = v(\alpha^{2t-1})$

2, Polynóm lokátorov

$\sigma(x) = x^t + \sigma_1 x^{t-1} + \sigma_2 x^{t-2} + \ldots + \sigma_{t-1} x + \sigma_t$

$(x^5 \alpha^4 + \underline{x^3} \alpha + \alpha^2)$ poloha chyby

$S_t + \sigma_1 S_{t-1} + \ldots + \sigma_t S_0 = 0$

$S_{t+1} + \sigma_1 S_t + \ldots + \sigma_t S_1 = 0$

$\vdots$

$S_{2t-1} + \sigma_1 S_{2t-2} + \ldots + \sigma_t S_{t-1} = 0$

$\sigma(x) \rightarrow \forall$ prvky $GF(q)$ $(\alpha^0 \ldots \alpha^6)$ pre $GF(8)$

$\sigma(\alpha^j) = 0 \Rightarrow X_i = \alpha^j$, $i = 1 \ldots t$

$e(x) = X_1 Y_1 + X_2 Y_2 + \ldots + X_t Y_t$ (Y - daj vžš α a $X = x^3$ napr.)

$S_i = \sum_{j=1}^{t} X_j^i Y_j$

ÚLOHY: DÁTUM:

NADPIS:

$$\boxed{(\alpha^0)^6 = \alpha^{0\cdot 6} = \alpha^0}$$

ZAPAMÄTAJTE SI:

$\boxed{PR.}$ $GF(8)$

$(7,3,5)$ $p(x) = x^3 + x + 1$

$$v_x = \alpha^2 x^6 + \alpha^1 x^5 + \alpha^6 x^4 + \alpha^4 x^3 + \alpha^1 x^2 + \alpha^3 x + \alpha^6$$

$g(x) = (x-\alpha^0)\cdot(x-\alpha^1)\cdot(x-\alpha^2)\cdot(x-\alpha^3)$ koreňov je $2t$ $t=2$

$$S_0 = v(\alpha^0) = \alpha^2 + \alpha^1 + \cancel{\alpha^6} + \alpha^4 + \alpha^1 + \alpha^3 + \cancel{\alpha^6} = \alpha^2 + \alpha^4 + \alpha^3 = \alpha^0$$

$$S_1 = v(\alpha^1) = \alpha^8 + \alpha^6 + \alpha^{10} + \alpha^7 + \alpha^3 + \alpha^4 + \alpha^6 = \alpha^7 + \alpha^8 + \alpha^{10} + \alpha^3 + \alpha^4 =$$
$$\alpha^0 + \alpha^1 + \alpha^3 + \alpha^3 + \alpha^4 = \alpha^0 + \alpha^1 + \alpha^4 = \underline{\underline{\alpha^6}}$$

$$S_2 = v(\alpha^2) = \alpha^4 + \alpha^{11} + \alpha^{14} + \alpha^{10} + \cancel{\alpha^5} + \cancel{\alpha^5} + \alpha^6 = \alpha^{11} + \alpha^{10} + \alpha^6 = \alpha^4 + \alpha^3 + \alpha^0 =$$
$$= \emptyset$$

$$S_3 = v(\alpha^3) = \alpha^{20} + \alpha^{16} + \alpha^{18} + \alpha^{13} + \alpha^7 + \alpha^6 + \alpha^6 = \alpha^6 + \alpha^2 + \alpha^4 + \alpha^6 + \alpha^0 =$$
$$= \alpha^2 + \alpha^4 + \alpha^0 = \alpha^3$$

S_Y

$$S_2 + \sigma S_1 + \sigma S_0 = 0$$
$$S_3 + \sigma S_2 + \sigma_2 S_1 = 0$$

$$0 + \sigma_1 \alpha^6 + \sigma_2 \alpha^0 = 0$$
$$\alpha^3 + \sigma_1 \cdot 0 + \sigma_2 \cdot \alpha^6 = 0$$

$$\sigma_1 \alpha^6 + \sigma_2 = 0$$
$$\alpha^3 + \sigma_2 \cdot \alpha = 0 \Rightarrow \sigma_2 = \frac{\alpha^3}{\alpha^6} = \underline{\underline{\alpha^4}}$$

$$\sigma_1 = \frac{\alpha^4}{\alpha^6} = \alpha^5$$

ÚLOHY:

$$\sigma(x) = x^2 + \alpha^5 x^1 + \alpha^4$$

DÁTUM:

PODNADPISY: NADPIS: ZAPAMÄTAJTE SI

$$\boxed{G(\alpha^0) = \alpha^0 + \alpha^6 + \alpha^4 = \emptyset} \qquad X_1 = \alpha^0$$

$$G(\alpha^1) = \alpha^2 + \alpha^6 + \alpha^4 = \alpha^5$$

$$G(\alpha^2) = \alpha^4 + \alpha^7 + \alpha^4 = \alpha^0$$

$$G(\alpha^3) = \alpha^6 + \alpha^8 + \alpha^4 = \alpha^0$$

$$\boxed{G(\alpha^4) = \alpha^8 + \alpha^9 + \alpha^4 = \emptyset} \qquad X_2 = \alpha^4$$

$$G(\alpha^5) = \alpha^{10} + \alpha^{10} + \alpha^4 = \alpha^4$$

$$G(\alpha^6) = \underset{=\alpha^5}{\alpha^{12}} + \underset{=\alpha^4}{\alpha^{11}} + \alpha^4 = \alpha^5$$

$$S_0 = Y_1 (X_1)^0 + Y_2 (X_2)^0 = Y_1 \alpha^0 + Y_2 \alpha^0 = \alpha^0$$

$$S_1 = Y_1 (X_1)^1 + Y_2 (X_2)^1 = Y_1 \alpha^0 + Y_2 \alpha^4 = \alpha^6$$

$$Y_2(\alpha^0 + \alpha^4) = \alpha^0 + \alpha^6$$

$$\boxed{Y_2 = \alpha^4}$$

$$Y_1 \alpha^0 + \alpha^4 \alpha^0 = \alpha^0$$

$$\boxed{Y_1 = \frac{\alpha^0 + \alpha^4}{\alpha^0} = \alpha^5}$$

$$e(x) = (X_1) Y_1 + (X_2) Y_2$$

$$= x^0 \alpha^5 + x^4 \alpha^4$$

$$c(x) = v(x) + e(x)$$

$$\begin{array}{cc} x^4 & x^0 \\ \alpha^6 & \alpha^6 \\ \alpha^4 & \alpha^5 \\ \hline \alpha^3 & \alpha^1 \end{array}$$

ÚLOHY: DÁTUM:

PR. 2

NADPIS:

$\alpha = u - \varepsilon + 1$

$\varepsilon = -\frac{\alpha - 1}{2} = \frac{u - \varepsilon}{2}$

ZAPAMÄTAJTE SI:

(735) $\quad P(x) = x^3 + x + 1 \quad GF(8)$

$$V(y) = \alpha^5 x^6 + \overset{\alpha^1}{\alpha^2} x^5 + \alpha^4 x^4 + \alpha^7 x^3 + \alpha^0 x^2 + \alpha^0 x + \alpha^5 \to \alpha^0$$

$$g(y) = (x - \alpha^0) \cdot (x - \alpha^1)(x - \alpha^2)(x - \alpha^3)$$

$$S_0 = \cancel{\alpha^5} + \alpha^2 + \alpha^4 + \alpha^7 + \cancel{\alpha^0} + \cancel{\alpha^0} + \cancel{\alpha^5} = \alpha^2 + \alpha^4 + \alpha^7 = \alpha^3$$

$$S_1 = v(\alpha^1) = \cancel{\alpha^4} + \alpha^0 + \cancel{\alpha^7} + \cancel{\alpha^4} + \cancel{\alpha^2} + \cancel{\alpha^7} + \alpha^5 = \alpha^0 + \alpha^2 + \alpha^5 = \alpha^7$$

$$S_2 = v(\alpha^2) = \alpha^3 + \alpha^5 + \dots = \alpha^5$$

$$S_3 = v(\alpha^3) = \alpha^2 + \cancel{\alpha^3} + \cancel{\alpha^2} + \alpha^3 + \alpha^6 + \cancel{\alpha^3} + \alpha^5 = \alpha^3 + \alpha^6 + \alpha^5 = \alpha^0$$

DÚ.

dopoč!

$G(y) = x^2 + \alpha^4 + \alpha^5$

$G(\alpha^0) = 0 \qquad x_1 = \alpha^0$

$G(\alpha^5) = 0 \qquad x_2 = \alpha^5$

$Y_1 = \alpha^4$

$Y_2 = \alpha^4$

$G(y) = x^2 +$

ÚLOHY: DÁTUM:

17.04.2009 ~~Chyb~~ Teória informácie

NADPIS:

pravdepodobnosť chyby

$P_b = 10^{-3}$, $n = 7$ 7bit. slovo

$P_s = ?$ P_s - pravdepodobnosť chyby S_i

$(1-P_b)^n = (1-10^{-3})^7 = 0,993$

$P_s = 1-(1-P_b)^n = 0,007$

alebo

$$\boxed{P_s = 1 - \sum_{i=0}^{t} \binom{n}{i} P_b^i (1-P_b)^{n-i}}$$

$$P_s = 1 - \underbrace{\binom{7}{0}}_{=1} \underbrace{P_b^0}_{=1} (1-P_b)^7 = 1-(1-P_b)^7$$

$(7,4,3)$ $P_b = 10^{-3}$, $n = 7$

$P_s = ?$ $t = 1$

$$P_s = 1 - \sum_{i=0}^{t=1} \binom{n}{i} P_b^i (1-P_b)^{n-i} = \cancel{1 - \sum_{i=0}^{t=1} \binom{7}{0} P_b^i (1-P_b)^{7-0}} + \binom{7}{1} P_b^1 (1-P_b)$$

$$P_s = 1 - \left[\binom{7}{0} P_b^0 (1-P_b)^{7-0} + \binom{7}{1} P_b^1 (1-P_b)^6 \right]$$

$$P_s = 1 - \left[(1-P_b)^7 + 7 \cdot P_b^1 (1-P_b)^6 \right] = 2,09 \cdot 10^{-5}$$

$P_D = P_D^1 + P_D^2$

ÚLOHY:

$0,993 + 7 P_b \cdot (1-P_b)^6$

ZAPAMÄTAJTE SI:

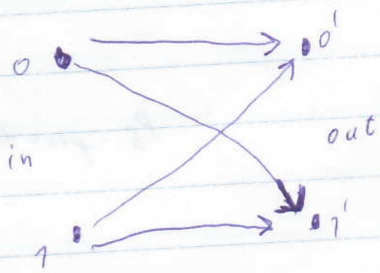
DÁTUM:

ZAPAMÄTAJTE SI:

PODNADPISY:

NADPIS:

BSC



$$\boxed{P(A) = \sum_{i=1}^{N} p(H_i)\, p(A/H_i) \qquad P(A/H_i) = \frac{P(A \cap H_i)}{P(H_i)}}$$

$P_b = 0{,}01$

$P(0) = 0{,}3$

$P(1) = 0{,}7$

$P(0') = ?$

$P(1') = ?$

$$P(0') = P(0)\, \underbrace{p(0'/0)}_{1-P_b} + P(1)\, \underbrace{p(0'/1)}_{P_b}$$

$$P(0') = 0{,}3(1-10^{-2}) + 0{,}7 \cdot 10^{-2} = 0{,}304$$

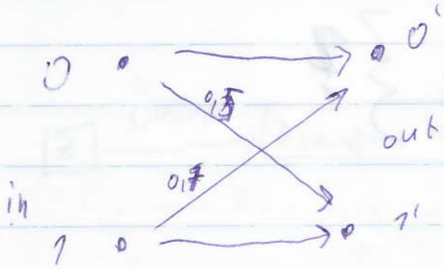
$$P(1') = 1 - P(0') = 0{,}696$$

$$= \underbrace{p(1)\, p(1'/1)}_{1-P_b} + \underbrace{p(0)\, p(1'/0)}_{P_b}$$

DÁTUM:

ÚLOHY:

ZAPAMÄTAJTE SI: PODNADPISY: NADPIS: ZAPAMÄTAJTE SI:



$P(1 \to 0') = 0{,}7$

$P(0 \to 1') = 0{,}5$

$P_0 = 0{,}4$

$P_1 = 0{,}6$

$P(0/0') = ?$

pravdepod. že ak prijmeme 0 tak sme ju aj vyslali

$P(0/0') = ?$

$P(0'/1) = 0{,}7$

$P(1'/0) = 0{,}5$

$$P(0/0') = \frac{P(0 \cap 0')}{P(0')} = \frac{P(0)\,p(0'/0)}{P(0)\,p(0'/0) + p(1)\,p(0'/1)}$$

$$\overset{0{,}5}{p(1'/0)} = 1 - \overset{0{,}5}{p(0'/0)}$$

$$\underset{0{,}3}{p(1'/1)} = 1 - \underset{0{,}7}{p(0'/1)}$$

$$= \frac{0{,}4 \cdot 0{,}5}{0{,}4 \cdot 0{,}5 + 0{,}6 \cdot 0{,}7} = \underline{\underline{0{,}323}}$$

ÚLOHY: DÁTUM:

PODNADPISY: NADPIS: ZAPAMÄTAJTE SI

	d	p
A	1000	0,25
B	150	0,1
C	10 000	0,05
D	100	0,45
E	500	0,15

$\sum = 1$

$\bar{d}$, $var(d)$ } ?

H

$$\bar{d} = \sum p_i d_i = 0{,}25 \cdot 10^3 + 0{,}1 \cdot 150 + \dots = \underline{\underline{885}}$$

$$var(d) = \sum p_i (d_i - \bar{d})^2 = 0{,}25 (10^3 - 885)^2 + \dots =$$

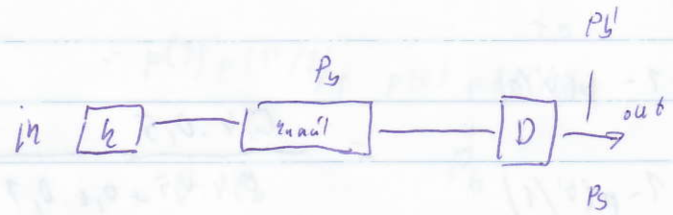
$$= 4\,511\,025$$

$$H = -\sum p_i \log_2 p_i = -(0{,}25 \log_2 0{,}25 + \dots) =$$

$$\log_a x = \frac{\log_y x}{\log_y a} = \frac{\ln x}{\ln a} = \frac{\log_{10} x}{\log_{10} a}$$

P_S

P_b'

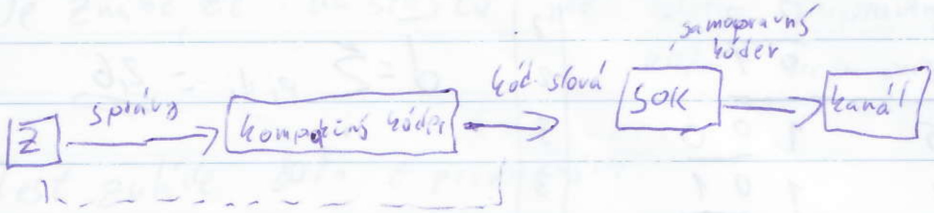


$$P_b' = \frac{d_{min}}{n} P_S$$

ÚLOHY: DÁTUM:

DK 24.04.2009

NADPIS:



priemerná dĺžka $\rightarrow \bar{d}$

A	0,999	00	0
B	0,0001	01	1
C	0,0001	10	1
D	0,0008	11	1
	$\Sigma 1$		

$$\bar{d} = \sum_{i=1}^{n} p_i d_i$$

$$\sum_{i=1}^{n} p_i = 1$$

$$\bar{d} \geq \bar{d}_{min} = - \frac{\sum p_i \log_2 p_i}{\log_2 L}$$

L - počet symbolov v abecede

$(\{0,1\} \Rightarrow L = 2)$

Príklad.

	p_i
A	0,3
B	0,25
C	0,15
D	0,1
E	0,1
F	0,05
G	0,03
H	0,02

$\bar{d}_{min} = ?$

-> Zakódovať blok. kódom konšt. , $\bar{d}$

-> -II- Shanon - Fano , $\bar{d}$

-> -II- Huffman , $\bar{d}$

A 000
B 001
C 010
D 011
E 100
F 101
G 110
H 111

$\bar{d} = 3$

$$\bar{d}_{min} = - \frac{0,3 \cdot \log_2 0,3 + 0,25 \log_2 0,25 + 0,15 \log_2 0,15 +}{\log_2 2}$$

$$+ \frac{0,1 \log_2 0,1 + 0,1 \log_2 0,1 + 0,05 \log_2 0,05 +}{\log_2 2}$$

$$\frac{0,03 \cdot \log_2 0,03 + 0,02 \log_2 0,02}{\log_2 2} = 2,578$$

$$\boxed{\log_a x = \frac{\log_n x}{\log_n a}}$$

ÚLOHY:

ZAPAMÄTAJTE SI:

DÁTUM:

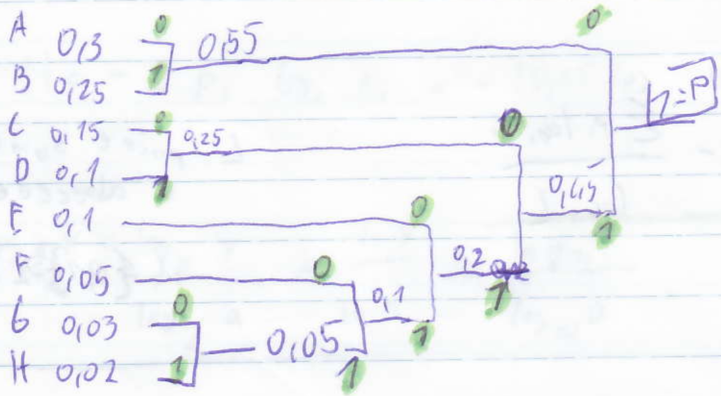
PODNADPISY: NADPIS: → Shanon – FanO

ZAPAMÄTAJTE SI

			d_i
A	0,3	0 0	2
B	0,25	0 1	2
C	0,15	1 0 0	3
D	0,1	1 0 1	3
E	0,1	1 1 0	3
F	0,05	1 1 1 0	4
G	0,03	1 1 1 1 0	5
H	0,02	1 1 1 1 1	5

$$\bar{d} = \sum_i p_i d_i = 2{,}6$$

Huffman



A 00
B 01
C 100
D 101
E 110
F 1110
G 11110
H 11111

$\bar{d} = 2{,}6$

Huffman
taký alebo lepší ako Fano

$0{,}2 \log_2 0{,}2 + 0{,}3 \log_2 0{,}3$

$$\frac{0{,}2 \log_{10} 0{,}2}{\log_{10} 2} + \frac{0{,}3 \log_{10} 0{,}3}{\log_{10} 2}$$

ÚLOHY: DÁTUM:

PODNADPISY: NADPIS: ZAPAMÄTAJTE SI:

Je známe, že : na skúšku ide 60% pripravených študentov
40% nepripravených

Test zvládne 80% z pripravených
10% z nepripravených

Aké množstvo informácie o pripravenosti študentov dáva vykonanie testu?

A $p(a_1) = 0{,}6$ B
$p(a_2) = 0{,}4$

úsp: $p(b_1) = ?$
neúsp: $p(b_2) = ?$

$p(b_1/a_1) = 0{,}8$

$p(b_2/a_2) = 0{,}2$

$p(b_1/a_2) = 0{,}1$

$p(b_2/a_2) = 0{,}9$

$p(b_1) = p(a_1/b_1) + p(a_2/b_1)$

$p(a_i b_j)$

	b_1	b_2	
a_1	0,48	0,12	0,6
a_2	0,04	0,36	0,4

$p(a_2, b_1)$

$p(b_1) = 0{,}52$ $p(b_2) = 0{,}48$

$$\boxed{p(a_i; b_j) = p(a_i)\, p(b_j/a_i)}$$

$H(A) = -\sum p(a_i) \log_2 P(a_i) = 0{,}97095$

$H(B) = -\sum p(b_i) \log_2 P(b_i) = 0{,}99885$

$I(A,B) = H(A) + H(B) - H(A,B)$

A, B nezávislé $I(A,B) \stackrel{!}{=} 0$

$H(A,B) = H(A) + H(B)$

ÚLOHY: DÁTUM:

A, B - závislé : $H(A,B) = -\sum_i \sum_j p(a_i b_j) \log_2 (a_i, b_j)$

PODNADPISY: NADPIS: ZAPAMÄTAJTE SI:

$H(A,B) = 1{,}59171$

$I(A,B) = 0{,}37809$ [bit]

Odpoveď: dáva nám to 0,378 [bitu] či je študent pripravený alebo nie.

A 0,8 $\bar{d}_{min} = 0{,}7219 = \sum p_i \log_2 p_i$

B 0,2

Hufmanov kód

A 0,8 0 $\bar{d} = 1$

B 0,2 1

$P(A,B) = p(A)\, p(B)$

Doma rozpísať !!!

AA	0,64	0
AB	0,16	11
BA	0,16	100
BB	0,04	101

$\bar{d}_{min} = 1{,}4438$

$\bar{d}_{min_{/znak}} = 0{,}7219$

$\bar{d} = 1{,}56$

$\bar{d}_{/znak} = 0{,}78$

AAA
AAB
⋮
BBB

$\bar{d} = 0{,}728$

ÚLOHY: DÁTUM: