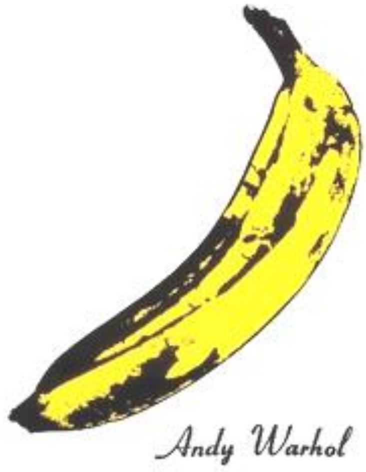
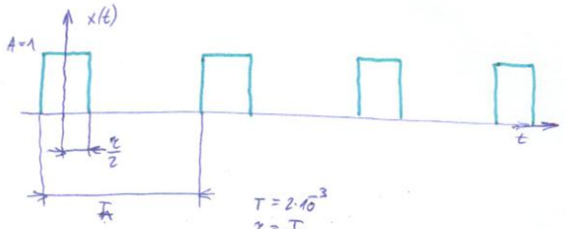
ADSS1

Zopár príkladov na FR a FT z miniloročných skúšok



ADSS 2009 OT

1. Pre daný periodický signál na obrázku



$T = 2\cdot10^{-3}$
$\tau = \frac{T}{2}$

a) Analyticky vyjadrite signál x(t) a vypočítajte spektrum signálu a nakreslite jeho fázové a amplitúdové spektrum.

b) pre zadaný signál zvoľte medznú frekvenciu $f_m = 1kHz$, určte vzorkovaciu frekvenciu f_{vz}.
- nakreslite navzorkovaný signál a jeho spektrum.

a) FR

$$c_n = \frac{1}{T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} x(t)\cdot e^{-jn\omega t}\,dt = \frac{A}{T}\left[\frac{e^{-jn\omega t}}{-jn\omega}\right]_{-\frac{\tau}{2}}^{\frac{\tau}{2}} = \frac{A}{T}\cdot\frac{1}{-jn\omega}\cdot\left[e^{-jn\omega\frac{\tau}{2}} - e^{jn\omega\frac{\tau}{2}}\right] = {}^{/\cdot(-1)}$$

$$= \frac{A}{T}\cdot\frac{1}{jn\omega}\cdot\left[e^{jn\omega\frac{\tau}{2}} - e^{-jn\omega\frac{\tau}{2}}\right] = \frac{A}{T}\frac{2j}{jn\omega}\cdot\left[\frac{e^{jn\omega\frac{\tau}{2}} - e^{-jn\omega\frac{\tau}{2}}}{2j}\right] = \frac{A\tau}{T}\cdot\frac{\sin(n\omega\frac{\tau}{2})}{n\omega\frac{\tau}{2}} =$$

$$= \frac{A\tau}{T}\, si\left(n\omega\frac{\tau}{2}\right) = \frac{1\cdot10^{-3}}{2\cdot10^{-3}}\cdot si\left(n\frac{2\pi}{2\cdot10^{-3}}\cdot\frac{2\cdot10^{-3}}{2\cdot2}\right) = \frac{1}{2}\, si\left(n\frac{\pi}{2}\right)$$

$$a_0 = c_0 = \frac{1}{T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} x(t)\,dt = \frac{A}{T}\left[t\right]_{-\frac{\tau}{2}}^{\frac{\tau}{2}} = \frac{A}{T}\tau = \frac{1}{2}$$

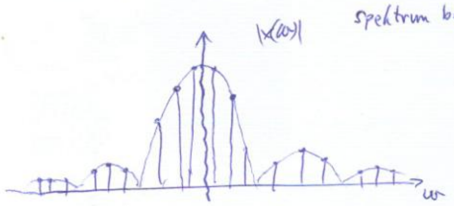
$si\left(n\frac{\pi}{2}\right) = 0$

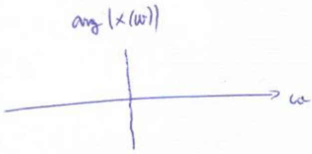
$n\frac{\pi}{2} = \pi$

$n\pi = 2\pi$

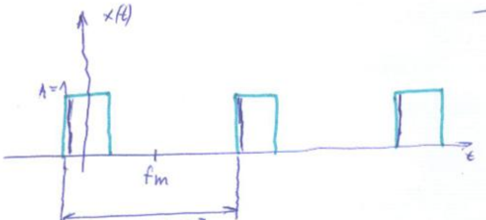
$n = 2$

spektrum bude čiarové neperiodické

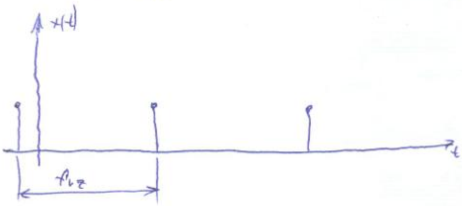


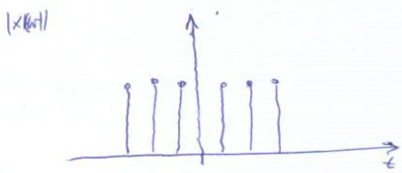


b) $f_{vz} \geq 2f_m = 2kHz$



spektrum je diskrétne periodické

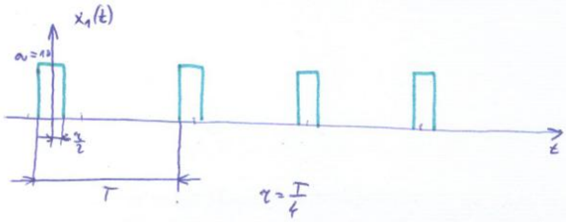




ADSS 2010 PT

1\. Pre dané periodické signály odvoďte vzťah pre spektrum a nakreslite ich amplitúdové a fázové spektrum.

a)



$\tau = \frac{T}{4}$

Je to periodický signál => FR

$$c_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x_1(t)\cdot e^{-jn\omega_1 t}\,dt = \frac{A}{T}\left[\frac{e^{-jn\omega_1 t}}{-jn\omega_1}\right]_{-\frac{\tau}{2}}^{\frac{\tau}{2}} = \frac{A}{T}\cdot\frac{1}{-jn\omega_1}\cdot\left[e^{-jn\omega_1\frac{\tau}{2}} - e^{+jn\omega_1\frac{\tau}{2}}\right] /(-1) =$$

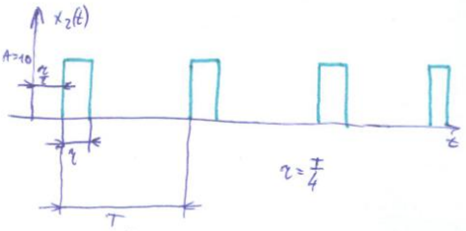
$$= \frac{A}{T}\cdot\frac{1}{jn\omega_1}\left[e^{jn\omega_1\frac{\tau}{2}} - e^{-jn\omega_1\frac{\tau}{2}}\right] = \frac{A}{T}\cdot\frac{2j}{jn\omega_1}\cdot\left[\frac{e^{jn\omega_1\frac{\tau}{2}} - e^{-jn\omega_1\frac{\tau}{2}}}{2j}\right] = \frac{2A}{T}\cdot\frac{1}{n\omega_1}\cdot\frac{\sin\left(n\omega_1\frac{\tau}{2}\right)\cdot n\omega_1\frac{\tau}{2}}{n\omega_1\frac{\tau}{2}} =$$

$$= \frac{2A}{T}\cdot\frac{1}{n\omega_1}\cdot\frac{n\omega_1\tau}{2}\cdot si\left(n\omega_1\frac{\tau}{2}\right) = \frac{A}{T}\tau\cdot si\left(n\omega_1\frac{\tau}{2}\right)$$

$$a_0 = c_0 = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x_1(t)\,dt = \frac{A}{T}\cdot\left[t\right]_{-\frac{\tau}{2}}^{\frac{\tau}{2}} = \frac{A}{T}\tau = \frac{10}{T}\,\frac{T}{4} = \frac{10}{4} = 2{,}5$$

b)

~~$c_n = \int_{-\frac{T}{2}}^{\frac{T}{2}} \ldots\, dt$~~



$\tau = \frac{T}{4}$

$$c_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x_2(t)\cdot e^{-jn\omega_2 t}\,dt = \frac{A}{T}\left[\frac{e^{-jn\omega_2 t}}{-jn\omega_2}\right]_{\frac{\tau}{2}}^{\frac{3}{2}\tau} = \frac{A}{T}\cdot\frac{1}{-jn\omega_2}\cdot\left[e^{-jn\omega_2\frac{3}{2}\tau} - e^{-jn\omega_2\frac{\tau}{2}}\right] =$$

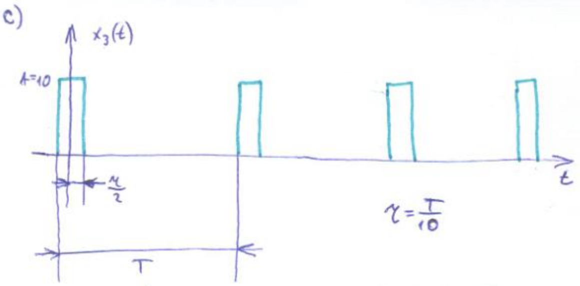
$$= \frac{A}{T}\cdot\frac{e^{-jn\omega_2\tau}}{-jn\omega_2}\cdot\left[e^{-jn\omega_2\frac{\tau}{2}} - e^{+jn\omega_2\frac{\tau}{2}}\right] /(-1) = \frac{A}{T}\cdot\frac{e^{-jn\omega_2\tau}}{jn\omega_2}\left[e^{jn\omega_2\frac{\tau}{2}} - e^{-jn\omega_2\frac{\tau}{2}}\right] = \frac{A}{T}\,\frac{2j\cdot e^{-jn\omega_2\tau}}{jn\omega_2}\cdot\left(\frac{e^{jn\omega_2\frac{\tau}{2}} - e^{-jn\omega_2\frac{\tau}{2}}}{2j}\right) =$$

$$= \frac{2A}{T}\cdot\frac{e^{-jn\omega_2\tau}}{n\omega_2}\cdot\sin\left(n\omega_2\frac{\tau}{2}\right) = \frac{2A}{T}\cdot\frac{e^{-jn\omega_2\tau}}{n\omega_2}\,\frac{\sin\left(n\omega_2\frac{\tau}{2}\right)}{n\omega_2\frac{\tau}{2}}\cdot n\omega_2\frac{\tau}{2} = \frac{2A}{T}\cdot\frac{n\omega_2\tau}{n\omega_2 2}\cdot e^{-jn\omega_2\tau}\cdot si\left(n\omega_2\frac{\tau}{2}\right) =$$

$$= \frac{A\tau}{T}\cdot e^{-jn\omega_2\tau}\cdot si\left(n\omega_2\frac{\tau}{2}\right)$$

$$a_0 = c_0 = \frac{1}{T}\int_{+\frac{\tau}{2}}^{\frac{3}{2}\tau} x_2(t)\,dt = \frac{A}{T}\left[t\right]_{\frac{\tau}{2}}^{\frac{3}{2}\tau} = \frac{A}{T}\tau = \frac{A}{T}\,\frac{T}{4} = \frac{A}{4} = \frac{10}{4} = 2{,}5$$

c)



$\tau = \frac{T}{10}$

$$c_n = \frac{1}{T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} x_3(t)\cdot e^{-jn\omega_3 t}\,dt = \frac{A}{T}\left[\frac{e^{-jn\omega_3 t}}{-jn\omega_3}\right]_{-\frac{\tau}{2}}^{\frac{\tau}{2}} = \frac{A}{T}\cdot\frac{1}{-jn\omega_3}\cdot\left[e^{-jn\omega_3\frac{\tau}{2}} - e^{+jn\omega_3\frac{\tau}{2}}\right] \overset{/(-1)}{=}$$

$$= \frac{A}{T}\cdot\frac{1}{jn\omega_3}\left[e^{jn\omega_3\frac{\tau}{2}} - e^{-jn\omega_3\frac{\tau}{2}}\right] = \frac{A}{T}\cdot\frac{2j}{jn\omega_3}\left[\frac{e^{jn\omega_3\frac{\tau}{2}} - e^{-jn\omega_3\frac{\tau}{2}}}{2j}\right] = \frac{2A}{T}\cdot\frac{1}{n\omega_3}\cdot\frac{\sin\left(n\omega_3\frac{\tau}{2}\right)}{n\omega_3\frac{\tau}{2}}\cdot n\omega_3\frac{\tau}{2} =$$

$$= \frac{A\tau}{T}\cdot \mathrm{Si}\left(n\omega_3\frac{\tau}{2}\right)$$

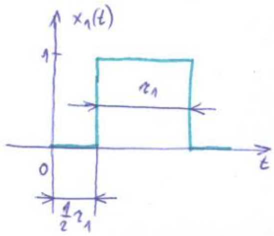
$$a_0 = c_0 = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x_3(t)\,dt = \frac{A}{T}\left[t\right]_{-\frac{\tau}{2}}^{\frac{\tau}{2}} = \frac{A}{T}\tau = \frac{10}{T}\frac{T}{10} = 1$$

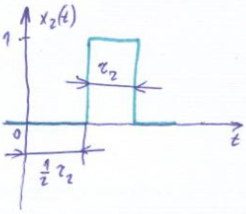
Spektrá budú čiarové diskrétne neperiodické

a)

ADSS 2006 + 2008 RT + 2009 RT

2. Pre dané priebehy v časovej oblasti:





$\tau_1 = 2\tau_2$

a) analyzujte ich reprezentáciu vo frekvenčnej oblasti
b) vyjadrite a nakreslite spektrálne funkcie
c) vyjadrite a nakreslite spektrálnu fázovú funkciu pre $x_1(t)$

Je to neperiodický priebeh ⇒ FT

a) $$X_1(\omega) = \int_{-\infty}^{\infty} x_1(t)\cdot e^{-j\omega t}\,dt = \int_{\frac{1}{2}\tau_1}^{\frac{3}{2}\tau_1} 1\cdot e^{-j\omega t}\,dt = \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{\frac{1}{2}\tau_1}^{\frac{3}{2}\tau_1} = \frac{1}{-j\omega}\left[e^{-j\omega\frac{3}{2}\tau_1} - e^{-j\omega\frac{1}{2}\tau_1}\right] \Big/ \cdot(-1) =$$

$$= \frac{1}{j\omega}\cdot\left[e^{-j\omega\frac{1}{2}\tau_1} - e^{-j\omega\frac{3}{2}\tau_1}\right] = \frac{1}{j\omega}\cdot e^{-j\omega\tau_1}\cdot\left[e^{+j\omega\frac{\tau_1}{2}} - e^{-j\omega\frac{\tau_1}{2}}\right] = \frac{1}{j\omega}\cdot 2j\cdot e^{-j\omega\tau_1}\cdot\left[\frac{e^{+j\omega\frac{\tau_1}{2}} - e^{-j\omega\frac{\tau_1}{2}}}{2j}\right] =$$

$$= \frac{2}{\omega}\cdot e^{-j\omega\tau_1}\cdot\frac{\sin(\omega\frac{\tau_1}{2})}{\omega\frac{\tau_1}{2}}\cdot\omega\frac{\tau_1}{2} = \frac{2}{\omega}\cdot\frac{\omega\tau_1}{2}\cdot si\left(\omega\frac{\tau_1}{2}\right)\cdot e^{-j\omega\tau_1} = \tau_1\cdot si\left(\omega\frac{\tau_1}{2}\right)\cdot e^{-j\omega\tau_1}$$

$si\left(\omega\frac{\tau_1}{2}\right) = 0$

$\omega\frac{\tau_1}{2} = \pi$

$\omega = \frac{2\pi}{\tau_1}$

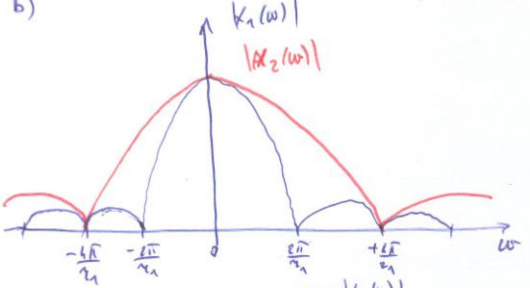
a) $$X_2(\omega) = \tau_2\cdot si\left(\omega\frac{\tau_2}{2}\right)\cdot e^{-j\omega\tau_2} = \frac{1}{2}\tau_1\, si\left(\omega\frac{\tau_1}{4}\right)\cdot e^{-j\omega\frac{\tau_1}{2}}$$

$si\left(\omega\frac{\tau_1}{4}\right) = 0$

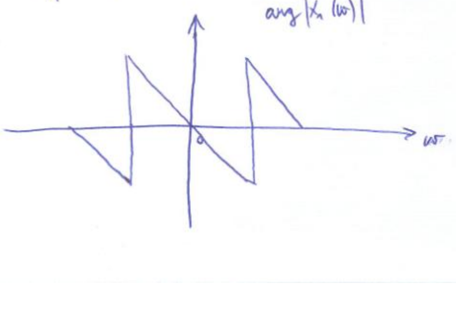
$\omega\frac{\tau_1}{4} = \pi$

$\omega = \frac{4\pi}{\tau_1}$

b) Spektrum bude spojité neperiodické

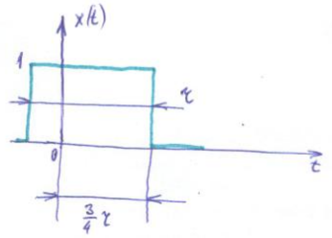


c)



TKS 2006

1. Priebeh v časovej oblasti:



a) analyzujte jeho reprezentáciu vo frekvenčnej oblasti (analyticky)
b) vyjadrite a nakreslite amplitúdovú (magnitúdovú) spektrálnu funkciu
c) vyjadrite a nakreslite jeho fázovú spektrálnu funkciu.

a) je to neperiodický signál ⇒ FT

$$X(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt = \int_{-\frac{1}{4}\tau}^{\frac{3}{4}\tau} 1 \cdot e^{-j\omega t} dt = \left[\frac{e^{-j\omega t}}{-j\omega} \right]_{-\frac{1}{4}\tau}^{\frac{3}{4}\tau} = \frac{1}{-j\omega} \cdot \left[e^{-j\omega \frac{3}{4}\tau} - e^{+j\omega \frac{1}{4}\tau} \right] / \cdot (-1) =$$

$$= \frac{1}{j\omega} \left[e^{j\omega \frac{1}{4}\tau} - e^{-j\omega \frac{3}{4}\tau} \right] = \frac{1}{j\omega} \cdot e^{-j\omega \frac{\tau}{2}} \cdot \left[e^{j\omega \frac{\tau}{4}} - e^{-j\omega \frac{\tau}{4}} \right] = \frac{1}{j\omega} \cdot 2j \cdot e^{-j\omega \frac{\tau}{2}} \cdot \left[\frac{e^{j\omega \frac{\tau}{4}} - e^{-j\omega \frac{\tau}{4}}}{2j} \right] =$$

$$= \frac{2}{\omega} \cdot e^{-j\omega \frac{\tau}{2}} \cdot \frac{\sin\left[\omega \frac{\tau}{4}\right]}{\omega \frac{\tau}{4}} \cdot \omega \frac{\tau}{4} = \frac{2}{\omega} \cdot \omega \frac{\tau}{4} \cdot e^{-j\omega \frac{\tau}{2}} \cdot si\left(\omega \frac{\tau}{4}\right) = \underline{\underline{\frac{1}{2} \tau \cdot e^{-j\omega \frac{\tau}{2}} \cdot si\left(\omega \frac{\tau}{4}\right)}}$$

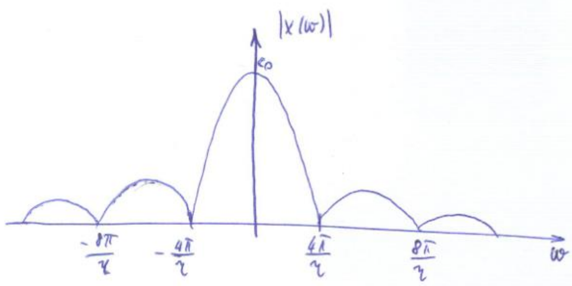
$$si\left(\omega \frac{\tau}{4}\right) = 0$$

$$\omega \frac{\tau}{4} = \pi$$

$$\omega \tau = 4\pi$$

$$\omega = \frac{4\pi}{\tau}$$

b)



Spektrum bude spojité neperiodické

c)