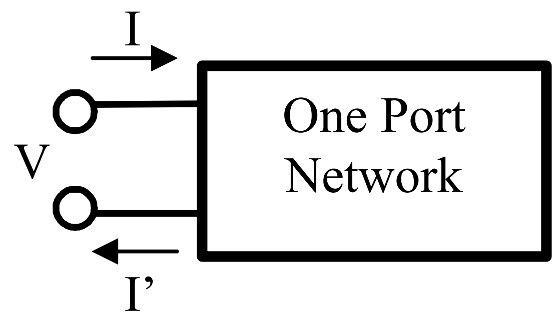
Two-Port Networks

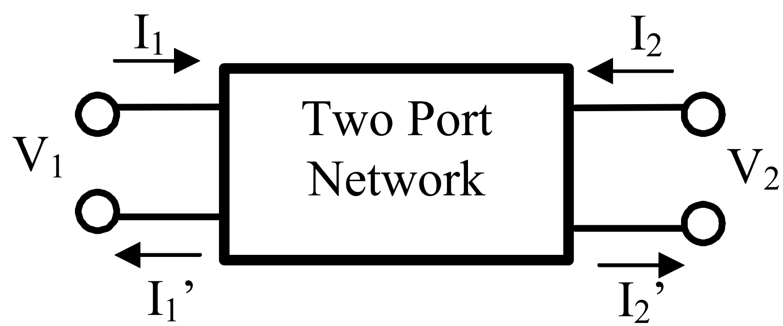
- One-port network:

$$V = Z \cdot I$$



$$I = I'$$

- Two-port network:



$$\begin{cases} I_1 = I_1' \\ I_2 = I_2' \end{cases}$$

$$V_1 = Z_{11} \cdot I_1 + Z_{12} \cdot I_2$$
$$V_2 = Z_{21} \cdot I_1 + Z_{22} \cdot I_2$$

$$[V] = [Z] \cdot [I]$$

How to Determine Z Parameters?

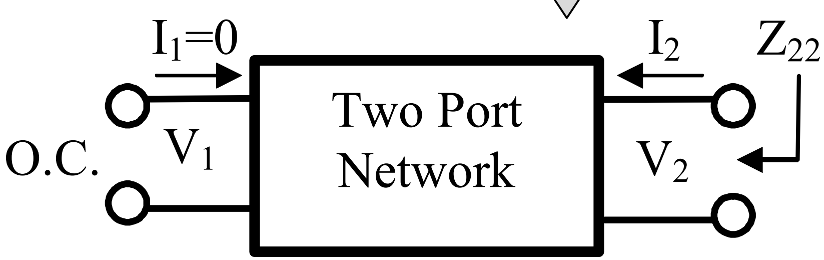
$$V_1 = Z_{11} \cdot I_1 + Z_{12} \cdot I_2$$
$$V_2 = Z_{21} \cdot I_1 + Z_{22} \cdot I_2$$

$I_2 = 0$ →

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$I_1 = 0$ →

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \qquad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$



Reciprocity:

$$Z_{12} = Z_{21}$$

Y Parameters

$$[V]=[Z]\cdot[I] \Rightarrow [I]=[Z]^{-1}\cdot[V] \Rightarrow [Y]=[Z]^{-1}$$

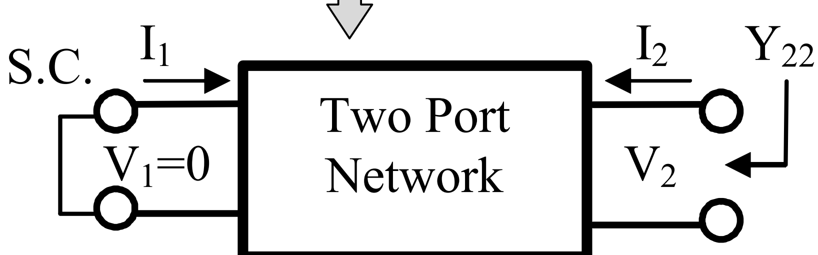
$$I_1 = Y_{11}\cdot V_1 + Y_{12}\cdot V_2$$
$$I_2 = Y_{21}\cdot V_1 + Y_{22}\cdot V_2$$

$V_2 = 0$:

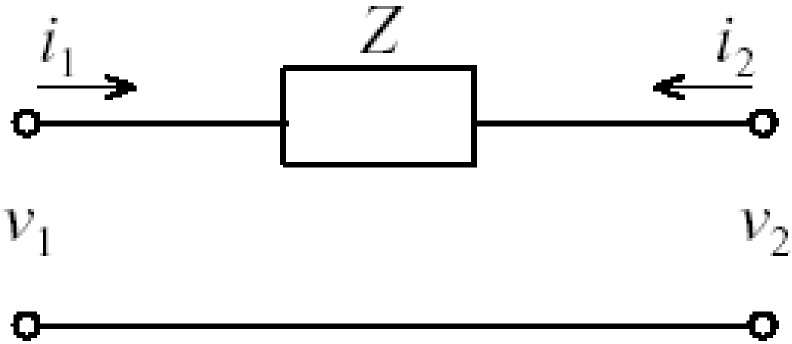
$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$V_1 = 0$:

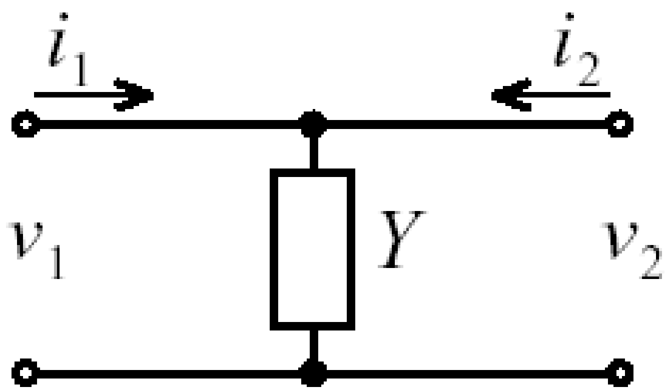
$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \qquad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$



Simple Networks

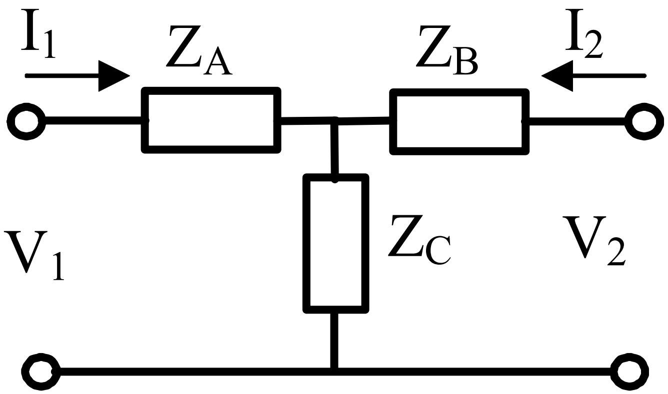


$\Rightarrow$ $[Z],[Y]-???$



$\Rightarrow$ $[Z],[Y]-???$

Example: T-Network



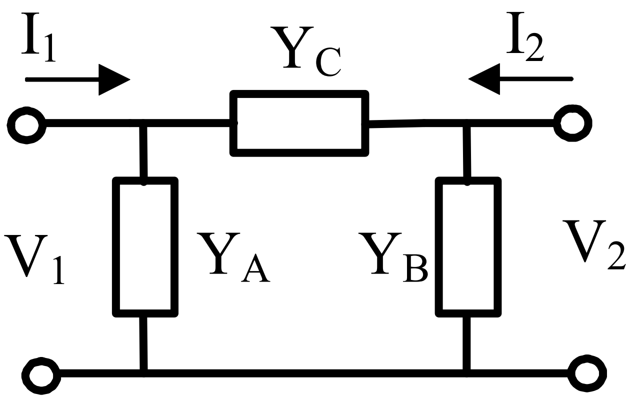
$I_2 = 0$:

$$\begin{cases} Z_{11} = \left.\dfrac{V_1}{I_1}\right|_{I_2=0} = Z_A + Z_C \\ Z_{21} = \left.\dfrac{V_2}{I_1}\right|_{I_2=0} = Z_C \end{cases}$$

$I_1 = 0$:

$$\begin{cases} Z_{12} = Z_{21} = Z_C \\ Z_{22} = \left.\dfrac{V_2}{I_2}\right|_{I_1=0} = Z_B + Z_C \end{cases}$$

Example: Pi-Network



$V_2 = 0$ ⇒

$$\begin{cases} Y_{11} = \left.\dfrac{I_1}{V_1}\right|_{V_2=0} = Y_A + Y_C \\ Y_{21} = \left.\dfrac{I_2}{V_1}\right|_{V_2=0} = -Y_C \end{cases}$$

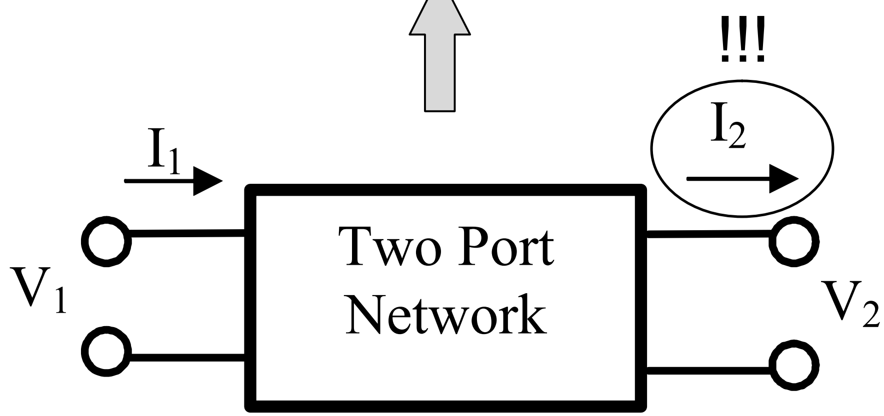
$V_1 = 0$ ⇒

$$\begin{cases} Y_{12} = Y_{21} = -Y_C \\ Y_{22} = \left.\dfrac{I_2}{V_2}\right|_{V_1=0} = Y_B + Y_C \end{cases}$$

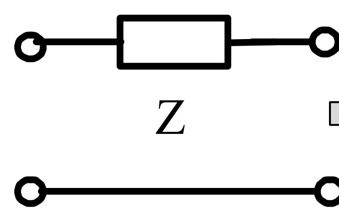
ABCD Parameters

$$V_1 = A \cdot V_2 + B \cdot I_2$$
$$I_1 = C \cdot V_2 + D \cdot I_2$$

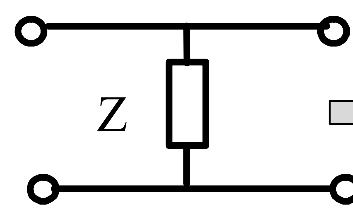
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$



1)  $[ABCD] - ?$

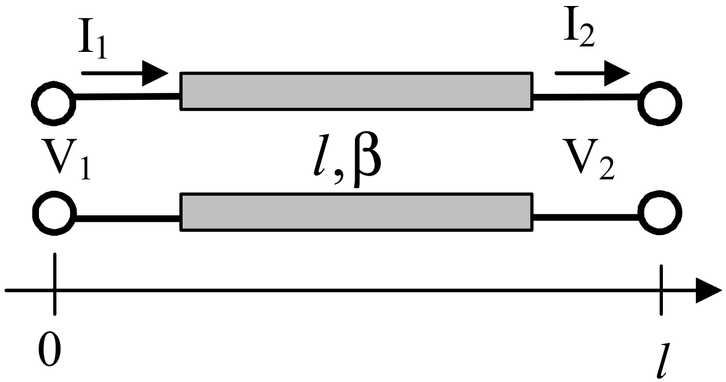
2) 

$[ABCD] - ?$

3) 

$[ABCD] - ?$

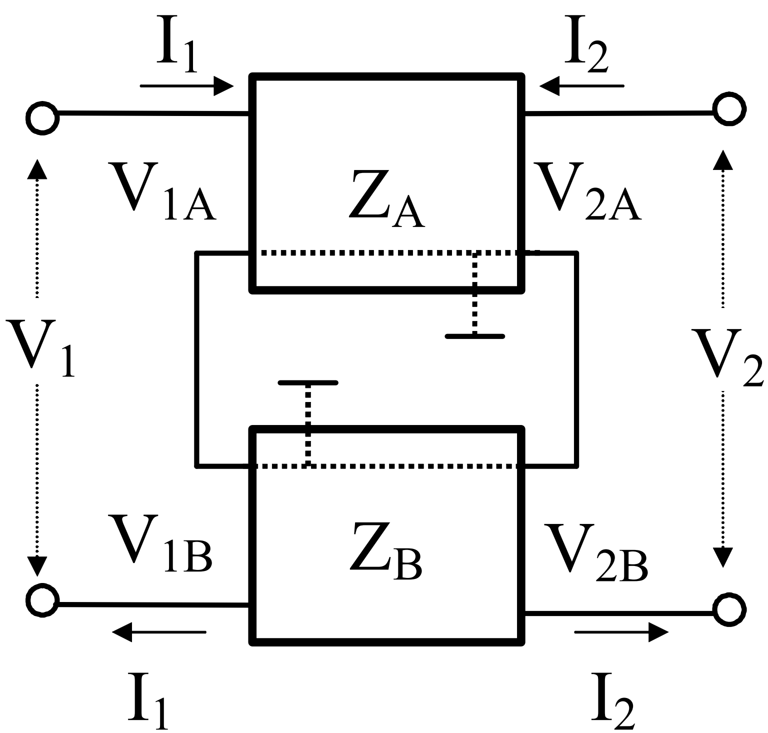
ABCD Parameters of TL



$$\begin{cases} V_1 = V^+ + V^-, \quad I_1 = \dfrac{1}{Z_0}\left(V^+ - V^-\right) \\ V_2 = V^+ e^{-j\beta l} + V^- e^{j\beta l} \\ I_2 = \dfrac{1}{Z_0}\left(V^+ e^{-j\beta l} - V^- e^{j\beta l}\right) \end{cases}$$

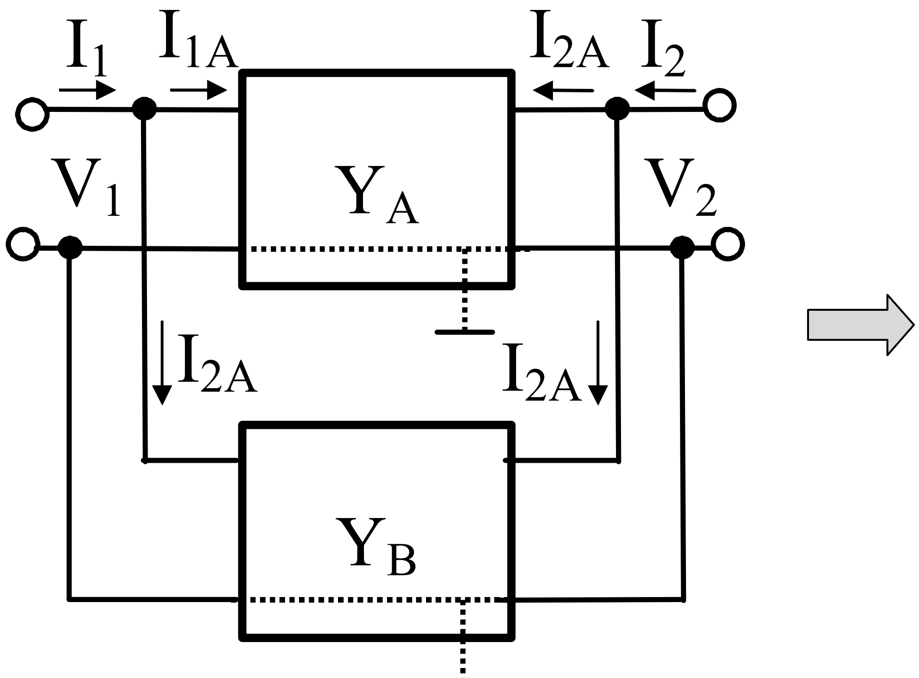
$$\begin{cases} V_2 = V_1 \cos\beta l + jI_1 Z_0 \sin\beta l \\ I_2 = j\dfrac{V_1}{Z_0}\sin\beta l + I_1 \cos\beta l \end{cases} \Rightarrow \left[ABCD\right] = \begin{bmatrix} \cos\beta l & jZ_0 \sin\beta l \\ jY_0 \sin\beta l & \cos\beta l \end{bmatrix}$$

Series Connection of Two-Port Networks



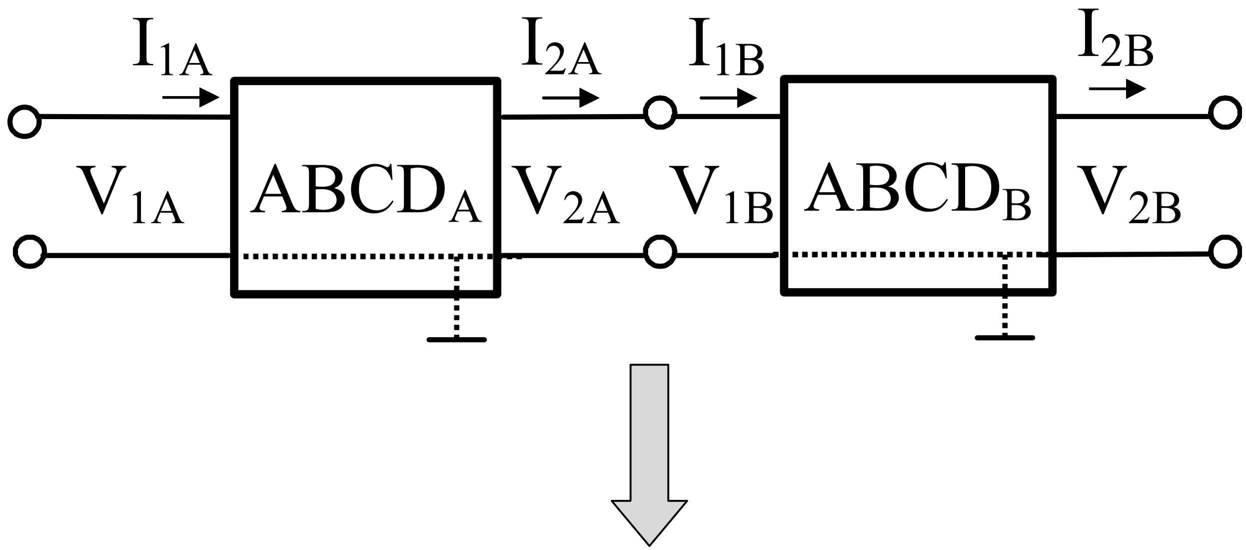
$$[Z]=[Z_A]+[Z_B]$$

Parallel Connection of Two-Port Networks



$$\left[Y\right]=\left[Y_A\right]+\left[Y_B\right]$$

Cascade Connection of Two-Port Networks



$$[ABCD] = [ABCD_A] + [ABCD_B]$$